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Community Development with Externalities and Corrective Taxation*

Abstract

This paper studies the impact of granting a community the authority to tax development when growth imposes negative externalities on existing residents. Taxes are chosen in each period by the residents who are fully forward-looking. Residents' policy choices reflect not only the desire to counter negative externalities but also their wish to raise tax revenues and the value of their homes. There exists an equilibrium in which taxes are gradually lowered to close to optimal levels, resulting in falling housing prices and increasing community size. In addition, there exist equilibria in which taxes are set much too high and development is permanently stalled. In these equilibria, residents anticipate that lowering taxes will cause a sharp fall in the value of their homes. This multiplicity of equilibrium means that, for a broad range of initial conditions, allowing residents to tax development can increase or decrease social welfare. Regulating growth with zoning generates even worse outcomes, but allowing the community to charge developers impact fees does better.

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1 Introduction

Community development is an important topic in public and urban economics. The large literature on the subject identifies a number of negative externalities associated with the development of a community. These include increased air and water pollution, traffic congestion, overtaxing of natural amenities, and the dilution of public wealth. In the presence of such externalities, free entry will produce too much development.

The textbook remedy for such externalities is a Pigouvian tax on development. Appropriately set, such a tax can align private and social incentives and assure that communities develop optimally. Despite this, U.S. states have not generally provided their localities with the authority to tax development (Altshuler and Gomez-Ibanez 1993). Rather, localities regulate development with growth-control measures such as land-use and building regulations, zoning, and limiting building permits. Such policies do not raise revenue but create a "regulatory tax" (Glaeser, Gyourko, and Saks 2005b) that acts like a Pigouvian tax to dampen development.[1] Some localities employ impact fees and other exactions that do raise resources from developers, but these are legally constrained in a way that makes them distinct from taxes.[2]

This raises the question of what would happen if localities were allowed to tax development? Would this result in an improvement in social welfare relative to free entry? Furthermore, how would the outcome compare with those emerging under the regulatory policies that localities actually use? This paper explores these questions.

The paper studies a dynamic model of a single community. The community starts out with a stock of housing and initial residents who own this housing. Undeveloped land is available and new housing can be built on this land. Undeveloped land is owned by landowners who reside outside the community and new construction is supplied by competitive developers. Land has a heterogeneous return in its non-building use, leading to an upward-sloping supply curve of building land. There is a pool of potential residents with heterogeneous desires to live in the community, generating a downward-sloping demand curve for housing. There is turnover, with

[1] Rosen and Katz (1981) provide a well-known description of the various growth-control measures used by communities in the San Francisco Bay Area.

[2] An exaction is a requirement that developers, as a condition for receiving a building permit, expend resources for the provision of public facilities or services. Exactions may be in-kind or monetary and monetary exactions are known as impact fees. The so-called rational nexus standard requires that exactions can only be used to finance expenditures attributable to the development in question. For more detail see Altshuler and Gomez-Ibanez (1993) and Rosenberg (2006).

households entering and exiting the pool each period, so that the market for housing is always active. The possibility that current residents may leave the pool makes them care about the value of their homes. When living in the community, all residents obtain a payoff that is decreasing in the number of residents, creating a negative externality. The community can levy a tax on new construction and the revenues are shared equally by all residents. Policy decisions are made by the residents who are fully forward-looking. The community's initial housing stock is sufficiently small that some development is socially optimal.

The study of this model produces a number of insights. First, it identifies a time inconsistency problem in that future residents would like to lower taxes below the level that would result if the initial residents could choose policies for all future periods. Second, it shows that this time inconsistency problem results in interesting dynamic patterns of development when policies are chosen in each period by the current residents. On the one hand, an equilibrium exists in which taxes are gradually lowered, resulting in falling housing prices and increasing community size. On the other, there exists equilibria in which taxes remain high and development is stalled. Third, the range of equilibria that can arise with sequential policy-making means that, in the long run, the tax can be close to its optimal level or much too high. This implies that, for a broad range of initial conditions, allowing residents to tax development can either increase or decrease social welfare relative to free entry. Fourth, social welfare when residents can tax development is higher than when they are authorized to control growth by zoning undeveloped land for building, but lower than when they are allowed to levy impact fees on developers.

The paper is a companion to Barseghyan and Coate (2021) which introduces the basic model of community development used here. In that paper, there is a positive externality from development created by the cost-sharing of a durable public good. The focus is on how residents invest in the public good and finance this investment with an eye to attracting potential residents to the community. This leads to a theory of community development by the accumulation of "public wealth". By contrast, this paper focuses on a community seeking to limit development because of negative externalities. It assumes that residents can directly tackle these externalities by taxing new construction. In addition, it allows for an upward sloping supply curve of building land, as opposed to the flat curve assumed in Barseghyan and Coate (2021). To incorporate these innovations, this paper abstracts from public goods, debt, and general taxation, and models externalities in a reduced form manner.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature. Section 3 outlines the model. Section 4 establishes two benchmarks by describing socially optimal development and development with free entry. Section 5 explains what would happen if the initial residents could choose policies for all future periods. Section 6 explores development with sequential policy-making and describes the different types of equilibria that arise. Section 7 addresses the question of how authorizing residents' to tax development impacts social welfare. Section 8 compares taxing development with zoning and impact fees, and Section 9 concludes.

2 Related literature

This paper relates to four distinct literatures: i) that studying the political determination of policies regulating community development; ii) that on the durable good monopoly problem; iii) that seeking to undertake policy analyses which account for the public choice critique of normative public economics; and iv) that on dynamic political economy. We discuss each in turn.

2.1 Political economy models of development regulation

The importance of development regulating policies in practice motivates a literature studying the political economy of such policies in various models of community development.[3] Early work in this literature employs static, multi-community models in which communities first select policies and then households choose where to live. Land is typically assumed owned by absentee landlords and households are renters. A nice example is Helsley and Strange (1995) who consider a spatial model with competing communities.[4] A fixed population of households with identical preferences choose in which community to live. A "passive" community is available that does not regulate its size and serves as a repository for households crowded out of the other communities. In each community, higher populations create negative externalities via congestion. The paper considers both the case in which communities regulate their size directly by land-use regulation and that in which they employ entry fees. Communities choose their policies to maximize landowner profits. Entry fee revenues are included in these profits. With either policy, communities choose to deter

[3] A good review of the evidence on the importance of development regulating policies for the housing market is provided by Gyourko and Molloy (2015).

[4] Other good examples are Brueckner (1995) and Brueckner and Lai (1996).

entry and this reduces aggregate welfare.

A more recent paper in this vein is Hilber and Robert-Nicoud (2013) who develop a multi-community model in which each community's planning board chooses a regulatory tax. Households have heterogeneous preferences over communities and some communities have better amenities than others. Planning boards choose their regulatory taxes to maximize land rents plus revenues. In equilibrium, communities with better amenities have higher populations and impose higher regulatory taxes. Empirical support for this prediction is provided using cross-sectional data from U.S. metro areas. Regulatory taxes distort the allocation of households across communities.

Further advancing the multi-community tradition, Albouy et al (2019) develop a model in which the number of active communities is endogenous. Potential communities are heterogeneous in their suitability for production. Moreover, in each potential community, there is a positive agglomeration externality in production and a negative congestion externality. This means that there is a socially optimal number of communities and an ideal size for each of these communities. Communities control their size via land-use regulation and seek to maximize resident payoffs under the assumption that residents share all local land rents. In equilibrium, there are too many communities and their populations are too small. This is because communities do not take into account non-residents' payoffs. In particular, they do not take into account that excluded potential residents have to live in less suitable communities.[5]

While yielding important insights, static models have difficulty capturing the incentives of current homeowners to protect the value of their properties. Such incentives seem important in practice. Indeed, in his influential book the *Homevoter Hypothesis*, William Fischel argues that protecting home values is the key motivator of local government behavior (Fischel 2001). Another strand of the literature has therefore explored dynamic models that try to capture the redistributional conflict between current and future homeowners.

Glaeser, Gyourko, and Saks (2005a) analyze the decision of a zoning authority to approve a development project seeking to build new homes in a community. Existing residents oppose the project both because of congestion externalities and the negative impact on home prices. The developer obtains the rents created by selling the new homes and thus wants to see it approved.

[5] Duranton and Puga (2019) develop a similar model, but imbed it in a dynamic framework with one period lived generations. Each new generation inherits the locations of the previous generation. Over time, communities become more productive and thus become more attractive for households born into rural areas (or less productive communities). Those households born into a community choose a regulatory tax to limit newcomers. The distortions that arise are similar to those in Albouy et al (2019).

The zoning authority has its own preferences regarding the project, but is also swayed by the time and money the existing residents and developer devote to influencing it. The paper analyzes the determinants of the equilibrium probability that the project is approved.

Ortalo-Magne and Prat (2014) study the supply of building permits in an overlapping generations model with a single city and a rural area. When young, households choose whether to locate in the city or the rural area. If they locate in the city, they have to decide whether to rent or own a home. Housing is infinitely durable and building new homes in the city requires developers to have building permits. In each period, the residents of the city choose how many permits to issue. Residents receive an exogenous fraction of the rents the permits create for developers. The paper characterizes the smallest city size which supports an equilibrium in which no additional permits are issued and shows that this size is smaller than optimal. This result is driven by the incentives of home-owners to increase the value of their homes by restricting supply. Home-owners care about the value of their homes because they sell them when old.

All the work reviewed here shares a common focus: understanding how the availability of a particular regulatory policy instrument impacts equilibrium allocations and social welfare. This paper follows in this tradition. The work differs in the underlying economic environment considered and the particular policy instrument studied. Regarding the latter, a key distinction is whether the policy can raise revenues for the community (i.e., entry fees, permits, etc.) or simply restricts land use (i.e., zoning and other land use regulations). This paper studies the simplest type of revenue raising instrument - a tax on development. In some respects, the environment considered here is simpler than in much of the literature. There is a single community, a single policy instrument, and the policy is determined solely by the interests of the residents - all of whom own identical homes. On the other hand, the policy is continuous and can be adjusted as the community develops. This allows study of the evolution of the policy over time and its impact on the dynamics of development, which is the novel feature of the analysis.

2.2 The durable good monopoly problem

The durable good monopoly problem has been analyzed by Bulow (1982), Coase (1972), Gul, Sonnenschein, and Wilson (1986), Stokey (1979, 1981), and many other distinguished economists. In this problem, a monopoly producer of a durable good faces a fixed population of potential consumers with heterogeneous valuations of the good. The time horizon is infinite and, in each period, the monopoly has to decide how much to produce. The residents' problem is similar in the sense that new houses are a durable good and, via their choice of taxes, residents effectively control how many are produced. Moreover, residents face a pool of potential residents with heterogeneous preferences for living in the community.

Nonetheless, the residents' problem differs from the durable good monopoly problem in three significant ways. First, because residents may be living in the community in the future, they care about more than just the revenues from taxing new construction: they also care about the negative externalities it generates. Second, there is resident turnover. This implies that the set of potential consumers of the durable (i.e., new housing) is partially refreshed each period.[6] Crucially, it also means that current residents may leave, which implies they care about the value of their homes (i.e., the value of goods already produced). Concern about home values explains why, for example, equilibrium may involve no new construction. Third, the revenues from taxing new construction are shared among the residents. This sharing will be anticipated by potential residents and will influence the demand to live in the community. It will also impact how the current residents evaluate the costs and benefits of additional new construction. These differences mean that our results differ from those associated with the durable good monopoly problem, as we will point out along the way.

2.3 Policy analyses accounting for political determination

The standard approach in normative public economics assumes that the policy analyst can determine the level of corrective instruments such as taxes, subsidies, and regulations, and of spending on public programs. Intervention with an instrument or program is then deemed desirable if welfare at the socially optimal level exceeds welfare at the status quo. Writers in the public choice tradition argue that this approach is flawed because it ignores policy determination via the political process (see Buchanan 1962, Buchanan and Vanberg 1988, and, for further discussion, Besley and Coate 2003). The assumption is that, in reality, once a new policy or program is introduced, the way in which it is operated will be beyond the control of the analyst. According to this view, when assessing the desirability of a particular instrument or program, the analyst should anticipate the political process and recommend its introduction only if welfare at the politically

[6] There are papers in the durable good monopoly literature that allow for new consumers to enter (for example, Board 2008 and Sobel 1991). What is different here is that the new consumers who enter replace old ones who leave and the leavers who have previously purchased homes put them up for sale creating additional supply.

determined levels of the policy is higher than welfare at the status quo. This paper adds to the small body of work providing policy analyses that proceed as the critique prescribes.[7]

2.4 Dynamic political economy

The dynamic political economy literature develops and analyzes infinite horizon positive models of policy-making with forward-looking decision makers.[8] Many interesting issues arise from recognizing the dynamic linkage of policies across periods. Such linkages arise directly, as with public investment or debt, or indirectly because current policy choices impact citizens' private investment decisions. The model studied here features a state variable determined by the market (the housing stock) but shaped by residents' policy decisions (the choice of tax). It also features a changing group of decision-makers, as the size of the community is growing. A nice feature of the analysis from the perspective of this literature is that it obtains closed form solutions for dynamic political equilibria.

3 Model

Consider a community such as a small town or village. This community can be thought of as one of a number in a particular geographic area. The time horizon is infinite and periods are indexed by $t = 0, ..., \infty$. There is a pool of potential residents of size 1. These can be thought of as households who for exogenous reasons (employment opportunities, family ties, etc.) need to live in the geographic area in which the community is located and are potentially open to living in the community. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter θ. This desire, for example, may be determined by a household's idiosyncratic taste for the community's natural amenities. The preference parameter θ is uniformly distributed on $[0, \overline{\theta}]$. Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident remains one in the subsequent period is $\mu \in (0, 1]$. Thus, in each period, a

[7] Other examples include Rodrik's (1986) analysis of tariffs versus production subsidies in an international trade setting, Krusell, Quadrini, and Rios-Rull's (1996) analysis of income versus consumption taxation in a neo-classical macro model, and Finkelshtain and Kislev's (1997) analysis of price versus quantity regulation of pollution.

[8] Examples of this style of work are Azzimonti (2011), Battaglini and Coate (2008), Bowen, Chen, and Eraslan (2015), Brinkman, Coen-Pirani, and Sieg (2018), Coate and Morris (1999), Hassler, Rodriguez Mora, Storesletten, and Zilibotti (2003), and Krusell and Rios-Rull (1999).

fraction $1-\mu$ of households leave the pool and are replaced by an equal number of new ones.

The community consists of land area L. Units of land can be used for housing or an alternate use. Each house requires one unit of land. In their alternate use, units of land yield a heterogeneous return π, where π is uniformly distributed on $[\underline{\pi}, \overline{\pi}]$. At the beginning of period 0, the community has a housing stock H_0, implying that H_0 units of land are already being used for housing. This housing is built on those units of land with the lowest productivity in their alternate use. Thus, land with productivity between $\underline{\pi}$ and $\pi(H_0)$ is being used for housing where the function $\pi(H)$ is defined as

$$\pi(H) \equiv \underline{\pi} + (\overline{\pi} - \underline{\pi}) \frac{H}{L}. \tag{1}$$

The only way to live in the community is to own a house. New houses can be built on the $L - H_0$ units of land not already used for housing. Houses are homogeneous and infinitely durable.[9] A stock of housing H accommodates a fraction H of the pool of potential residents. The cost of constructing a house is C and new construction is supplied by competitive developers. Undeveloped land is owned by landowners who reside outside of the community.

A competitive housing market operates in each period. Demand comes from new households moving into the community, while supply comes from owners leaving the community and new construction. The price of a unit of building land in period t is denoted R_t and the price of houses is denoted P_t. The stock of houses at the beginning of period t is denoted H_t. New construction in period t is therefore $H_{t+1} - H_t$. The community levies a tax τ_t on new construction in period t. This tax is paid by developers on every new house they build. The revenues of the tax are divided equally between all households who reside in the community at the end of the period. Thus, each household receives an amount $\tau_t(H_{t+1} - H_t)/H_{t+1}$. The determination of the building land and housing prices and the level of new construction will be explained below.

When living in the community in period t, a household with preference parameter θ and consumption x_t obtains a period payoff of $\theta + x_t + S(H_{t+1})$ if the number of households is H_{t+1}. The function $S(H)$ represents the surplus associated with living in the community. Intuitively, this can be thought of as determined by the net benefits of the public spending undertaken by the community on behalf of its residents, along with population-related costs such as congestion and

[9] The assumption of infinitely durable housing is common in the urban economics literature and is justified by the fact that buildings in developed countries display considerable longevity.

pollution. We assume that over the relevant range of housing levels, $S(H)$ is equal to $S - sH$ where S and s are non-negative. Thus, there is a negative externality associated with higher population. This linear specification allows us to both obtain analytical solutions and capture the strength of the externality in a simple way.

Households and landowners discount future payoffs at rate β and can borrow and save at rate $\rho = 1/\beta - 1$. This assumption means that agents are indifferent to the intertemporal allocation of their consumption. Each household receives an exogenous income stream the present value of which is sufficient to purchase housing in the community and to pay any tax obligations.[10] When not living in the community, a household's per period payoff (net of the consumption benefits from income) is normalized to 0.[11]

The timing of the model is as follows. At the beginning of any period t, the community starts with a stock of houses H_t. The action begins with the existing residents choosing the tax on new construction τ_t. Then, households who were in the pool of potential residents in the previous period learn whether they will be remaining and new households join. Those in the pool decide whether to live in the community and existing residents no longer in the pool prepare to leave it. The housing market opens and the equilibrium prices of building land R_t and housing P_t are determined along with new construction or, equivalently, next period's housing stock H_{t+1}. New residents buy houses and move into the community and old ones sell up and leave. Landowners sell $H_{t+1} - H_t$ units of land. Developers build $H_{t+1} - H_t$ new houses and pay taxes $\tau_t(H_{t+1} - H_t)$. Residents enjoy a surplus $S(H_{t+1})$ and share the revenues from the tax. Next period begins with the new housing stock H_{t+1} and the process begins anew.

3.1 Housing market equilibrium

We now explain how the housing market determines land and housing prices and new construction. We begin with the demand for housing. At the outset of any period t, households fall into two groups: those who resided in the community in period $t - 1$ and those who did not, but could in period t. Households in the first group own homes, while the second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff of P_t.

[10] The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

[11] Note that 0 is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

The remaining households in the first group and all those in the second must decide whether to live in the community. This decision will depend on their preference parameter θ, current and future housing prices, expected surplus, and their share of tax revenues. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of any period.[12] This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the price of housing in the next period.

To see how this plays out, consider a household with preference parameter θ deciding whether to live in the community in period t. As explained below, we will be focusing on equilibria in which prices in any period just depend on the housing stock at the beginning of that period. Thus, let $P(H_{t+1})$ denote the anticipated equilibrium price of housing in period $t+1$ when the housing stock at the beginning of that period is H_{t+1}. Then, if the new construction tax is τ_t, the price of housing is P_t, and H_{t+1} households will be living in the community, the household will choose to reside in the community if and only if

$$\theta + S(H_{t+1}) + \frac{\tau_t(H_{t+1} - H_t)}{H_{t+1}} \geq P_t - \beta P(H_{t+1}). \quad (2)$$

The left hand side represents the benefit from locating in the community in period t and the right hand side the cost. The latter assume that the household buys a house at the beginning of period t and sells it back in period $t+1$.

Given (2) and the fact that household preferences are uniformly distributed over $[0, \overline{\theta}]$, the equilibrium price of housing P_t in the current period must satisfy the market clearing condition

$$H_{t+1} = 1 - \frac{P_t - \beta P(H_{t+1}) - \left(S(H_{t+1}) + \frac{\tau_t(H_{t+1}-H_t)}{H_{t+1}}\right)}{\overline{\theta}}. \quad (3)$$

This implies that the equilibrium housing price is

$$P_t = (1 - H_{t+1})\overline{\theta} + S(H_{t+1}) + \frac{\tau_t(H_{t+1} - H_t)}{H_{t+1}} + \beta P(H_{t+1}). \quad (4)$$

Turning to the supply side of the housing market, the assumption that houses are infinitely durable implies that

$$H_{t+1} \geq H_t. \quad (5)$$

[12] It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community.

Moreover, because the supply of new construction is perfectly elastic at a price equal to the land price plus the construction cost plus the tax, it must also be the case that

$$P_t \leq R_t + C + \tau_t \text{ } (= \text{if } H_{t+1} > H_t). \tag{6}$$

In terms of the land market, the demand for building land in period t is $H_{t+1} - H_t$. The market will allocate the land with the lowest return in its alternate use to building. Thus, at the beginning of time t, land with productivity between $\underline{\pi}$ and $\pi(H_t)$ has already been used for housing. For equilibrium in the land market it must be the case that the owner of land of productivity $\pi(H_{t+1})$ must be just indifferent between selling it for building land or keeping it. This requires that the price of building land must satisfy

$$R_t = \pi(H_{t+1}) + \beta R(H_{t+1}), \tag{7}$$

where $R(H_{t+1})$ is the anticipated equilibrium price of building land in period $t+1$ if the housing stock at the beginning of that period is H_{t+1}.

To see why (7) must hold, note first that if there is new construction in period $t+1$, so that H_{t+1} is less than H_{t+2} then the marginal landowner in period t would certainly sell his land in period $t+1$ if he kept it in period t. It follows that the right hand side of (7) describes his payoff from keeping his land in period t. If there is no new construction in period $t+1$ then the marginal owner of land of productivity $\pi(H_{t+1})$ will be the marginal owner for ever and it must be the case that $R_t = R(H_{t+1}) = \pi(H_{t+1})/(1-\beta)$, which implies that (7) holds.

Given that period $t+1$'s housing and building land prices are described by the functions $P(H_{t+1})$ and $R(H_{t+1})$, any policy τ_t and housing market tuple (H_{t+1}, P_t, R_t) is consistent with housing and land market equilibrium if and only if (4), (5), (6), and (7) are satisfied.

3.2 Policy choice

Next we turn to residents' choice of policy in period t. As explained above, the timing of the model is first that the existing residents choose the new construction tax τ_t, and then the housing market determines new construction H_{t+1} and the prices of housing P_t and building land R_t. Obviously, when residents choose policy they will anticipate how they impact the housing market. Rather than deriving the relationship between the housing market equilibrium and the tax, and then analyzing the optimal policy, it is easier to think of residents as directly choosing new construction and the housing and building land prices, along with the policy, but subject to the constraint that their choice be consistent with housing market equilibrium. Thus, given H_t, we will assume that residents choose $(\tau_t, H_{t+1}, P_t, R_t)$ but subject to the market equilibrium constraints (4), (5), (6), and (7).

Note further that there is no loss of generality in assuming that τ_t is chosen so that it equals $P_t - C - R_t$. If H_{t+1} exceeds H_t, then this must be true in equilibrium (see (6)). If H_{t+1} equals H_t, then no tax revenue is being raised and τ_t can be lowered to $P_t - C - R_t$ with no implications for the housing market equilibrium. This observation permits removing τ_t from the set of choice variables. It also permits ignoring constraint (6) since it will automatically be satisfied whether or not H_{t+1} exceeds H_t.

While period t residents differ in their desires to live in the community θ, they will have identical preferences over policies and hence there is no collective choice problem to resolve. To understand this, note that, given H_t, period t residents will have preferences between $(1-H_t)\overline{\theta}$ and $\overline{\theta}$.[13] The market will allocate housing to those in the pool of potential residents with the highest θ and that the supply of housing can only expand. It follows that these residents will all anticipate living in the community as long as they stay in the pool. Accordingly, residents have no difference in policy preferences resulting from different time horizons. Furthermore, the preference θ enters as an additive term and does not impact the surplus $S(H)$ or the marginal value of consumption. Thus, all residents trade off population, tax revenue, and housing prices in the same way.

In light of this, the period t residents' policy problem can be written as

$$\max_{(H_{t+1}, P_t, R_t)} \left\{ \begin{array}{c} (1-\mu)P_t + \mu\left[S(H_{t+1}) + \frac{(P_t - C - R_t)(H_{t+1} - H_t)}{H_{t+1}} + \beta V(H_{t+1})\right] \\ s.t.\ (4),\ (5),\ \&\ (7) \end{array} \right\}. \tag{8}$$

This objective function reflects the fact that, with probability $1-\mu$, a resident leaves the pool and sells its house, and, with probability μ, it remains in the pool and continues to live in the community. The value function $V(H_{t+1})$ measures the continuation payoff of a household who is residing in the community at the beginning of period $t+1$ net of the future benefits coming from the additive preference θ. As discussed below, it is determined by the policies that future residents are expected to choose. Next period's prices $P(H_{t+1})$ and $R(H_{t+1})$ which enter in constraints (4)

[13] In periods $t = 1, ..., \infty$ this follows from the fact that, in equilibrium, the households with the highest preference for living in the community purchase houses in the community in the previous period. We assume that this condition also characterizes the initial distribution of residents in period 0.

and (7) are also determined by next period's residents.

3.3 Equilibrium defined

The model gives rise to a dynamic game between the different cohorts of residents. The equilibrium concept we use to solve this game is Markov perfect equilibrium. In such an equilibrium, strategies in any period just depend on the state at the beginning of that period. The state is the housing stock. Because all that matters is the state, when defining equilibrium it is not necessary to index strategies by the time period t. All we need do is distinguish the state at the beginning of a period, which we denote H, and the state at the beginning of the next period, which we denote H'. The latter will be determined by the policy choices made during the period.

In a Markov perfect equilibrium, the residents in a period in which the state is H choose policies (H', P, R) to maximize their payoff under the assumption that future policies will be chosen according to policy rules that depend only on whatever the state is at that time. In equilibrium, the policies residents choose must be consistent with the policy rules that govern future policies. Formally, an *equilibrium* consists of a housing rule $H'(H)$, a housing price rule $P(H)$, a building land price rule $R(H)$, and a value function $V(H)$ satisfying two conditions. The first condition is that, for all states $H \in [H_0, 1]$, the policies prescribed by the equilibrium policy rules solve problem (8). The second condition is that the value function is consistent with the equilibrium policy rules. Thus, for all states $H \in [H_0, 1]$, the value function satisfies the equality

$$V(H) = (1-\mu)P(H) + \mu \left[S(H'(H)) + \frac{(P(H) - C - R(H))\,(H'(H) - H)}{H'(H)} + \beta V(H'(H)) \right]. \tag{9}$$

With an equilibrium, we can trace out the dynamic evolution of the community. In period 0, the number of residents will be $H_1 = H'(H_0)$, the price of housing will be $P(H_0)$ and the price of building land will be $R(H_0)$. The tax on new construction will be $P(H_0) - C - R(H_0)$. Similarly, in period 1, the number of residents will be $H_2 = H'(H_1)$, the price of housing will be $P(H_1)$ and the price of building land will be $R(H_1)$. The tax on new construction will be $P(H_1) - C - R(H_1)$. Continuing on in this way, we can see exactly how the community develops both with respect to its housing stock and its prices and policies.

4 Two benchmarks for comparison

This section describes two benchmarks with which we will compare the equilibrium: the development path that would be optimal for a utilitarian social planner and that which would arise with free entry (i.e., when residents do not have access to corrective taxation). The section also lays out the assumptions we will impose on the model's parameters.

4.1 Optimal community development

A utilitarian planner wishes to maximize the discounted sum of the aggregate payoffs of the different pools of potential residents and the landowners. The assumption that utility is linear in consumption, implies that the planner is indifferent between transfers of consumption between agents. Accordingly, there is no loss of generality in simply assuming that, in any period, the cost of new construction is financed by lump-sum taxation of the pool of potential residents and landowners.

Given the community's initial stock of housing H_0, the planner chooses a time path for new construction or, equivalently, a sequence $\{H_{t+1}\}_{t=0}^{\infty}$. The planner must respect the feasibility constraint created by durable housing (5). In any period t, the planner will allocate the households in the pool with the highest θ to the H_{t+1} houses and use the lowest π land for building. Given that θ is uniformly distributed on $[0,\overline{\theta}]$, this implies that households with preferences between $(1-H_{t+1})\,\overline{\theta}$ and $\overline{\theta}$ will be assigned to live in the community. Similarly, land with productivity between $\underline{\pi}$ and $\pi(H_{t+1})$ will be used for building. Accordingly, the planner's objective function is

$$\sum_{t=0}^{\infty}\beta^t\left[\int_{(1-H_{t+1})\overline{\theta}}^{\overline{\theta}}\theta\frac{d\theta}{\overline{\theta}}+H_{t+1}S(H_{t+1})+L\int_{\pi(H_{t+1})}^{\overline{\pi}}\pi\frac{d\pi}{\overline{\pi}-\underline{\pi}}-C(H_{t+1}-H_t)\right]. \qquad (10)$$

The first two terms represent the benefits received by the households assigned to the community. Recall that households not assigned receive a payoff that is normalized to zero. The third term is the return to land not used for housing and the final term is the costs of new construction.

There is no social benefit from delaying development, so that, if the community's initial housing stock is not so large to make the feasibility constraint bind, the solution will be to raise housing to the socially optimal level in the initial period. Differentiating the objective function, we see that the socially optimal housing level satisfies the first order condition

$$(1-H^o)\,\overline{\theta}+S(H^o)+H^oS'(H^o)=\pi(H^o)+C(1-\beta). \qquad (11)$$

The left hand side represents the social benefit from assigning an additional household to the community. The term $(1-H^o)\,\overline{\theta}$ is the preference of the marginal household for living in the community and $S(H^o)$ reflects the surplus accruing to the marginal household. The term $H^o S'(H^o)$ reflects the impact of adding the household on the surpluses of the other residents, and reflects the externality created by the additional household. The right hand side is the per-period cost of an additional house which includes both the construction cost and the opportunity cost of the land on which the house is built.

Using the assumptions that $S(H)$ and $\pi(H)$ are linear, we can solve (11) for the socially optimal housing level. Letting this be denoted H^o, we have

$$H^o = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{\overline{\theta} + 2s + \frac{\overline{\pi}-\underline{\pi}}{L}}. \tag{12}$$

We make assumptions on the parameters and the community's initial housing level to ensure that the socially optimal housing level is larger than the initial housing stock H_0 but does not use up the available land L or potential residents (i.e., is less than $\min\{L, 1\}$).

Assumption 1 (i)

$$(1-H_0)\,\overline{\theta} + S(H_0) + H_0 S'(H_0) > \pi(H_0) + C(1-\beta).$$

(ii)

$$(1-\min\{L,1\})\,\overline{\theta} + S(\min\{L,1\}) < \pi(\min\{L,1\}) + C(1-\beta).$$

Assumption 1(ii) is stronger than needed to ensure that H^o is less than $\min\{L, 1\}$. This is because it also serves to ensure that the free entry equilibrium housing level defined in the next sub-section is less than $\min\{L, 1\}$. Then, we have:

Proposition 1 *Under Assumption 1, the optimal community development plan is to construct $H^o - H_0$ new houses in the initial period. Thereafter, no more housing should be constructed.*

4.2 Development with free entry

If residents do not have access to a new construction tax, there is nothing for residents to do and development is just determined by the market. All new construction will take place in the initial period. To find the level, note that the inverse demand curve for housing is $\left[(1-H)\,\overline{\theta} + S(H)\right]/(1-\beta)$. This reflects the willingness to pay of the marginal buyer to live in the community when it has housing level H. For housing levels larger than H_0, the inverse supply curve of housing is $C+\pi(H)/(1-\beta)$. This represents the cost of producing the marginal house in the community when it has housing level H. Assuming that demand is sufficiently strong to induce some development, the level of housing that equates demand and supply, denoted H^e, satisfies

$$(1-H^e)\,\overline{\theta}+S(H^e)=\pi(H^e)+C(1-\beta). \tag{13}$$

Using the assumptions that $S(H)$ and $\pi(H)$ are linear, we can solve (13) to obtain

$$H^e=\frac{\overline{\theta}+S-C(1-\beta)-\underline{\pi}}{\overline{\theta}+s+\frac{\overline{\pi}-\underline{\pi}}{L}}. \tag{14}$$

Reflecting the negative externality, H^e is larger than H^o.

Proposition 2 *Under Assumption 1, community development with free entry involves the construction of H^e-H_0 new houses in the initial period. Thereafter, no more housing is constructed.*

The only difference between the socially optimal development plan and that which emerges with free entry, lies in the extent of new construction that occurs in the initial period. The textbook solution to the excessive development that arises with free entry is to impose a corrective tax $sH^o/(1-\beta)$ on new construction. With such a tax, the inverse supply curve of housing becomes $C+sH^o/(1-\beta)+\pi(H)/(1-\beta)$ and demand equals supply at the socially optimal housing level H^o.

5 The initial residents' optimal plan

In equilibrium, the initial residents simply get to choose the policies for period 0. Here we ask what would they choose if they could commit the community to following a complete development plan? This is interesting in its own right, since what happens with commitment is the usual starting point in dynamic policy problems. It is also key for developing intuition for the case with sequential policy-making. In particular, understanding the time consistency of this plan, lends insight into how development will be modified in the sequential case.

A complete development plan for the initial residents is described by $\{H_{t+1},P_t,R_t\}_{t=0}^{\infty}$. The optimal plan maximizes the objective function

$$\sum_{t=0}^{\infty}(\mu\beta)^t\left\{(1-\mu)P_t+\mu\left[S(H_{t+1})+\frac{(P_t-C-R_t)(H_{t+1}-H_t)}{H_{t+1}}\right]\right\}, \tag{15}$$

subject to satisfying the constraints of market equilibrium in each period t.[14] The difference between this and the equilibrium problem (8) is that the initial residents get to directly choose the entire sequence of policies rather than having only an indirect influence through their determination of the period 1 housing stock.

To describe the solution, first let

$$H^* = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{\overline{\theta}(1 + \frac{1-\mu}{1-\mu\beta}) + 2s + \frac{\overline{\pi}-\underline{\pi}}{L}}. \tag{16}$$

How the initial housing level H_0 compares with H^* determines whether or not development takes place. Note that H^* is less than or equal to H^o with the equality holding only when μ equals 1. Second, for all housing levels H less than or equal to H^*, let

$$\mathcal{H}(H) = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{2\left(\overline{\theta} + s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)} + \frac{\frac{\overline{\pi}-\underline{\pi}}{L} + \frac{\mu(1-\beta)}{1-\mu\beta}\overline{\theta}}{2\left(\overline{\theta} + s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)} H. \tag{17}$$

This function describes the housing level that actually arises when development takes place (which is $\mathcal{H}(H_0)$). The function is linear with a positive intercept and a slope less than 1. Furthermore, $\mathcal{H}(H^*)$ equals H^*, so that for housing levels less than H^*, $\mathcal{H}(H)$ exceeds H. Finally, for all H, let

$$\mathcal{B}(H) = (1-H)\overline{\theta} + S(H) - (\pi(H) + C(1-\beta)). \tag{18}$$

The function $\mathcal{B}(H)$ describes the net private benefit that would be obtained by the marginal household from living in the community if residents did not have access to a tax. We can now state:

Proposition 3 *Suppose that Assumption 1 is satisfied. Then the initial residents' optimal development plan has the following form.*

(i) If $H_0 \geq H^*$, no development takes place and the tax on new construction is $\mathcal{B}(H_0)/(1-\beta)$ in each period.

(ii) If $H_0 < H^*$, $\mathcal{H}(H_0) - H_0$ new houses are provided in the initial period, and, thereafter, no more are provided. The tax on new construction is $\mathcal{H}(H_0)\mathcal{B}(\mathcal{H}(H_0))/H_0(1-\beta)$ in the initial period and $\mathcal{B}(\mathcal{H}(H_0))/(1-\beta)$ thereafter.

[14] To this problem we need to add the transversality condition that $\lim_{t\to\infty} \beta^t P_t = 0$. This prevents the initial residents using an increasing sequence of taxes to create a housing price bubble in which potential residents buy a house in the community expecting its price will rise in the next period. This price rise is caused by a higher future tax.

Thus, the commitment solution takes one of two possible forms. If the initial housing stock exceeds H^*, there is no development. To achieve this outcome, the residents impose a time invariant tax that deters new construction. If the initial housing stock is below H^*, there is development, all of which occurs in the initial period.[15] The tax is higher in the initial period than thereafter, reflecting the fact that new residents in period 0 share in the tax revenues the development generates. In either of its two forms, the commitment solution involves too little development. This is immediate when there is no development, since the initial housing stock is smaller than optimal by Assumption 1. When there is development, the result follows from the fact that $\mathcal{H}(H_0)$ is smaller than H^*, which is in turn smaller than the optimal level H^o.

The proof of Proposition 3 is in the Appendix. While the initial residents' problem may look complicated, the proof shows it can be reduced to a problem involving only a single choice variable: the amount of development to undertake in the initial period. Development impacts both the utility from living in the community and the price of housing. These two are related, since the housing price reflects the utility from living in the community. The impact of development on the utility from living in the community depends on the tax revenue it generates and the costs stemming from the negative externality. The impact on the price depends on these two considerations and an additional negative factor: development implies going down the demand curve in the sense that households with lower preference θ must be drawn into the community.[16]

To understand the incentives of the initial residents in choosing development it is helpful to start with the case in which μ equals 1. In this case, the initial residents anticipate remaining in the community forever, implying that all they care about is the impact of development on the utility from living in the community. The optimal amount of development for the initial residents balances the benefits of the tax revenue it generates with the costs created by the negative externality. The residents ignore the impact of going down the demand curve and thus the amount of development exceeds that which maximizes the period 0 housing price.[17] As μ falls below 1, residents care

[15] In the durable good monopoly problem, the commitment solution involves the monopoly producing only in the initial period (Stokey 1979).

[16] The proof of Proposition 3 develops expressions for what we are terming the utility from living in the community and the period 0 housing price. The utility from living in the community consists of the discounted sum of the stream of surpluses $\{S(H_{t+1})\}_{t=0}^{\infty}$ along with the discounted sum of the stream of per-capita tax revenues. The period 0 price of housing equals the utility from living in the community plus the discounted sum of the preferences of the marginal household $\{(1 - H_{t+1})\overline{\theta}\}_{t=0}^{\infty}$. Increasing development in any period therefore has a larger negative impact on the period 0 housing price than it does on the utility from living in the community.

[17] It is common in the urban economics literature to assume that policy-makers seek to maximize the value of

more and more about the period 0 housing price because of the increasing likelihood they will leave the community as soon as they have chosen period 0 policies. This leads them to reduce the amount of development and, in the limit, they choose the amount of development that maximizes the period 0 housing price.

When μ equals 1, the optimal amount of development for the initial residents is necessarily positive. This follows from the Proposition because H^* equals H^o when μ equals 1 and the initial housing stock H_0 is less than H^o by Assumption 1. Intuitively, it reflects the fact that when H_0 is less than H^o the tax revenues that can be extracted from a small amount of development will necessarily exceed the externality cost. Nonetheless, the amount of development will fall short of the socially optimal amount. This is because holding back development boost tax revenues by familiar monopoly logic. This incentive to hold back development is higher the smaller is H_0.

Once μ falls below 1, not only will the initial residents prefer a smaller level of development, but it could be optimal for the initial residents not to develop at all. This follows from the Proposition because H^* is less than H^o and hence H_0 could exceed H^* without violating Assumption 1. When no development is optimal it is because of its impact on the period 0 housing price. The period 0 housing price will reflect any tax revenues that accrue to residents in period 0 and, as noted, at least for a small amount of development, these will outweigh the cost of the negative externality. However, development requires going down the demand curve, and this can lead to a net negative impact of development on price. When this negative impact outweighs the net gains from tax revenues over externality costs, no development is optimal.

5.1 Time consistency

We say that the initial residents' solution $\{H_{t+1}, P_t, R_t\}_{t=0}^{\infty}$ is *time consistent* if, for all $t \geq 1$, $\{H_{z+1}, P_z, R_z\}_{z=t}^{\infty}$ is an optimal plan for those residents in the community at the beginning of period t, given the initial housing stock H_t. To assess time consistency, we need to understand what optimal plans for future residents look like. The optimal plan for the period t residents will solve the same problem as for the period 0 residents, except that the community's housing stock will be H_t. It will therefore have the same form as that described in Proposition 3. Accordingly, if H_t is less than H^* the period t residents will increase the housing stock to $\mathcal{H}(H_t)$ in period t, while if H_t is greater than H^*, the period t residents will keep the housing stock constant. Given

residents' property. This paper illustrates a setting where maximizing the value of property is not the same as maximizing residents' payoffs.

this, we can now establish:

Proposition 4 *Suppose that Assumption 1 is satisfied. Then, the initial residents' optimal development plan is time consistent if and only if $H_0 \geq H^*$.*

Thus, the commitment solution is time inconsistent when it involves development. Once the population has expanded in the initial period, next period's residents would want to expand it further. Intuitively, the reason they would favor such an expansion and the initial residents would not, is that they do not internalize the negative consequences for the initial period. Specifically, the initial residents set the tax in all periods after period 0 at a level to choke off new construction. Future residents have an incentive to lower this tax to obtain some tax revenue. The initial residents would not approve of such a future reduction because it would adversely impact initial period tax revenues by encouraging some potential residents to delay entry.[18]

The proof of Proposition 4 is simple. If H_0 is less than H^* the initial residents' optimal plan involves increasing the housing stock to $\mathcal{H}(H_0)$ in period 0. Thereafter, there is no new construction. Now consider the period 1 residents. We know that $\mathcal{H}(H_0)$ is less than H^*. Accordingly, they will wish to increase the housing stock to $\mathcal{H}(\mathcal{H}(H_0))$ in period 1. By contrast, if H_0 exceeds H^* the initial residents' optimal plan involves no development. The period 1 residents therefore face the same trade-off as the period 0 residents and want to do the same thing.

6 Sequential policy-making

We now turn to the development paths that emerge when policies are chosen sequentially. Sequential policy-making seems the most interesting case to consider since policy-making is an ongoing process and there is no obvious way that current policy-makers can bind the tax choices of future policy-makers.

6.1 Equilibrium with no development

Given Propositions 3 and 4, it is natural to expect that if H_0 exceeds H^*, there will exist an equilibrium in which the outcome is the same as in the commitment case. Indeed, it is straightforward to show that if H_0 is greater than H^*, we can find an equilibrium in which $H'(H) = H$

[18] This logic is similar to that explaining why the commitment solution is not time consistent in the durable good monopoly problem (Coase 1972).

and $P(H) = \left[(1-H)\overline{\theta} + S(H)\right]/(1-\beta)$. This yields:[19]

Proposition 5 *Suppose that Assumption 1 is satisfied. Then, if $H_0 \geq H^*$, there exists an equilibrium in which no development takes place and the tax on new construction is $\mathcal{B}(H_0)/(1-\beta)$ in each period.*

6.2 Equilibrium with gradual development

If H_0 is less than H^*, the equilibrium outcome must differ from that arising under the initial residents' optimal plan. We begin by looking for an equilibrium in which the housing stock increases gradually over time. This is a natural thing to expect given the nature of the time inconsistency problem. Conveniently, we find such an equilibrium in which the housing rule $H'(H)$ is linear.[20]

To describe this equilibrium, first let γ solve the equation

$$\gamma = \frac{\frac{\overline{\theta}\mu}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}. \tag{19}$$

This equation defines the slope of the equilibrium housing rule. It is a cubic equation and has a unique solution on the interval $(0, 1)$. Next, let

$$\xi = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}. \tag{20}$$

This equation defines the intercept of the housing rule. Finally, let

$$H^{**} = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{\overline{\theta}\left(1 + \frac{1-\mu}{1-\mu\beta\gamma}\right) + 2s + \frac{\overline{\pi}-\underline{\pi}}{L}}. \tag{21}$$

This is the steady state to which the housing level converges in our equilibrium. Notice that $H^{**} \in (H^*, H^o)$ when μ is less than 1 but that all three housing levels are equal when μ equals 1.

The housing rule in our equilibrium is then

$$H'(H) = \begin{cases} \xi + \gamma H & \text{if } H \in [H_0, H^{**}) \\ H & \text{if } H \in [H^{**}, 1] \end{cases}. \tag{22}$$

[19] The equilibrium described in Proposition 5 is analogous to the stationary equilibrium identified in Ortalo-Magne and Prat (2014).

[20] In the durable goods monopoly problem, Stokey (1981) finds an equilibrium in which the total production of the good (new plus stock) is a linear function of the existing stock (see Stokey 1981 Theorem 4 and Gul, Sonneschein and Wilson 1986 Example 1). In this equilibrium, price is also a linear function of the existing stock and the monopoly's value function is a quadratic function. By contrast, in our equilibrium, price is a strictly convex function of the housing stock and the residents' value function is not quadratic. This reflects the fact that revenues are shared by the residents.

When H is less than H^{**}, the housing rule is linear with a positive intercept and a slope less than 1. The definition of H^{**} implies that $H'(H)$ exceeds H for H in this range. Thus, housing is increasing on $[H_0, H^{**})$ and converges asymptotically to H^{**}.

For the housing rule $H'(H)$ defined in (22), define the sequence $\langle H_t(H)\rangle_{t=0}^{\infty}$ inductively as follows: $H_0(H) = H$, $H_t(H) = H'(H_{t-1}(H))$ for all $t \geq 1$. The interpretation is that $H_t(H)$ is the housing level that will prevail in t periods time if H is the housing level selected this period and future residents follow the housing rule $H'(H)$. Then the housing price rule is

$$P(H) = \sum_{t=0}^{\infty} \beta^t \left[(1 - H_t(H'(H)))\overline{\theta} + S(H_t(H'(H)))\right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_t(H'(H)) - H)\,\mathcal{B}(H_t(H'(H)))}{H}, \tag{23}$$

the building land price rule is

$$R(H) = \sum_{t=0}^{\infty} \beta^t \pi(H_t(H'(H))), \tag{24}$$

and the value function is given by

$$V(H) = P(H) - \mu \left[\sum_{t=0}^{\infty} (\mu\beta)^{t-1} (1 - H_t(H'(H)))\overline{\theta}\right]. \tag{25}$$

We can now state:

Proposition 6 *Suppose that Assumption 1 is satisfied and that $H_0 < H^{**}$ where H^{**} is as defined in (21). Then, $\{H'(H), P(H), R(H), V(H)\}$ as defined in (22), (24), (23), and (25), is an equilibrium. In this equilibrium, development takes place in each period and the housing stock converges asymptotically to H^{**}. The tax on new construction in period t is decreasing over time and converges to $\mathcal{B}(H^{**})/(1-\beta)$.*

The interesting feature of this equilibrium is the paths of development and taxes it implies. Community development is driven by a gradual lowering of new construction taxes. In each period, residents lower the tax to stimulate new construction and harvest the revenues this generates. The housing level H^{**} to which the community converges is higher than the level that arises in the commitment solution but is still smaller than socially optimal. However, it does approach the social optimum as the probability residents leave the community vanishes. Irrespective of the eventual level of housing, the gradual development creates inefficient delay.

To prove this Proposition, we first show that substituting the two price constraints (4) and (7) into the objective function reduces the residents' problem described in (8) to one that involves a single choice variable - next period's housing level H' - and only one constraint: namely, that housing cannot decrease. In addition, a solution to the unconstrained version of this problem can be used to create a solution to the constrained problem if the unconstrained solution satisfies appropriate conditions. We then find an unconstrained solution using the strategy of "guess and verify". We derive a first order condition that characterizes the residents' optimal housing choice. We conjecture that the housing rule is linear and the first order condition reveals that the optimal housing choice is indeed a linear function of the current housing level H under this assumption. This allows us to solve for the intercept and slope of the housing rule. We then go back and verify that this solution satisfies the conditions that allow us to create a solution to the constrained problem. The solution we create is described in (22). The implied price rules and value function are as described in (24), (23), and (25). Finally, we establish the claims about the time path of taxes. All this is detailed in the Appendix.

The residents' choice of next period's housing level H' impacts development in all future periods by determining the future housing levels $\langle H_t(H')\rangle_{t=1}^{\infty}$. The development sequence determines the utility from living in the community and the current price of housing. The utility from living in the community depends on the stream of tax revenues generated by development and the costs stemming from the negative externality. As in the commitment case, the price depends on these two considerations and the fact that development implies going down the demand curve in the sense that households with lower preference θ must be drawn into the community. As discussed in the previous section, the value of μ determines the weight residents put on the utility of living in the community versus the current price of housing.

When μ equals 1, residents' choice of development balances the positive impact on the discounted stream of tax revenues with the discounted stream of externality costs. The intercept of the equilibrium housing rule is lower than that of the function $\mathcal{H}(H)$ describing the commitment solution (i.e., (17)) and the slope is higher. Intuitively, the lower intercept reflects the fact that current residents, realizing that future residents will want to develop more than they would like, hold back current development to induce lower levels in the future. Of course, they only have the ability to delay development rather than reduce the long run level, but delay they do. The higher slope reflects the fact that marginal tax revenues are less sensitive to the existing housing stock than in the commitment case because they accrue over the entire future. The fact that the housing level eventually reaches H^o reflects the fact at any lower housing level the revenues generated by a marginal increase in new construction exceeds the externality cost. Thus, until this level is reached, the current residents always have an incentive to develop a little more.[21]

As μ falls below 1, the residents put more weight on the current price of housing and the equilibrium choice of development balances the positive impact on the discounted stream of tax revenues with costs associated with the externality and a lower housing price. In terms of the equilibrium housing rule, simulations reveal that the intercept increases as μ falls and the slope flattens. This suggests the incentive to hold back development decreases as μ falls. This makes sense intuitively because the residents are less likely to be living in the community in the future.

6.3 Equilibrium with stalled development

As we have argued, gradual development is the natural outcome to expect given the nature of the time consistency problem. However, it is not the only possibility. We now identify an equilibrium in which development takes place in the initial period and thereafter is stalled.[22] Further development does not take place because residents anticipate that more development will cause a precipitous fall in the price of housing.

For $\widetilde{H}$ in the interval $[H_0, H^{**})$, consider potential equilibria of the form

$$H'(H) = \begin{cases} \widetilde{H} & \text{if } H \in [H_0, \widetilde{H}] \\ \xi + \gamma H & \text{if } H \in (\widetilde{H}, H^{**}) \\ H & \text{if } H \in [H^{**}, 1] \end{cases} , \tag{26}$$

$$P(H) = \sum_{t=0}^{\infty} \beta^t \left[(1 - H_t(H'(H)))\overline{\theta} + S(H_t(H'(H))) \right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_t(H'(H)) - H)\,\mathcal{B}(H_t(H'(H)))}{H}, \tag{27}$$

$$R(H) = \sum_{t=0}^{\infty} \beta^t \pi(H_t(H'(H))), \tag{28}$$

and

$$V(H) = P(H) - \mu \left[\sum_{t=0}^{\infty} (\mu\beta)^{t-1} (1 - H_t(H'(H)))\overline{\theta} \right]. \tag{29}$$

[21] This logic is analogous to the reasoning behind the famous result in the durable good monopoly problem that output and price converge to competitive levels (the Coase Conjecture).

[22] There is no analogue to the stalled equilibrium in the durable good monopoly problem. This is because, as we show below, the underlying logic rests on the residents caring about the value of their properties.

Here, $\langle H_t(H)\rangle_{t=0}^{\infty}$ is defined in the same way as the previous sub-section, except that it uses the housing rule $H'(H)$ defined in (26) (as opposed to (22)).

In such an equilibrium, the housing level increases to $\widetilde{H}$ when the current level is below $\widetilde{H}$. When the current level exceeds $\widetilde{H}$, the housing level equals the solution of the previous sub-section. In the implied development path, the housing level increases to $\widetilde{H}$ in the initial period and then remains there. This is so despite the fact that any small increase of the housing stock beyond $\widetilde{H}$ would cause the housing stock to eventually grow all the way to H^{**}. The tax is $\widetilde{H}\mathcal{B}(\widetilde{H})/H_0(1-\beta)$ in the initial period and $\mathcal{B}(\widetilde{H})/(1-\beta)$ thereafter.

Proposition 7 *Suppose that Assumption 1 is satisfied and that $\mu < 1$. Then there exists $\underline{H} < H^*$ such that if $H_0 \in (\underline{H}, H^*)$, there exist equilibria of the form described in (26), (28), (27), and (29) in which the steady state housing level $\widetilde{H}$ is less than the level arising in the commitment solution $\mathcal{H}(H_0)$.*

This proposition establishes that, when μ is less than 1, there exist stalled equilibria for a range of initial housing levels. Moreover, in these equilibria, the housing level that emerges is actually less than the level that would be chosen in the commitment solution. The implication of this finding is that sequential policy-making will not necessarily produce more development than would emerge under the commitment solution. *Indeed, it may actually enhance the under-supply of housing.* The logic underlying this is appealing. In the commitment solution, the initial residents expand housing knowing that they can limit the amount they get. With sequential policy-making, residents do not develop because they cannot control the expansion that will occur after they have developed.[23]

To prove this Proposition, we first show that for a given $\widetilde{H}$ in the interval $[H_0, H^{**})$, there exists an equilibrium of the form described in (26), (27), (28) and (29) with steady state housing level $\widetilde{H}$ if $\widetilde{H}$ satisfies two conditions. The first guarantees that with a housing stock in the interval $[H_0, \widetilde{H})$, the residents prefer to increase the housing stock to $\widetilde{H}$ than to increase it to some smaller level. The second guarantees that with housing stock in the interval $[H_0, \widetilde{H}]$ the residents prefer $\widetilde{H}$ to any higher level. We then show that there exists $\underline{H}$ less than H^* such that if $H_0 \in (\underline{H}, H^*)$ there must be values of $\widetilde{H}$ in the interval $(H_0, \mathcal{H}(H_0))$ satisfying these two conditions. The details

[23] The stalled equilibrium relates to a conjecture in Ortalo-Magne and Prat (2014). They note: "without stationarity, there could be equilibria with an extremely small city. Intuitively, even a small size increase today could create the "expectation" of large increases in the future. Any deviation today would trigger a collapse in house prices. Hence, current generations do not modify the city size, even when it is extremely low." (p.163).

can be found in the Appendix.

Proposition 7 raises the question of what happens when μ equals 1? The following Proposition addresses this.

Proposition 8 *Suppose that Assumption 1 is satisfied and that $\mu = 1$. Then there does not exist an equilibrium of the form described in (26), (28), (27), and (29).*

The proof of this Proposition proceeds by contradiction. It establishes that if there were such an equilibrium, residents would always benefit from deviating at the housing level $\widetilde{H}$ by choosing to develop. Specifically, the proof shows that the payoff from choosing to expand housing to the level $H'(\widetilde{H})$ where $H'(\cdot)$ is the housing rule associated with the gradual equilibrium (i.e., that defined in (22)) exceeds that from keeping housing constant at $\widetilde{H}$. Given equilibrium play, such an expansion gives rise to further development in the future, but the tax revenue benefits from this development exceed the externality costs. Intuitively, this is because housing levels remain smaller than H^o throughout the expansion and the additional tax revenue extracted from an increase in housing below this level always exceeds the externality cost.[24]

Proposition 8 makes clear that the key driver of stalled development is residents' concern over the price of housing. In a stalled equilibrium, choosing to develop beyond the housing level $\widetilde{H}$ sets in motion a path of future development. This future development implies going down the demand curve to draw in households with lower preferences to live in the community. This drives down future housing prices and, since the current price of housing reflects its future value, causes a fall in the current value of homes. The desire to avoid this leads residents not to develop.

7 Welfare implications of a development tax

Does allowing residents to tax development increase or decrease welfare? Answering this requires comparing the welfare level arising when residents can tax development to that associated with free entry. Since sequential policy-making is the most realistic assumption, the equilibria that arise in this case are the natural point of comparison. Given the complexity of welfare under the equilibria that arise with sequential policy-making, we employ numerical methods to make the comparison. We vary the community's initial housing stock H_0 and the size of the negative

[24] Viewed together, Propositions 7 and 8 might suggest discontinuous behavior in the set of stalled equilibria at $\mu = 1$. However, this is not the case. As μ converges to 1, the size of the interval of $\widetilde{H}$ values for which a stalled equilibrium exists converges to 0.

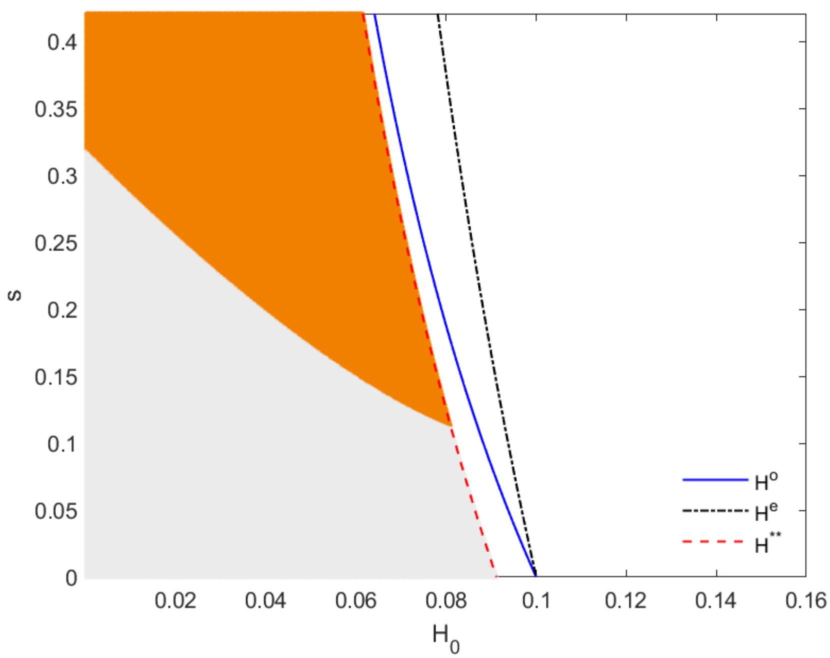

Figure 1: Gradual Equilibrium versus Free Entry

externality s and set the remaining parameters equal to the following values

Parameter	$\bar{\theta}$	β	μ	C	S	$\underline{\pi}$	$\frac{\overline{\pi}-\underline{\pi}}{L}$
Value	1	.95	.95	17	0	0	0.5

There is nothing special about these particular values. A similar picture emerges under all the parameter choices we have studied.

Figure 1 compares welfare in the equilibrium with gradual development to that arising under free entry. The community's initial housing stock H_0 is measured on the horizontal axis and the externality s is measured on the vertical. The blue (solid), black (dotted), and green (dashed) lines depict, respectively, the housing levels H^o, H^e, and H^{**} associated with any given externality level s. Notice that the divergence between H^o and H^e increases with s, while the divergence between H^o and H^{**} decreases. For the gradual equilibrium to exist, we require that the initial housing stock H_0 is smaller than H^{**}. The (H_0, s) pairs for which the comparison can be made therefore lie to the left of the green (dashed) line. The orange (dark) shaded area represents (H_0, s) pairs for

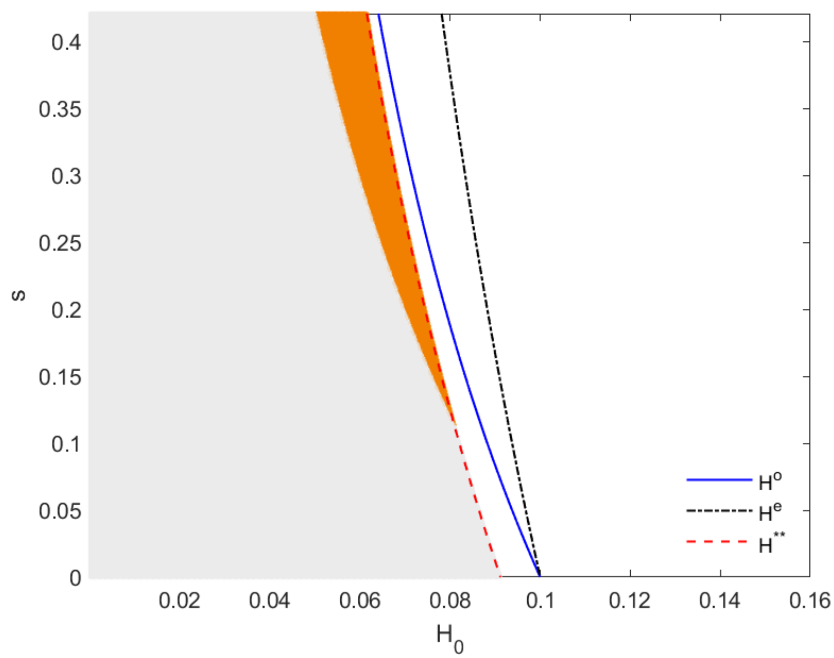

Figure 2: Worst Stalled Equilibrium versus Free Entry

which welfare is higher in the equilibrium with gradual development and the gray (light) shaded area represents (H_0, s) pairs for which welfare is higher under free entry. As expected, welfare is higher with free entry when the externality is small but higher in the gradual equilibrium when the externality is large. The line separating the two regions is decreasing in H_0. This reflects the fact, while a smaller initial housing stock does not alter the long run size of the community, it does reduce the size of the community along the transition. This increases the welfare loss arising from delayed development.

Turning to the stalled equilibrium, there are multiple equilibria of this form each characterized by an associated steady state housing level. Like the commitment solution, these equilibria have the same timing as the free entry equilibrium in the sense that all new construction occurs in the initial period. Thus, for a welfare analysis, we just need to compare the amount of new construction in the two solutions. Obviously, the results will depend on which equilibrium is considered. Equilibria in which development stalls after a larger burst of new construction in the initial period will perform better. For our current purposes, we are interested in worst case

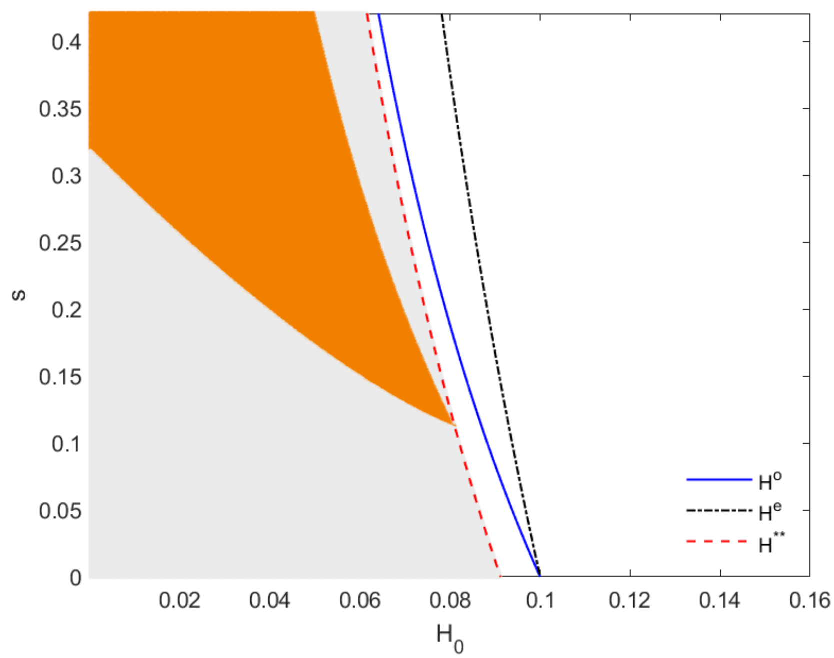

Figure 3: Region of Ambiguous Effect of Corrective Taxation

scenarios. Thus, we consider the stalled equilibrium with the smallest steady state housing level $\widetilde{H}$.

Figure 2 compares welfare in the worst stalled equilibrium to that arising under free entry. For a stalled equilibrium to exist, we again require that the initial housing stock H_0 is smaller than H^{**}. The (H_0, s) pairs for which the comparison can be made are therefore exactly as in Figure 1. The orange (dark) shaded area represents (H_0, s) pairs for which welfare is higher in the worst stalled equilibrium and the gray (light) shaded area represents (H_0, s) pairs for which welfare is higher under free entry. The key point to note is that even when the externality is large, free entry dominates, unless H_0 is very close to H^{**}. Since $\widetilde{H}$ must exceed H_0, the latter condition forces the steady state housing level in the worst stalled equilibrium to also be very close to H^{**}.

Pulling together the information from Figures 1 and 2, we see that, for a large range of (H_0, s) pairs, there exists an equilibrium - the worst stalled equilibrium - which is worse than free entry and an equilibrium - the one with gradual development - that is better. The orange (dark) shaded area in Figure 3 illustrates this set of points. For this set, the answer to the question "does allowing residents to tax development increase welfare?" is "maybe but maybe not". All that can be said with confidence is that allowing residents to tax development will reduce welfare when externalities are small.[25]

8 A development tax versus the alternatives

This section compares the development paths that arise with corrective taxation to those that emerge when the community is authorized to control development with zoning and when it can levy impact fees.

8.1 Zoning

The simplest type of growth-control policy is zoning. With zoning, the use of undeveloped land is controlled by regulating whether or not it can be developed.[26] In principle, it is possible to achieve the same level of development with either taxes or zoning. However, zoning creates rents for landowners whose land is zoned for building, rather than revenue for the community. When residents control policy, this difference matters. When choosing taxes, residents weigh the benefits of the revenues that would come from allowing more development with the costs associated with the externality and the negative impact on the value of their homes. With zoning, there is no equivalent benefit from relaxing zoning and hence no incentive to allow any development at all.[27]

Equilibrium with zoning simply involves the initial residents implementing a zoning ordinance preventing building on all undeveloped land. This policy would be maintained by all future cohorts of residents. This coincides with what happens with taxes only when equilibrium involves residents choosing a tax high enough to choke off all development (Proposition 5). Clearly, outside of this case, zoning will be even worse for welfare than taxing development. This is the case whichever equilibrium emerges with a development tax.

Development under zoning could look more similar to that with corrective taxation if both

[25] Given that free entry generates the optimal development plan when there are no externalities, this is not a surprising finding. However, it does underscore the point that, while the distortions associated with free entry vanish as externalities become small, those arising from residents choosing corrective taxes do not.

[26] Zoning also commonly regulates the size of homes that can be constructed. For political economy analyses of this type of zoning see Barseghyan and Coate (2016), Calabrese, Epple, and Romano (2007), Fernandez and Rogerson (1997), and Hamilton (1975).

[27] The point that policies, such as impact fees, that allow localities to extract resources from developers may encourage more development than zoning has been made by many authors (see, for example, Altshuler and Gomez-Ibanez 1993, Burge 2010, Burge and Ihlanfeldt 2006, and Gyourko 1991).

residents and landowners have input into the policy-making process. Suppose, in the spirit of Glaeser, Gyourko and Saks (2005a) and Hilber and Robert-Nicoud (2013), that policy was set by a community planning board that weighed the interests of both homeowners and owners of undeveloped land. In this case, we conjecture that similar results to those described in this paper might emerge with zoning. If this is the case, then the different equilibria that arise could be interpreted as different predictions of what might happen when land-use is regulated by zoning.[28]

This is something that would be useful to clarify in future work. It is by no means an easy extension, since the objective function of such a planning board is considerably more complicated than that of the residents.[29]

Of course, for a gradual equilibrium to emerge under zoning, the community planning board would have to continually change zoning plans by allowing more and more land to be developed. It would be interesting to know whether such incremental change in zoning plans is observed in practice. If zoning plans are difficult to change, this may enable the initial residents and landowners to lock in their development plans. Accordingly, outcomes with zoning and a community planning board might look more like those associated with the commitment solution than with sequential policy-making. Again, this is something that would be useful to clarify in future work.

8.2 Impact fees

The key feature of impact fees is that they can only be used to finance expenditures "attributable" to the development in question. An expenditure is attributable to the development if it is to be used by the new residents (for example, new public infrastructure) or to mitigate adverse impacts of the development on the community (Altshuler and Gomez-Ibanez 1993). Our model captures externalities in a reduced form manner and is not rich enough to consider the additional infrastructure requirements created by development or how externalities may be mitigated.[30]

[28] In particular, similar communities might end up with more or less restrictive zoning plans. The amount of undeveloped land and the price of housing will be higher in the more restrictive communities. The price of building land and aggregate welfare will be lower. This would be consistent with Turner et al (2014) who find large negative effects of land-use regulation on both the value of land and welfare.

[29] As discussed in Hilber and Robert-Nicoud (2013), it is natural to think of political power shifting over time from owners of undeveloped land to homeowners as the relative sizes of the two groups diverge over time as undeveloped land is converted to homes. With such a political model, it may be possible to explore the dynamic vision of city development advanced by Molotch (1976). He argues that initially the political process is captured by landowners who favor development, but then as the community matures, residents gain the upper hand and limit development because of the negative externalities it creates.

[30] Brueckner (1997) provides an interesting analysis of impact fees in a model that captures the additional infrastructure requirements created by development.

Nonetheless, the spirit of the requirement may be captured by supposing that the fee imposed on a developer building a new house should not exceed the discounted present value of the externalities imposed on other households by the construction of that house. After all, such a fee would be sufficient to fully compensate (which is a form of mitigation) each household in the community for the negative externality. Formally, this would mean that, in any period t, the tax τ_t must be less than $sH_{t+1}/(1-\beta)$. Under such a restriction, equilibrium involves residents setting the fee equal to $sH^o/(1-\beta)$ in the initial period and keeping it constant thereafter. All development would occur in the initial period and the size of the community would be H^o. This is the optimal community development plan described in Proposition 1.

Of course, this represents a best case scenario in which the impact fees developers are charged perfectly reflect the adverse impacts they impose on the community. In practice, fees could be systematically set at levels greater or less than this amount.[31] In the former case, impact fees still have the edge over a development tax provided that the legal rules that govern their use somewhat constrain their levels. In the latter case, long run welfare could be higher with a development tax than with impact fees if the gradual equilibrium emerges. This said, if impact fees are systematically set too low, this suggests that policy reflects more than the interests of the residents.

9 Conclusion

The textbook remedy for the negative externalities from development is a corrective tax. This paper investigates how a community might develop if it could levy such a tax. It assumes that taxes are chosen in each period by resident homeowners who choose policy to maximize their future payoffs. In equilibrium, residents' policy choices reflect not only their desire to counter the negative externalities from development but also their wish to raise tax revenues and the value of their homes.

The paper shows that there exists an equilibrium in which the community gradually develops. This development is fueled by a continual reduction in taxes over time. This reduction is driven by the desire of each cohort of residents to raise additional revenues. Lowering taxes a little encourages more development, which generates a little more revenue. Residents do not take into

[31] Burge (2010) suggests that, in the U.S., localities generally set impact fees too low.

account that their actions lower the revenue that earlier cohorts may raise. The size of the community in this equilibrium converges to a steady state which, while smaller than the socially optimal level, approaches it when residents are unlikely to leave the community. This means that the long run level of the corrective tax can be close to the Pigouvian level.

Complicating the picture, the paper also shows there exist equilibria with stalled development. In these equilibria, development takes place only in the initial period and then stops. Development is stalled by the rational fear of residents that further development will result in a precipitous fall in the price of housing. The extent of development in these equilibria can be much less than optimal, implying that the steady state level of the corrective tax is much too high.

This multiplicity of equilibria, makes it difficult to answer the basic question of whether it would be desirable to authorize communities to tax development. When externalities are large, granting a community the power to tax can improve welfare relative to free entry if the equilibrium with gradual development arises. However, it can have the opposite effect if a stalled equilibrium arises. All that can be said, is that, when externalities are small, free entry dominates whichever equilibrium arises.

These findings suggest a weak case for allowing communities to tax development to tackle negative externalities. It is true that, if the alternative to taxing development is not free entry but zoning, taxing is preferable. This is because the revenues that it brings communities may encourage residents to allow more development than with zoning. However, impact fees also provide a revenue incentive, but have the additional feature that the fee cannot exceed the cost the development is imposing on the community. This prevents impact fees from being used by existing residents to extract surplus from landowners and future residents. In this way, impact fees guarantee a welfare gain over free entry. The bottom line conclusion therefore is that communities should be allowed to levy impact fees on developers rather than to tax development.

References

Albouy, D., K. Behrens, F. Robert-Nicoud and N. Seegert, (2019), "The Optimal Distribution of Population Across Cities," *Journal of Urban Economics*, 110, 102-113.

Altshuler, A. and J. Gomez-Ibanez, (1993), *Regulation for Revenue: The Political Economy of Land Use Exactions*, Brookings Institution: Washington, DC and Lincoln Institute of Land Policy: Cambridge, MA.

Azzimonti, M., (2011), "Barriers to Investment in Polarized Societies," *American Economic Review*, 101, 2182-2204.

Barseghyan, L. and S. Coate, (2016), "Property Taxation, Zoning, and Efficiency in a Dynamic Tiebout Model," *American Economic Journal: Economic Policy*, 8, 1-38.

Barseghyan, L. and S. Coate, (2021), "Community Development by Public Wealth Accumulation," *Journal of Urban Economics*, 121, 103297.

Battaglini, M. and S. Coate, (2008), "A Dynamic Theory of Public Spending, Taxation, and Debt," *American Economic Review*, 98, 201-236.

Besley, T. and S. Coate, (2003), "On the Public Choice Critique of Welfare Economics," *Public Choice*, 114, 253-273.

Board, S., (2008), "Durable-Goods Monopoly with Varying Demand," *Review of Economic Studies*, 75, 391-413.

Bowen, R., Y. Chen and H. Eraslan, (2015), "Mandatory versus Discretionary Spending: the Status Quo Effect," *American Economic Review*, 104, 2941-2974.

Brinkman, J., D. Coen-Pirani and H. Sieg, (2018), "The Political Economy of Municipal Pension Funding," *American Economic Journal: Macroeconomics*, 10, 215-246.

Brueckner, J., (1995), "Strategic Control of Growth in a System of Cities," *Journal of Public Economics*, 57, 393-416.

Brueckner, J., (1997), "Infrastructure Financing and Urban Development: The Economics of Impact Fees," *Journal of Public Economics*, 66, 383-407.

Brueckner, J. and F-C. Lai, (1996), "Urban Growth Controls with Resident Landowners," *Regional Science and Urban Economics*, 26, 125-143.

Buchanan, J., (1962), "Politics, Policy, and the Pigouvian Margins," *Economica*, 29, 17-28.

Buchanan, J. and V. Vanberg, (1988), "The Politicization of Market Failure," *Public Choice*, 57, 101-113.

Bulow, J., (1982), "Durable-Goods Monopolists," *Journal of Political Economy*, 90, 314-332.

Burge, G., (2010), "The Effects of Development Impact Fees on Local Fiscal Conditions," in G. Ingram and Y-H Hong (eds), *Municipal Revenues and Land Policies*, Lincoln Institute of Land Policy: Cambridge, MA.

Burge, G. and K. Ihlanfeldt, (2006), "Impact Fees and Single-Family Home Construction," *Journal of Urban Economics*, 60, 284-306.

Calabrese, S., D. Epple and R. Romano, (2007), "On the Political Economy of Zoning," *Journal of Public Economics*, 91, 25-49.

Coase, R., (1972), "Durability and Monopoly," *Journal of Law and Economics*, 15, 143-149.

Coate, S. and S. Morris, (1999), "Policy Persistence," *American Economic Review*, 89, 1327-1336.

Duranton, G. and D. Puga, (2019), "Urban Growth and its Aggregate Implications," NBER Working Paper #26591.

Fernandez, R. and R. Rogerson, (1997), "Keeping People Out: Income Distribution, Zoning, and the Quality of Public Education," *International Economic Review*, 38, 23-42.

Finkelshtain, I. and Y. Kislev, (1997), "Prices versus Quantities: The Political Perspective," *Journal of Political Economy*, 105, 83-100.

Fischel, W., (2001), *The Homevoter Hypothesis: How Home Values Influence Local Government Taxation, School Finance and Land-Use Policies*, Harvard University Press: Cambridge, MA.

Glaeser, E., J. Gyourko and R. Saks, (2005a), "Why Have Housing Prices Gone Up?" *American Economic Review*, 95, 329-333.

Glaeser, E., J. Gyourko and R. Saks, (2005b), "Why is Manhattan so Expensive? Regulation and the Rise of Housing Prices," *Journal of Law and Economics*, 48, 331-369.

Gul, F., H. Sonnenschein and R. Wilson, (1986), "Foundations of Dynamic Monopoly and the Coase Conjecture," *Journal of Economic Theory*, 39, 155-190.

Gyourko, J., (1991), "Impact Fees, Exclusionary Zoning, and the Density of New Development," *Journal of Urban Economics*, 30, 242-256.

Gyourko, J. and R. Molloy, (2015), "Regulation and Housing Supply," in G. Duranton, J.V. Henderson and W. Strange (eds), *Handbook of Regional and Urban Economics Volume 5*, North Holland: Amsterdam.

Hamilton, B., (1975), "Zoning and Property Taxation in a System of Local Governments," *Urban Studies*, 12, 205-211.

Hassler, J., J. Rodriguez Mora, K. Storesletten and F. Zilibotti, (2003) "The Survival of the Welfare State," *American Economic Review*, 93, 87-112.

Helsley, R. and W. Strange, (1995), "Strategic Growth Controls," *Regional Science and Urban Economics*, 25, 435-460.

Hilber, C. and F. Robert-Nicoud, (2013), "On the Origins of Land Use Regulations: Theory and Evidence from US Metro Areas," *Journal of Urban Economics*, 75, 29-43.

Krusell, P., V. Quadrini and J-V Rios-Rull, (1996), "Are Consumption Taxes Really Better than Income Taxes?" *Journal of Monetary Economics*, 37, 475-503.

Krusell, P. and J-V Rios-Rull, (1999), "On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model," *American Economic Review*, 89, 1156-1181.

Molotch, H., (1976), "The City as a Growth Machine: Toward a Political Economy of Place," *American Journal of Sociology*, 82(2), 309-332.

Ortalo-Magne, F. and A. Prat, (2014), "On the Political Economy of Urban Growth: Homeownership versus Affordability," *American Economic Journal: Microeconomics*, 6, 154-181.

Rodrik, D., (1986), "Tariffs, Subsidies, and Welfare with Endogenous Policy," *Journal of International Economics*, 21, 285-299.

Rosen, K. and L. Katz, (1981), "Growth Management and Land Use Controls: The San Francisco Bay Area Experience," *Journal of American Real Estate and Urban Economics Association*, 9, 321-343.

Rosenberg, R., (2006), "The Changing Culture of American Land Use Regulation: Paying for Growth with Impact Fees," *SMU Law Review*, 59, 177-263.

Sobel, J., (1991), "Durable Goods Monopoly with Entry of New Consumers," *Econometrica*, 59, 1455-1485.

Stokey, N., (1979), "Intertemporal Price Discrimination," *Quarterly Journal of Economics*, 93, 355-371.

Stokey, N., (1981), "Rational Expectations and Durable Goods Pricing," *The Bell Journal of Economics*, 12, 112-128.

Tiebout, C., (1956), "A Pure Theory of Local Public Expenditures," *Journal of Political Economy*, 64, 416-424.

Turner, M., A. Haughwout and W. van der Klaauw, (2014), "Land Use Regulation and Welfare," *Econometrica,* 82(4), 1341-1403.

10 Appendix

10.1 Proof of Proposition 3

The initial residents' problem is

$$\max_{\{H_{t+1},P_t,R_t\}_{t=0}^{\infty}} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} (\mu\beta)^t \left\{(1-\mu)P_t + \mu \left[S(H_{t+1}) + \frac{(P_t-C-R_t)(H_{t+1}-H_t)}{H_{t+1}}\right]\right\} \\ s.t. \text{ for all } t \; P_t = (1-H_{t+1})\overline{\theta} + S(H_{t+1}) + \frac{(P_t-C-R_t)(H_{t+1}-H_t)}{H_{t+1}} + \beta P_{t+1} \\ H_t \leq H_{t+1} \;\&\; R_t = \pi(H_{t+1}) + \beta R_{t+1} \end{array} \right\}.$$

We also add the transversality condition that $\lim_{t\to\infty} \beta^t P_t = 0$. We approach this problem through a process of successive simplification. Our first simplifying observation concerns the objective function.

Fact 1. *Suppose that the sequence of policies $\{H_{t+1}, P_t, R_t\}_{t=0}^{\infty}$ satisfies in each period t the market equilibrium condition*

$$P_t = (1-H_{t+1})\overline{\theta} + S(H_{t+1}) + \frac{(P_t-C-R_t)(H_{t+1}-H_t)}{H_{t+1}} + \beta P_{t+1} \tag{30}$$

and the transversality condition that $\lim_{t\to\infty} \beta^t P_t = 0$. Then, the initial residents' objective function satisfies

$$\begin{array}{c} \sum_{t=0}^{\infty} (\mu\beta)^t \left\{(1-\mu)P_t + \mu \left[S(H_{t+1}) + \frac{(P_t-C-R_t)(H_{t+1}-H_t)}{H_{t+1}}\right]\right\} \\ = P_0 - \sum_{t=0}^{\infty} (\mu\beta)^t \mu(1-H_{t+1})\overline{\theta}. \end{array} \tag{31}$$

Proof of Fact 1. From the market equilibrium condition, we have that

$$S(H_{t+1}) + \frac{(P_t-C-R_t)(H_{t+1}-H_t)}{H_{t+1}} = P_t - (1-H_{t+1})\overline{\theta} - \beta P_{t+1}.$$

Thus, for any period $t \geq 0$, we have that

$$\begin{aligned} & \sum_{z=0}^{t} (\mu\beta)^z \left\{(1-\mu)P_z + \mu \left[S(H_{z+1}) + \frac{(P_z-C-R_z)(H_{z+1}-H_z)}{H_{z+1}}\right]\right\} \\ = & \sum_{z=0}^{t} (\mu\beta)^z \left\{(1-\mu)P_z + \mu \left[P_z - (1-H_{z+1})\overline{\theta} - \beta P_{z+1}\right]\right\}. \end{aligned}$$

Expanding the right hand side, we have that

$$\begin{aligned}
& \sum_{z=0}^{t} (\mu\beta)^z \left\{(1-\mu)P_z + \mu\left[P_z - (1-H_{z+1})\overline{\theta} - \beta P_{z+1}\right]\right\} \\
= \; & (1-\mu)P_0 + \mu\left[P_0 - (1-H_1)\overline{\theta} - \beta P_1\right] \\
& +\mu\beta\left\{(1-\mu)P_1 + \mu\left[P_1 - (1-H_2)\overline{\theta} - \beta P_2\right]\right\} \\
& +(\mu\beta)^2\left\{(1-\mu)P_2 + \mu\left[P_2 - (1-H_3)\overline{\theta} - \beta P_3\right]\right\} \\
& +..... + (\mu\beta)^t\left\{(1-\mu)P_t + \mu\left[P_t - (1-H_{t+1})\overline{\theta} - \beta P_{t+1}\right]\right\}.
\end{aligned}$$

Note that the P_1 term in the first line on the right hand side of the equality cancels with that in the second line. Similarly, the P_2 term in the second line cancels with the P_2 term in the third line, etc. Thus, we have

$$\begin{aligned}
& \sum_{z=0}^{t} (\mu\beta)^z \left\{(1-\mu)P_z + \mu\left[P_z - (1-H_{z+1})\overline{\theta} - \beta P_{z+1}\right]\right\} \\
= \; & P_0 - \sum_{z=0}^{t} (\mu\beta)^z \mu(1-H_{z+1})\overline{\theta} - (\mu\beta)^{t+1} P_{t+1}.
\end{aligned}$$

The transversality condition implies that $\lim_{t\to\infty} (\mu\beta)^{t+1} P_{t+1} = 0$. Thus,

$$\begin{aligned}
& \sum_{t=0}^{\infty} (\mu\beta)^t \left\{(1-\mu)P_t + \mu\left[P_t - (1-H_{t+1})\overline{\theta} - \beta P_{t+1}\right]\right\} \\
= \; & P_0 - \sum_{t=0}^{\infty} (\mu\beta)^t \mu(1-H_{t+1})\overline{\theta}.
\end{aligned}$$

■

This result reveals that the initial residents care about the period 0 housing price and a term that is weighted by μ and depends negatively on the discounted sum of the preferences of the marginal households in each period. We can therefore write the problem as:

$$\max_{\{H_{t+1},P_t,R_t\}_{t=0}^{\infty}} \left\{ \begin{array}{c} P_0 - \sum_{t=0}^{\infty} (\mu\beta)^t \mu(1-H_{t+1})\overline{\theta} \\ s.t. \text{ for all } t \geq 0 \\ P_t = (1-H_{t+1})\overline{\theta} + S(H_{t+1}) + \frac{(P_t - C - R_t)(H_{t+1}-H_t)}{H_{t+1}} + \beta P_{t+1} \\ H_t \leq H_{t+1} \ \& \ R_t = \pi(H_{t+1}) + \beta R_{t+1} \end{array} \right\}. \tag{32}$$

Our next simplifying result provides a convenient expression for the price P_0.

Fact 2. *Suppose that the sequence of policies* $\{H_{t+1}, P_t, R_t\}_{t=0}^{\infty}$ *satisfies in each period* t *the market equilibrium condition (30) and the transversality condition that* $\lim_{t\to\infty} \beta^t P_t = 0$*. Then,*

$$P_0 = \sum_{t=0}^{\infty} \beta^t \left[(1 - H_{t+1})\overline{\theta} + S(H_{t+1})\right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_{t+1} - H_0)\,\mathcal{B}(H_{t+1})}{H_0}. \tag{33}$$

Proof of Fact 2. Note that P_t appears on the left and right hand side of the market equilibrium constraint (30). Solving the constraint for P_t reveals that

$$P_t = C + R_t + \frac{H_{t+1}\mathcal{B}(H_{t+1})}{H_t} + \beta \frac{H_{t+1}}{H_t} (P_{t+1} - R_{t+1} - C)$$

Thus,

$$P_t - R_t - C = \frac{H_{t+1}\mathcal{B}(H_{t+1})}{H_t} + \beta \frac{H_{t+1}\left[P_{t+1} - R_{t+1} - C\right]}{H_t}$$

It follows that if all the future market equilibrium constraints are satisfied, it must be the case that

$$\begin{aligned} P_0 - R_0 - C &= \frac{H_1\mathcal{B}(H_1)}{H_0} + \beta \frac{H_1\left[P_1 - R_0 - C\right]}{H_0} \\ &= \frac{H_1\mathcal{B}(H_1)}{H_0} + \beta \frac{H_2\mathcal{B}(H_2)}{H_0} + \beta^2 \frac{H_2\left[P_2 - R_2 - C\right]}{H_0}. \end{aligned}$$

Generalizing this logic, for all $t \geq 1$ we can write:

$$P_0 = R_0 + C + \sum_{z=0}^{t-1} \beta^z \frac{H_{z+1}\mathcal{B}(H_{z+1})}{H_0} + \beta^t \frac{H_t\left[P_t - R_t - C\right]}{H_0}.$$

The transversality condition implies that $\lim_{t\to\infty} \beta^t H_t\left[P_t - R_t - C\right] = 0$ and thus, we have that

$$P_0 = R_0 + C + \sum_{t=0}^{\infty} \beta^t \frac{H_{t+1}\mathcal{B}(H_{t+1})}{H_0}.$$

In addition, we have that

$$R_0 = \pi(H_1) + \beta R_1 = \sum_{t=0}^{\infty} \beta^t \pi(H_{t+1}).$$

Thus,

$$P_0 = \sum_{t=0}^{\infty} \beta^t \pi(H_{t+1}) + C + \sum_{t=0}^{\infty} \beta^t \frac{H_{t+1}\mathcal{B}(H_{t+1})}{H_0}.$$

Using the definition of $\mathcal{B}(H)$, we can write this as:

$$P_0 = \sum_{t=0}^{\infty} \beta^t \left[(1 - H_{t+1})\overline{\theta} + S(H_{t+1})\right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_{t+1} - H_0)\,\mathcal{B}(H_{t+1})}{H_0}.$$

■

Thus, the period 0 housing price depends on two terms. The first is the discounted sum of the surplus the marginal household receives from living in the community ignoring the cost of housing and tax revenues. The second term can be shown to equal the discounted sum of per-capita tax revenues. Using Fact 2, we can write the problem as

$$\max_{\{H_{t+1}\}_{t=0}^{\infty}} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \beta^t \left[(1-H_{t+1})\overline{\theta} + S(H_{t+1})\right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_{t+1}-H_0)\mathcal{B}(H_{t+1})}{H_0} \\ -\sum_{t=0}^{\infty} (\mu\beta)^t \mu(1-H_{t+1})\overline{\theta} \\ s.t.\ H_{t+1} \geq H_t \text{ for all } t \geq 0 \end{array} \right\}. \quad (34)$$

This substantially simplifies matters, since the choice variables are just the sequence of housing levels $\{H_{t+1}\}_{t=0}^{\infty}$. Note also that when $\mu = 1$, the objective function simplifies to

$$\sum_{t=0}^{\infty} \beta^t S(H_{t+1}) + \sum_{t=0}^{\infty} \beta^t \frac{(H_{t+1}-H_0)\,\mathcal{B}(H_{t+1})}{H_0}$$

This is what we mean in the text by the "utility from living in the community". It consists of the discounted sum of the stream of surpluses $\{S(H_{t+1})\}_{t=0}^{\infty}$ along with the discounted sum of the stream of per-capita tax revenues. By contrast, when $\mu = 0$, the objective function simplifies to

$$\sum_{t=0}^{\infty} \beta^t \left[(1-H_{t+1})\overline{\theta} + S(H_{t+1})\right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_{t+1}-H_0)\,\mathcal{B}(H_{t+1})}{H_0}$$

which is just the period 0 housing price. Note that this price depends not only on the utility from living in the community but also on the discounted sum of the preferences of the marginal households. The consequence of this is that a marginal increase in H_{t+1} leads to a greater reduction in the period 0 housing price than it does in the utility from living in the community. This is what we mean in the text by development necessitating going down the demand curve in the sense of drawing households with lower preference θ into the community.

Our next simplifying result shows that all new construction occurs in the initial period.

Fact 3 *Let* $\{H_{t+1}\}_{t=0}^{\infty}$ *solve the initial residents' problem. Then, for all* $t \geq 1$

$$H_{t+1} = H_1.$$

Proof of Fact 3 Suppose the contrary. Then there exist some period $t \geq 1$ such that $H_{t+1} > H_t$. It follows that we can increase the value of H_t marginally without violating the constraints. The change in the objective function is

$$\beta^{t-1}\left[-\left(\overline{\theta}+s\right) + \frac{(H_t - H_0)\,\mathcal{B}'(H_t) + \mathcal{B}(H_t)}{H_0}\right] + \mu\overline{\theta}\,(\mu\beta)^{t-1}.$$

It follows that

$$-\left(\overline{\theta}+s\right)+\frac{\left(H_t-H_0\right)\mathcal{B}'(H_t)+\mathcal{B}(H_t)}{H_0}+\mu^t\overline{\theta}\leq 0.$$

But we can also reduce the value of H_{t+1} marginally without violating the constraints. The change in the objective function resulting from such a change is

$$-\left(\beta^t\left[-\left(\overline{\theta}+s\right)+\frac{\left(H_{t+1}-H_0\right)\mathcal{B}'(H_{t+1})+\mathcal{B}(H_{t+1})}{H_0}\right]+\mu\overline{\theta}\left(\mu\beta\right)^t\right).$$

It follows that

$$-\left(\overline{\theta}+s\right)+\frac{\left(H_{t+1}-H_0\right)\mathcal{B}'(H_{t+1})+\mathcal{B}(H_{t+1})}{H_0}+\mu^{t+1}\overline{\theta}\geq 0.$$

Combining these two inequalities, we find that

$$\left(H_{t+1}-H_0\right)\mathcal{B}'(H_{t+1})+\mathcal{B}(H_{t+1})\geq\left(H_t-H_0\right)\mathcal{B}'(H_t)+\mathcal{B}(H_t).$$

However, we have that

$$\begin{aligned}\frac{d\left(\left(H-H_0\right)\mathcal{B}'(H)+\mathcal{B}(H)\right)}{dH} &= \left(H-H_0\right)\mathcal{B}''(H)+2\mathcal{B}'(H)\\ &= -2\left(\overline{\theta}+s+\frac{\overline{\pi}-\pi}{L}\right)<0.\end{aligned}$$

Since $H_{t+1}>H_t$, this implies that

$$\left(H_{t+1}-H_0\right)\mathcal{B}'(H_{t+1})+\mathcal{B}(H_{t+1})<\left(H_t-H_0\right)\mathcal{B}'(H_t)+\mathcal{B}(H_t),$$

which is a contradiction. ■

This result allows us to reduce the initial residents' problem to the following very simple problem involving only one choice variable

$$\max_{\{H_1\}}\left\{\begin{array}{c}\frac{(1-H_1)\overline{\theta}+S(H_1)}{1-\beta}+\frac{(H_1-H_0)\mathcal{B}(H_1)}{H_0(1-\beta)}-\frac{\mu\overline{\theta}(1-H_1)}{1-\mu\beta}\\ s.t.\ H_1\geq H_0\end{array}\right\}. \tag{35}$$

The first two terms of the objective function constitute the period 0 price. The first term is the discounted surplus from living in the community of the marginal household (ignoring any sharing of tax revenues) and the second term represents the per-capita tax revenues. Let H^* denote the housing level defined in (16) the text and let $\mathcal{H}(H)$ be the function defined in (17). Then, we have the following result.

Fact 4 *The optimal level of housing for the initial residents is H_0 if $H_0\geq H^*$. Otherwise, it is equal to $\mathcal{H}(H_0)$.*

Proof of Fact 4 The first derivative of the objective function in problem (35) is as follows:

$$-\frac{\left(\overline{\theta}+s\right)}{1-\beta}+\frac{\left(H_1-H_0\right)\mathcal{B}'(H_1)+\mathcal{B}(H_1)}{(1-\beta)H_0}+\frac{\mu}{1-\mu\beta}\overline{\theta}$$

The term $\frac{(H_1-H_0)\mathcal{B}'(H_1)+\mathcal{B}(H_1)}{(1-\beta)H_0}$ measures the change in per-capita tax revenue resulting from an increase in development or the *marginal tax revenue* from development. It equals

$$\frac{(1-H_1)\overline{\theta}+S(H_1)-C(1-\beta)-\pi(H_1)-\left(\overline{\theta}+s\right)\left(H_1-H_0\right)-\pi(H_1)}{(1-\beta)H_0}.$$

The second derivative of the objective function is

$$\frac{d\left(\frac{(H_1-H_0)\mathcal{B}'(H_1)+\mathcal{B}(H_1)}{(1-\beta)H_0}\right)}{dH_1}=-\frac{2\left(\overline{\theta}+s+\frac{\overline{\pi}-\underline{\pi}}{L}\right)}{H_0(1-\beta)}<0,$$

implying that the objective function is strictly concave. It follows that if

$$\frac{\mathcal{B}(H_0)}{(1-\beta)H_0}\leq\frac{\overline{\theta}+s}{1-\beta}-\frac{\mu}{1-\mu\beta}\overline{\theta}$$

then the optimal housing level is H_0. Intuitively, no development is optimal if the marginal tax revenue from development at H_0 is less than the marginal cost of development which is represented by the right hand side. This will be the case if

$$(1-H_0)\overline{\theta}+S(H_0)-C(1-\beta)-\underline{\pi}-(\overline{\pi}-\underline{\pi})\frac{H_0}{L}\leq\left(\overline{\theta}+s\right)H_0-H_0\frac{\mu(1-\beta)}{1-\mu\beta}\overline{\theta}$$

This is equivalent to $H_0\geq H^*$. Note that this inequality cannot hold if $\mu=1$, since in that case $H^*=H^o$ and Assumption 1 implies that $H_0<H^o$. Intuitively, this reflects the fact that marginal tax revenue from development at H_0 (which is $\mathcal{B}(H_0)/(1-\beta)H_0$) exceeds the marginal externality cost of development (which is $s/(1-\beta)$).

If $H_0<H^*$ then the first order condition for the optimal development level is

$$\frac{\left(H_1-H_0\right)\mathcal{B}'(H_1)+\mathcal{B}(H_1)}{(1-\beta)H_0}=\frac{\overline{\theta}+s}{1-\beta}-\frac{\mu}{1-\mu\beta}\overline{\theta}. \tag{36}$$

This says that the marginal tax revenue from development equals the marginal cost of development. Note that the latter is just $\frac{s}{1-\beta}$ when $\mu=1$, reflecting that the only cost of development to the initial residents if they will stay in the community forever is the negative externality. However, when $\mu=0$, the right hand side is equal to $\frac{\overline{\theta}+s}{1-\beta}$. This reflects the cost of the period 0 price declining as a result of development necessitating drawing in households with lower θ. Rearranging the first order condition, we see that the optimal housing level satisfies the equation

$$\mathcal{B}(H_1)+\mathcal{B}'(H_1)\left(H_1-H_0\right)=\left(\overline{\theta}+s\right)H_0-H_0\frac{\mu(1-\beta)}{1-\mu\beta}\overline{\theta}$$

This implies

$$\begin{aligned}
& (1-H_1)\overline{\theta} + S(H_1) - C(1-\beta) - \pi(H_1) - \left(\overline{\theta} + s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)(H_1 - H_0) \\
= \quad & \left(\overline{\theta} + s\right) H_0 - H_0 \frac{\mu(1-\beta)}{1-\mu\beta}\overline{\theta},
\end{aligned}$$

which implies that

$$H_1 = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{2\left(\overline{\theta} + s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)} + \frac{\left(\frac{\overline{\pi}-\underline{\pi}}{L} + \frac{\mu(1-\beta)}{1-\mu\beta}\overline{\theta}\right)}{2\left(\overline{\theta} + s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)} H_0. \tag{37}$$

This means that it equals $\mathcal{H}(H_0)$. ■

We have therefore established that the solution to the initial period residents' problem is as follows. If $H_0 < H^*$, $\mathcal{H}(H_0) - H_0$ new houses are provided in the initial period. Thereafter, no more housing is provided. From (33), the price of housing is

$$\frac{(1-\mathcal{H}(H_0))\overline{\theta} + S(\mathcal{H}(H_0))}{1-\beta} + \frac{(\mathcal{H}(H_0) - H_0)\,\mathcal{B}(\mathcal{H}(H_0))}{H_0(1-\beta)}$$

in the initial period and

$$\frac{(1-\mathcal{H}(H_0))\overline{\theta} + S(\mathcal{H}(H_0))}{1-\beta}$$

thereafter. The tax in the initial period is

$$\begin{aligned}
P_0 - C - R_0 &= \frac{(1-\mathcal{H}(H_0))\overline{\theta} + S(\mathcal{H}(H_0))}{1-\beta} + \frac{(\mathcal{H}(H_0) - H_0)\,\mathcal{B}(\mathcal{H}(H_0))}{H_0(1-\beta)} - C - \frac{\pi(\mathcal{H}(H_0))}{1-\beta} \\
&= \frac{\mathcal{H}(H_0)\mathcal{B}(\mathcal{H}(H_0))}{H_0(1-\beta)}
\end{aligned}$$

and thereafter is

$$P_t - C - R_t = \frac{\mathcal{B}(\mathcal{H}(H_0))}{1-\beta}.$$

If $H_0 \geq H^*$, no new houses are provided. The price of housing in all periods is $\left[(1-H_0)\overline{\theta} + S(H_0)\right]/(1-\beta)$ and the tax is $\mathcal{B}(H_0)/(1-\beta)$.

10.2 Proof of Proposition 6

10.2.1 Simplifying the problem

It is helpful to begin by writing the residents' problem (8) using the H', H notation discussed in Section 3.3.

$$\max_{(H',P,R)} \left\{ \begin{array}{c} (1-\mu)P + \mu\left[S(H') + \frac{(P-C-R)(H'-H)}{H'} + \beta V(H')\right] \\ s.t.\ P = (1-H')\overline{\theta} + S(H') + \frac{(P-C-R)(H'-H)}{H'} + \beta P(H') \\ H' \geq H \ \&\ R = \pi(H') + \beta R(H') \end{array} \right\}. \tag{38}$$

For any candidate housing rule $H'(H)$, define the sequence $\langle H_t(H)\rangle_{t=0}^{\infty}$ inductively as follows: $H_0(H) = H$, $H_t(H) = H'(H_{t-1}(H))$ for all $t \geq 1$. The interpretation is that $H_t(H)$ is the housing level that will prevail in t periods time if H is the housing level selected this period and future residents follow the housing rule $H'(H)$. Using this notation, we can follow the steps used to establish Facts 1 and 2 in the proof of Proposition 3 to rewrite problem (38) as follows:

$$\max_{H'} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \beta^t \left[(1-H_t(H'))\overline{\theta} + S(H_t(H'))\right] + \sum_{t=0}^{\infty} \beta^t \frac{\left(H_t(H')-H\right)\mathcal{B}(H_t(H'))}{H} \\ -\mu\left[\sum_{t=0}^{\infty} (\mu\beta)^t (1-H_t(H'))\overline{\theta}\right] \\ s.t.\ H' \geq H \end{array} \right\}. \tag{39}$$

This is analogous to (34) in the proof of Proposition 3. The interpretation of the objective function is the same. The first two terms constitute the price of housing. The first term is the discounted sum of the surplus the marginal household receives from living in the community ignoring the cost of housing and tax revenues. The second term can be shown to equal the discounted sum of per-capita tax revenues. The third term is weighted by μ and depends negatively on the discounted sum of the preferences of the marginal household in each period. When $\mu = 1$, the objective function simplifies to the utility from living in the community. When $\mu = 0$, the objective function is just the housing price. The difference between problems (34) and (39) is that in the former the residents' directly choose each period's housing level and in the latter the residents choose only this period's housing H' and this indirectly determines future housing levels through its impact on the sequence $\langle H_t(H')\rangle_{t=1}^{\infty}$.

10.2.2 A preliminary result

We begin by considering a relaxed version of problem (39) which ignores the constraint that housing cannot decrease. The following result establishes that a solution to this unconstrained problem can be used to create a solution to problem (39) if the unconstrained solution satisfies certain conditions. The proof can be found in the on-line Appendix.

Lemma 1 *Let $H_u(H)$ be a solution to the problem*

$$\max_{H'}\left\{\begin{array}{c}\sum_{t=0}^{\infty}\beta^t\left[(1-H_t(H'))\overline{\theta}+S(H_t(H'))\right]+\sum_{t=0}^{\infty}\beta^t\frac{\left(H_t(H')-H\right)\mathcal{B}(H_t(H'))}{H}\\ -\mu\left[\sum_{t=0}^{\infty}(\mu\beta)^t(1-H_t(H'))\overline{\theta}\right]\end{array}\right\}. \tag{40}$$

Suppose further that the housing rule $H_u(H)$ is increasing on the interval $[H_0,1]$, that $H_u(H_0)>H_0$, that $H_u(1)<1$, and that there exists a unique housing level $\widetilde{H}\geq H^*$ such that $H_u(\widetilde{H})=\widetilde{H}$. Then, there exists a solution of problem (39) in which

$$H'(H)=\left\{\begin{array}{c}H_u(H)\text{ if }H\in[H_0,\widetilde{H})\\ H\text{ if }H\in[\widetilde{H},1]\end{array}\right.. \tag{41}$$

10.2.3 Solving the unconstrained problem

We now turn to solving the unconstrained problem (40). The residents' optimal choice of H' can be characterized by maximizing with respect to H'. The first order condition is

$$\begin{array}{c}\sum_{t=0}^{\infty}\beta^t\frac{\mathcal{B}(H_t(H'))+\left(H_t(H')-H\right)\mathcal{B}'(H_t(H'))}{H}H_t'(H')\\ =(\overline{\theta}+s)\left(\sum_{t=0}^{\infty}\beta^tH_t'(H')\right)-\mu\overline{\theta}\sum_{t=0}^{\infty}(\mu\beta)^tH_t'(H'),\end{array} \tag{42}$$

which is analogous to (36) in the proof of Proposition 3. Again, this says that the marginal tax revenue from development (the left hand side) equals the marginal cost of development (the right hand side). The difference is that the condition takes into account how this period's development H' indirectly impacts development in all future periods (i.e., via $H_t'(H')$). We show in the on-line Appendix that (42) implies that the solution housing rule $H_u(H)$ satisfies the condition

$$\begin{array}{c}(1-H')\overline{\theta}+S-2sH'-C(1-\beta)-\underline{\pi}-(\frac{\overline{\pi}-\underline{\pi}}{L})H'\\ =\overline{\theta}(H'-\mu H)\left[1+\sum_{t=1}^{\infty}(\mu\beta)^tH_t'(H')\right]+(\frac{\overline{\pi}-\underline{\pi}}{L})(H'-H)\left[1+\sum_{t=1}^{\infty}\beta^tH_t'(H')\right]\end{array}. \tag{43}$$

We next conjecture that the solution housing rule will take a linear form so that $H_u(H) = \xi + \gamma H$ for some $\gamma \in [0, 1)$. Then, condition (43) implies that

$$\begin{aligned}
&\overline{\theta}(H' - \mu H)\left[1 + \sum_{t=1}^{\infty} (\mu\beta)^t H_t'(H')\right] + (\frac{\overline{\pi} - \underline{\pi}}{L})(H' - H)\left[1 + \sum_{t=1}^{\infty} \beta^t H_t'(H')\right] \\
= \quad & \frac{\overline{\theta}(H' - \mu H)}{1 - \mu\beta\gamma} + (\frac{\overline{\pi} - \underline{\pi}}{L})\frac{(H' - H)}{1 - \beta\gamma} \\
= \quad & (1 - H')\overline{\theta} + S - 2sH' - C(1 - \beta) - \underline{\pi} - (\frac{\overline{\pi} - \underline{\pi}}{L})H'.
\end{aligned}$$

Thus

$$\begin{aligned}
& (1 - H')\overline{\theta} + S - 2sH' - C(1 - \beta) - \underline{\pi} - (\frac{\overline{\pi} - \underline{\pi}}{L})H' \\
= \quad & \frac{\overline{\theta}(H' - \mu H)}{1 - \mu\beta\gamma} + (\frac{\overline{\pi} - \underline{\pi}}{L})\frac{(H' - H)}{1 - \beta\gamma},
\end{aligned}$$

which implies that

$$\begin{aligned}
& H'\left[\frac{\overline{\theta}}{1 - \mu\beta\gamma} + \frac{\overline{\pi} - \underline{\pi}}{L(1 - \beta\gamma)} + \frac{\overline{\pi} - \underline{\pi}}{L} + \overline{\theta} + 2s\right] \\
= \quad & \overline{\theta} + S - C(1 - \beta) - \underline{\pi} + H\left[\frac{\overline{\theta}\mu}{1 - \mu\beta\gamma} + \frac{\overline{\pi} - \underline{\pi}}{L(1 - \beta\gamma)}\right].
\end{aligned}$$

This means that

$$H_u(H) = \left(\frac{\overline{\theta} + S - C(1 - \beta) - \underline{\pi}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}\right) + \left(\frac{\frac{\overline{\theta}\mu}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}\right)H, \tag{44}$$

which confirms the conjecture. It must then be the case that (ξ, γ) solves the pair of equations

$$\gamma = \frac{\frac{\overline{\theta}\mu}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}, \tag{45}$$

and

$$\xi = \frac{\overline{\theta} + S - C(1 - \beta) - \underline{\pi}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}. \tag{46}$$

We claim that there exists a unique $\gamma \in (0, 1)$ that solves equation (45). To see this, note that the equation can be rewritten as:

$$\left(\frac{\overline{\pi} - \underline{\pi}}{L} + \overline{\theta} + 2s\right)\gamma = \overline{\theta}\frac{\mu - \gamma}{1 - \mu\beta\gamma} + \frac{\overline{\pi} - \underline{\pi}}{L}\frac{1 - \gamma}{1 - \beta\gamma}.$$

The left hand side is increasing in γ, while the right hand side is decreasing. Hence, there can be at most one solution. Moreover, when $\gamma = 0$ the right hand side exceeds the left hand side, and when $\gamma = 1$, the opposite is true. Thus, there exists a solution in the interval $(0, 1)$.

Note that the steady state associated with the policy function (44) is

$$H^{**} = \xi + \gamma H^{**} \Rightarrow H^{**} = \frac{\xi}{1-\gamma}$$

and we know that

$$\begin{aligned} 1-\gamma &= 1 - \frac{\frac{\overline{\theta}\mu}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)}}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s} \\ &= \frac{\frac{\overline{\theta}(1-\mu)}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}{\frac{\overline{\theta}}{1-\mu\beta\gamma} + \frac{\overline{\pi}-\underline{\pi}}{L(1-\beta\gamma)} + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s}. \end{aligned}$$

Thus

$$H^{**} = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{\overline{\theta}\left(1 + \frac{1-\mu}{1-\mu\beta\gamma}\right) + \frac{\overline{\pi}-\underline{\pi}}{L} + 2s}. \tag{47}$$

10.2.4 The solution to the unconstrained problem satisfies Lemma 1

By Lemma 1, the solution to the unconstrained problem $H_u(H) = \xi + \gamma H$ can be used to create a solution to problem (38) if it satisfies certain conditions. These are that the housing rule $H_u(H)$ is increasing on the interval $[H_0, 1]$, that $H_u(H_0) > H_0$, that $H_u(1) < 1$, and that there exists a unique housing level $\widetilde{H} \geq H^*$ such that $H_u(\widetilde{H}) = \widetilde{H}$. These conditions are satisfied. The intercept of $H_u(H)$ is positive and its slope is less than one. The associated housing level $\widetilde{H}$ is given by

$$\widetilde{H} = \frac{\overline{\theta} + S - C(1-\beta) - \underline{\pi}}{\overline{\theta}\left(1 + \frac{1-\mu}{1-\mu\beta\gamma}\right) + \frac{\overline{\pi}-\underline{\pi}}{L} + \overline{\theta} + 2s} = H^{**}. \tag{48}$$

As noted in the text, $H^{**} \geq H^*$. It follows from Lemma 1 that the housing rule

$$H'(H) = \begin{cases} \xi + \gamma H \text{ if } H \in [H_0, H^{**}) \\ H \text{ if } H \in [H^{**}, 1] \end{cases}. \tag{49}$$

solves problem (39).

10.2.5 The equilibrium

The housing price rule implied by the housing rule (49) is

$$P(H) = \sum_{t=0}^{\infty} \beta^t \left[(1 - H_t(H'(H)))\overline{\theta} + S(H_t(H'(H)))\right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_t(H'(H)) - H)\,\mathcal{B}(H_t(H'(H)))}{H}, \tag{50}$$

and the associated building land price rule is

$$R(H) = \sum_{t=0}^{\infty} \beta^t \pi(H_t(H'(H))). \tag{51}$$

The value function is given by

$$V(H) = P(H) - \mu \left[\sum_{t=0}^{\infty} (\mu\beta)^t (1 - H_t(H'(H)))\overline{\theta}\right]. \tag{52}$$

Together, the functions $\{H'(H), P(H), R(H), V(H)\}$ constitute an equilibrium. These functions are as defined in (24), (23), and (25).

10.2.6 The equilibrium has the claimed properties

The housing rule (49) implies that development takes place in each period and the housing stock converges asymptotically to H^{**}. Turning to the price of housing, it is clear that it converges to $\left[(1 - H^{**})\overline{\theta} + S(H^{**})\right]/(1-\beta)$. To see that it is decreasing, note that

$$\begin{aligned} P'(H) \quad = \quad & -\sum_{t=0}^{\infty} \beta^t \left[\overline{\theta} + s\right] H_t'(H'(H)) \frac{dH'}{dH} \\ & + \sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H'(H))) + (H_t(H'(H)) - H)\,\mathcal{B}'(H_t(H'(H)))}{H} H_t'(H'(H)) \frac{dH'}{dH} \\ & - \sum_{t=0}^{\infty} \beta^t \frac{H_t(H'(H))}{H^2}. \end{aligned}$$

The first order condition (42) implies that

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H'(H))) + (H_t(H'(H)) - H)\,\mathcal{B}'(H_t(H'(H)))}{H} H_t'(H'(H)) \\ & \quad -(\overline{\theta} + s) \left(\sum_{t=0}^{\infty} \beta^t H_t'(H'(H))\right) = -\mu\overline{\theta} \sum_{t=0}^{\infty} (\mu\beta)^t H_t'(H'(H)). \end{aligned}$$

Thus, we have that

$$P'(H) = -\mu\overline{\theta} \sum_{t=0}^{\infty} (\mu\beta)^t H_t'(H'(H)) \frac{dH'}{dH} - \sum_{t=0}^{\infty} \beta^t \frac{H_t(H'(H))}{H^2} < 0. \tag{53}$$

The tax $\tau(H)$ is equal to $P(H) - C - R(H)$. Thus, we have that

$$\tau'(H) = P'(H) - R'(H)$$

$$= -\mu\overline{\theta}\sum_{t=0}^{\infty}(\mu\beta)^t H_t'(H'(H))\frac{dH'}{dH} - \sum_{t=0}^{\infty}\beta^t\frac{H_t(H'(H))}{H^2} - \sum_{t=0}^{\infty}\beta^t\pi'(H_t(H'(H)))H_t'(H'(H))\frac{dH'}{dH} < 0.$$

The tax converges to

$$P(H^{**}) - C - R(H^{**}) = \frac{(1-H^{**})\overline{\theta} + S(H^{**}) - C(1-\beta) - \pi(H^{**})}{1-\beta} = \frac{\mathcal{B}(H^{**})}{1-\beta}.$$

10.3 Proof of Proposition 7

10.3.1 The two conditions

We begin by presenting two conditions that are sufficient for there to exist a stalled equilibrium with a steady state housing level $\widetilde{H}$.

Lemma 2 *Let $\widetilde{H} \in [H_0, H^{**})$. Suppose that $\widetilde{H}$ satisfies the conditions*

$$(1-\widetilde{H})\overline{\theta} + S - 2\left(s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)\widetilde{H} - C(1-\beta) - \underline{\pi} \geq \overline{\theta}(\widetilde{H} - \mu H_0) - \frac{(\overline{\pi}-\underline{\pi})}{L}H_0, \tag{54}$$

and

$$\frac{(1-\widetilde{H})\overline{\theta} + S(\widetilde{H})}{1-\beta} - P(\widetilde{H}) \geq \mu\overline{\theta}\left[\sum_{t=1}^{\infty}(\mu\beta)^{t-1}H_t(\widetilde{H}) - \frac{\widetilde{H}}{1-\mu\beta}\right], \tag{55}$$

where the sequence $\left\langle H_t(\widetilde{H})\right\rangle_{t=1}^{\infty}$ is created using the housing rule $H'(H)$ in (22) and $P(H)$ is the price rule in (23). Then, there exists a solution of problem (38) of the form described in (26)-(29) with steady state housing level $\widetilde{H}$.

Proof of Lemma 2 Let $\widetilde{H} \in [H_0, H^{**})$ satisfy conditions (54) and (55). Then we need to show that $\{H'(H), P(H), R(H), V(H)\}$ as defined in (26), (28), (27), and (29), solve problem (38). Given the argument at the beginning of the proof of Proposition 6, it suffices to show that the housing rule $H'(H)$ defined in (26) solves the problem

$$\max_{H'}\left\{\begin{array}{c}\sum_{t=0}^{\infty}\beta^t\left[(1-H_t(H'))\overline{\theta} + S(H_t(H'))\right] + \sum_{t=0}^{\infty}\beta^t\frac{\left(H_t(H')-H\right)\mathcal{B}(H_t(H'))}{H} \\ -\mu\left[\sum_{t=0}^{\infty}(\mu\beta)^t(1-H_t(H'))\overline{\theta}\right] \\ s.t.\ H' \geq H\end{array}\right\}, \tag{56}$$

where the sequence $\langle H_t(H')\rangle_{t=0}^{\infty}$ as that generated by the housing rule $H'(H)$ defined in (26). We already know that $H'(H)$ is optimal on the interval $(\widetilde{H}, 1]$, so we just need to focus on the interval $[H_0, \widetilde{H}]$.

Consider some $H \in [H_0, \widetilde{H}]$. Then, $H'(H) = \widetilde{H} \geq H$ and the equilibrium payoff is

$$\frac{(1-\widetilde{H})\overline{\theta} + S(\widetilde{H})}{1-\beta} + \frac{(\widetilde{H}-H)}{H(1-\beta)}\mathcal{B}(\widetilde{H}) - \frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta}.$$

There are two types of deviations to consider. The first is to some smaller housing level $H' \in [H, \widetilde{H})$. Given equilibrium play, the consequences of such a choice would be to just delay the increase to $\widetilde{H}$ until the next period. The payoff from such a deviation would therefore be:

$$\begin{aligned}&(1-H')\overline{\theta} + S(H') + \beta\frac{(1-\widetilde{H})\overline{\theta} + S(\widetilde{H})}{1-\beta} + \frac{(H'-H)}{H}\mathcal{B}(H')\\&+\beta\frac{(\widetilde{H}-H)}{H(1-\beta)}\mathcal{B}(\widetilde{H}) - \mu\left[(1-H')\overline{\theta} + \frac{\mu\beta}{1-\mu\beta}(1-\widetilde{H})\overline{\theta}\right].\end{aligned}$$

The derivative of this payoff with respect to H' is

$$-\left(\overline{\theta}+s\right) + \frac{(H'-H)\mathcal{B}'(H') + \mathcal{B}(H')}{H} + \mu\overline{\theta}$$

If this derivative is positive on $[H, \widetilde{H})$, such a deviation cannot be profitable. Recall that $(H'-H)\mathcal{B}'(H') + \mathcal{B}(H')$ is decreasing in H'. Thus, this derivative is positive on $[H, \widetilde{H})$ if

$$-\left(\overline{\theta}+s\right) + \frac{(\widetilde{H}-H)\mathcal{B}'(\widetilde{H}) + \mathcal{B}(\widetilde{H})}{H} + \mu\overline{\theta} \geq 0.$$

We know that

$$(\widetilde{H}-H)\mathcal{B}'(\widetilde{H}) + \mathcal{B}(\widetilde{H}) = (1-\widetilde{H})\overline{\theta} + S - s\widetilde{H} - C(1-\beta) - \pi(\widetilde{H}) - \left(\widetilde{H}-H\right)\left[\overline{\theta} + s + \frac{\overline{\pi}-\underline{\pi}}{L}\right].$$

Thus, this condition amounts to

$$(1-\widetilde{H})\overline{\theta} + S - 2\left(s + \frac{\overline{\pi}-\underline{\pi}}{L}\right)\widetilde{H} - C(1-\beta) - \underline{\pi} \geq \overline{\theta}(\widetilde{H}-\mu H) - \frac{(\overline{\pi}-\underline{\pi})}{L}H.$$

Observe that this condition will be satisfied for any $H \in [H_0, \widetilde{H}]$ if (54) is satisfied. Accordingly, deviation to some smaller housing level $H' \in [H, \widetilde{H})$ is not profitable.

The second type of deviation is to some larger housing level $H' \in (\widetilde{H}, 1)$. First, consider deviations of this form in which $H' \leq H^{**}$. Given the equilibrium play following this deviation, the payoff from it can be written as

$$\begin{aligned}&(1-H')\overline{\theta} + S(H') + \frac{(H'-H)\,\mathcal{B}(H')}{H} + \sum_{t=1}^{\infty}\beta^t\left[(1-H_t(H'))\overline{\theta} + S(H_t(H'))\right]\\&+\sum_{t=1}^{\infty}\beta^t\frac{(H_t(H')-H)\,\mathcal{B}(H_t(H'))}{H} - \mu\left[(1-H')\,\overline{\theta} + \sum_{t=1}^{\infty}(\mu\beta)^t\,(1-H_t(H'))\,\overline{\theta}\right]\end{aligned}$$

Note that while the sequence $\langle H_t(H')\rangle_{t=1}^{\infty}$ in this expression is generated using the housing rule (26) it will equal that generated by the housing rule (22) since the two rules coincide on the interval $(\widetilde{H},1]$. To show that the deviation is not profitable, we therefore need to show that

$$\begin{aligned}
& \frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}+\frac{(\widetilde{H}-H)}{H(1-\beta)}\mathcal{B}(\widetilde{H})-\frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta} \\
\geq \quad & (1-H')\overline{\theta}+S(H')+\frac{(H'-H)\,\mathcal{B}(H')}{H}+\sum_{t=1}^{\infty}\beta^t\left[(1-H_t(H'))\overline{\theta}+S(H_t(H'))\right] \\
& +\sum_{t=1}^{\infty}\beta^t\frac{(H_t(H')-H)\,\mathcal{B}(H_t(H'))}{H}-\mu\left[(1-H')\,\overline{\theta}+\sum_{t=1}^{\infty}(\mu\beta)^t\,(1-H_t(H'))\,\overline{\theta}\right].
\end{aligned}$$

It is useful to observe that this is equivalent to:

$$\begin{aligned}
& \frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}-\frac{\mathcal{B}(\widetilde{H})}{1-\beta}+\frac{\widetilde{H}\mathcal{B}(\widetilde{H})}{H(1-\beta)}-\frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta} \\
\geq \quad & (1-H')\overline{\theta}+S(H')-\mathcal{B}(H')+\frac{H'\mathcal{B}(H')}{H}+\sum_{t=1}^{\infty}\beta^t\left[(1-H_t(H'))\overline{\theta}+S(H_t(H'))-\mathcal{B}(H_t(H'))\right] \\
& +\sum_{t=1}^{\infty}\beta^t\frac{H_t(H')\mathcal{B}(H_t(H'))}{H}-\mu\left[(1-H')\,\overline{\theta}+\sum_{t=1}^{\infty}(\mu\beta)^t\,(1-H_t(H'))\,\overline{\theta}\right].
\end{aligned}$$

This is equivalent to

$$\begin{aligned}
& \frac{1}{H}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H})-H'\mathcal{B}(H')-\sum_{t=1}^{\infty}\beta^t H_t(H')\mathcal{B}(H_t(H'))\right] \\
\geq \quad & \mu\overline{\theta}\left[H'+\sum_{t=1}^{\infty}(\mu\beta)^t\,H_t(H')-\frac{\widetilde{H}}{1-\mu\beta}\right]+\pi(H')+\sum_{t=1}^{\infty}\beta^t\pi(H_t(H'))-\frac{\pi(\widetilde{H})}{1-\beta}.
\end{aligned}$$

Condition (55) implies that

$$\frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}-P(\widetilde{H})\geq\mu\overline{\theta}\left[\sum_{t=1}^{\infty}(\mu\beta)^{t-1}\,H_t(\widetilde{H})-\frac{\widetilde{H}}{1-\mu\beta}\right],$$

or equivalently

$$\frac{\mathcal{B}(\widetilde{H})}{1-\beta}+C+\frac{\pi(\widetilde{H})}{1-\beta}-\frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta}\geq P(\widetilde{H})-\mu\sum_{t=1}^{\infty}(\mu\beta)^{t-1}\left(1-H_t(\widetilde{H})\right)\overline{\theta}=V(\widetilde{H})$$

where $V(H)$ is the value function in (25). Moreover, since $V(H)$ is the value function for problem

(39), we must have that

$$
\begin{aligned}
V(\widetilde{H}) &\geq (1-H')\overline{\theta} + S(H') + \frac{\left(H'-\widetilde{H}\right)\mathcal{B}(H')}{\widetilde{H}} + \sum_{t=1}^{\infty}\beta^t\left[(1-H_t(H'))\overline{\theta} + S(H_t(H'))\right] \\
&\quad + \sum_{t=1}^{\infty}\beta^t\frac{\left(H_t(H')-\widetilde{H}\right)\mathcal{B}(H_t(H'))}{\widetilde{H}} - \mu\left[(1-H')\,\overline{\theta} + \sum_{t=1}^{\infty}(\mu\beta)^t\,(1-H_t(H'))\,\overline{\theta}\right] \\
&= C(1-\beta) + \pi(H') + \frac{H'\mathcal{B}(H')}{\widetilde{H}} + \sum_{t=1}^{\infty}\beta^t\left[C(1-\beta) + \pi(H_t(H'))\right] \\
&\quad + \sum_{t=1}^{\infty}\beta^t\frac{H_t(H')\mathcal{B}(H_t(H'))}{\widetilde{H}} - \mu\left[(1-H')\,\overline{\theta} + \sum_{t=1}^{\infty}(\mu\beta)^t\,(1-H_t(H'))\,\overline{\theta}\right]
\end{aligned}
$$

Combining the two inequalities, we have that

$$
\frac{\mathcal{B}(\widetilde{H})}{1-\beta} + \frac{\pi(\widetilde{H})}{1-\beta} - \frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta} \geq
$$

$$
\pi(H') + \frac{H'\mathcal{B}(H')}{\widetilde{H}} + \sum_{t=1}^{\infty}\beta^t\pi(H_t(H')) + \sum_{t=1}^{\infty}\beta^t\frac{H_t(H')\mathcal{B}(H_t(H'))}{\widetilde{H}} - \mu\left[(1-H')\,\overline{\theta} + \sum_{t=1}^{\infty}(\mu\beta)^t\,(1-H_t(H'))\,\overline{\theta}\right]
$$

which is equivalent to

$$
\frac{1}{\widetilde{H}}\left[\frac{\widetilde{H}\mathcal{B}(\widetilde{H})}{1-\beta} - H'\mathcal{B}(H') - \sum_{t=1}^{\infty}\beta^t H_t(H')\mathcal{B}(H_t(H'))\right] \geq
$$

$$
\pi(H') + \sum_{t=1}^{\infty}\beta^t\pi(H_t(H')) - \frac{\pi(\widetilde{H})}{1-\beta} + \mu\overline{\theta}\left[H' + \sum_{t=1}^{\infty}(\mu\beta)^t\,H_t(H') - \frac{\widetilde{H}}{1-\mu\beta}\right]
$$

Since H' and $H_t(H')$ exceed $\widetilde{H}$, it follows that

$$
\begin{aligned}
&\frac{1}{\widetilde{H}}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - H'\mathcal{B}(H') - \sum_{t=1}^{\infty}\beta^t H_t(H')\mathcal{B}(H_t(H'))\right] \\
\geq\ &\mu\overline{\theta}\left[H' + \sum_{t=1}^{\infty}(\mu\beta)^t\,H_t(H') - \frac{\widetilde{H}}{1-\mu\beta}\right] + \pi(H') + \sum_{t=1}^{\infty}\beta^t\pi(H_t(H')) - \frac{\pi(\widetilde{H})}{1-\beta} > 0.
\end{aligned}
$$

Since $H \leq \widetilde{H}$, it therefore follows that

$$
\begin{aligned}
&\frac{1}{H}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - H'\mathcal{B}(H') - \sum_{t=1}^{\infty}\beta^t H_t(H')\mathcal{B}(H_t(H'))\right] \\
\geq\ &\frac{1}{\widetilde{H}}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - H'\mathcal{B}(H') - \sum_{t=1}^{\infty}\beta^t H_t(H')\mathcal{B}(H_t(H'))\right] \\
\geq\ &\mu\overline{\theta}\left[H' + \sum_{t=1}^{\infty}(\mu\beta)^t\,H_t(H') - \frac{\widetilde{H}}{1-\mu\beta}\right] + \pi(H') + \sum_{t=1}^{\infty}\beta^t\pi(H_t(H')) - \frac{\pi(\widetilde{H})}{1-\beta},
\end{aligned}
$$

as required.

A similar argument can be applied to rule out deviations to a larger housing level in which $H' > H^{**}$. The details can be found in the on-line Appendix. ■

10.3.2 Implications of the two conditions

The implications of condition (54) are straightforward. Under our Assumptions it will be satisfied if

$$\widetilde{H} \leq \frac{\overline{\theta}+S-C(1-\beta)-\underline{\pi}}{2\left[\overline{\theta}+s+\frac{\overline{\pi}-\underline{\pi}}{L}\right]} + \frac{\mu\overline{\theta}+\frac{\overline{\pi}-\underline{\pi}}{L}}{2\left[\overline{\theta}+s+\frac{\overline{\pi}-\underline{\pi}}{L}\right]} H_0 \tag{57}$$

Importantly for the Proposition, it is easy to see from (17) that condition (54) will be satisfied strictly by $\mathcal{H}(H_0)$ and hence for any smaller housing levels than $\mathcal{H}(H_0)$.

The implications of condition (55) are less obvious. It is convenient to define the function on the interval $[H_0, H^{**}]$

$$\varphi(H) \equiv \frac{(1-H)\overline{\theta}+S(H)}{1-\beta} + \mu\overline{\theta}\frac{H}{1-\mu\beta} - \left[P(H) + \mu\overline{\theta}\sum_{t=1}^{\infty}(\mu\beta)^{t-1} H_t(H)\right], \tag{58}$$

where the sequence $\langle H_t(H)\rangle_{t=1}^{\infty}$ is defined using the housing rule $H'(H)$ in (22). The second condition will be satisfied if $\varphi(\widetilde{H}) \geq 0$. Intuitively, $\varphi(\widetilde{H})$ represents the difference in payoffs experienced by the residents if the current housing level is $\widetilde{H}$ and they remain with this level rather than choosing the optimal deviation $H'(\widetilde{H})$ where $H'(H)$ is the housing rule defined in (22).

It is clear that $\varphi(H^{**}) = 0$ and, assuming $\mu < 1$, that $\varphi(H^*) > 0$. The latter follows because, with current housing level H^*, remaining with H^* is the solution that the residents would choose with commitment (recall that $\mathcal{H}(H^*)$ is just H^*). However, in the equilibrium, when $\mu < 1$, the housing level increases gradually to H^{**}. We show in the on-line Appendix that $\varphi(H)$ is concave and that $lim_{H\searrow 0}\varphi(H) = -\infty$. To understand this intuitively, note that when $H \searrow 0$ the per capita tax revenues generated from following the housing rule defined in (22) approach ∞. Accordingly, the payoff from following this housing rule tends to ∞, while the payoff from just keeping the housing stock constant at H tends to $\left(\overline{\theta}+S\right)/(1-\beta)$.

It follows from all this that, when $\mu < 1$, there exists a unique housing level $\underline{H} < H^*$ such that $\varphi(\underline{H}) = 0$. This housing level has the property that $\varphi(H)$ is non-negative in the range $[\underline{H}, H^{**}]$ and is negative everywhere else. Thus, condition (55) will be satisfied for all $\widetilde{H} \in [\underline{H}, H^{**}]$.

10.3.3 Completing the proof

We claim that $\underline{H}$ has the property described in the Proposition. We know that $\underline{H} < H^*$. Let $H_0 \in (\underline{H}, H^*)$ and take any $\widetilde{H} \in (H_0, \mathcal{H}(H_0))$. Note that $\widetilde{H}$ satisfies condition (54) since $\widetilde{H} <$

$\mathcal{H}(H_0)$. It also satisfies condition (55) because $\widetilde{H}$ belongs to the interval $[\underline{H}, H^*]$. Thus, with initial housing stock H_0 there exist an equilibrium of the form described in (26)-(29) with steady state housing level $\widetilde{H}$. Moreover, by construction, this steady state housing level is less than that which would be chosen in the initial residents' optimal plan $\mathcal{H}(H_0)$.

10.4 Proof of Proposition 8

Suppose there existed an equilibrium of the stalled form when $\mu = 1$. We claim that if the housing stock were $\widetilde{H}$ the residents would be better off choosing $H'(\widetilde{H})$ where $H'(H)$ is the housing rule defined in (22) than sticking with $\widetilde{H}$ - a contradiction. From (56), the payoff from sticking with $\widetilde{H}$ is simply

$$\frac{S(\widetilde{H})}{1-\beta}.$$

From (39), the payoff from deviating to $H'(\widetilde{H}) > \widetilde{H}$ is

$$\sum_{t=0}^{\infty} \beta^t S(H_t(H'(\widetilde{H}))) + \sum_{t=0}^{\infty} \beta^t \frac{\left(H_t(H'(\widetilde{H})) - H\right) \mathcal{B}(H_t(H'(\widetilde{H})))}{H} - \mu \left[\sum_{t=0}^{\infty} (\mu\beta)^t \left(1 - H_t(H'(\widetilde{H}))\right)\overline{\theta} \right].$$

Again, note that while the sequence $\langle H_t(H') \rangle_{t=1}^{\infty}$ in this expression is generated using the housing rule (26) it will equal that generated by the housing rule (22) since the two rules coincide on the interval $(\widetilde{H}, 1]$. So, we claim that

$$\begin{aligned} & S(H'(\widetilde{H})) + \frac{\left(H'(\widetilde{H}) - \widetilde{H}\right)}{\widetilde{H}} \mathcal{B}(H'(\widetilde{H})) + \sum_{t=1}^{\infty} \beta^t \left[S(H_t(H'(\widetilde{H}))) + \frac{\left(H_t(H'(\widetilde{H})) - \widetilde{H}\right)}{\widetilde{H}} \mathcal{B}(H_t(H'(\widetilde{H}))) \right] \\ > \; & \frac{S(\widetilde{H})}{1-\beta}. \end{aligned}$$

This is equivalent to

$$\begin{aligned} & \left[S(H'(\widetilde{H})) - S(\widetilde{H}) + \frac{\left(H'(\widetilde{H}) - \widetilde{H}\right)}{\widetilde{H}} \mathcal{B}(H'(\widetilde{H})) \right] \\ & + \sum_{t=1}^{\infty} \beta^t \left[S(H_t(H'(\widetilde{H}))) - S(\widetilde{H}) + \frac{\left(H_t(H'(\widetilde{H})) - \widetilde{H}\right)}{\widetilde{H}} \mathcal{B}(H_t(H'(\widetilde{H}))) \right] > 0. \end{aligned}$$

Using the functional form for $S(H)$, this in turn is equivalent to

$$\left[\frac{\mathcal{B}(H'(\widetilde{H}))}{\widetilde{H}} - s \right] \left(H'(\widetilde{H}) - \widetilde{H} \right) + \sum_{t=1}^{\infty} \beta^t \left[\frac{\mathcal{B}(H_t(H'(\widetilde{H})))}{\widetilde{H}} - s \right] \left(H_t(H'(\widetilde{H})) - \widetilde{H} \right) > 0$$

We know that

$$\mathcal{B}'(H) = -\left(\overline{\theta} + s + \frac{\overline{\pi} - \underline{\pi}}{L}\right) < 0$$

and that for all t $H_t(H'(\widetilde{H})) < H^o$. Thus, it suffices to show that $\mathcal{B}(H^o) \geq sH^o$. This is equivalent to

$$(1 - H^o)\overline{\theta} + S - 2sH^o - C(1-\beta) - \pi(H^o) \geq 0.$$

But, by definition

$$(1 - H^o)\,\overline{\theta} + S(H^o) + H^o S'(H^o) - C(1-\beta) - \pi(H^o) = 0.$$

■