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Appendix (For Online Publication)

to

Community Development with Externalities and Corrective Taxation

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On-line Appendix

Omitted details of the Proof of Proposition 6

Proof of Lemma 1

We need to show that the housing rule $H'(H)$ in (41) solves

$$\max_{H'} \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \beta^t \left[(1 - H_t(H'))\overline{\theta} + S(H_t(H')) \right] + \sum_{t=0}^{\infty} \beta^t \frac{\left(H_t(H') - H\right)\mathcal{B}(H_t(H'))}{H} \\ -\mu \left[\sum_{t=0}^{\infty} (\mu\beta)^t (1 - H_t(H'))\overline{\theta} \right] \\ s.t.\ H' \geq H \end{array} \right\}.$$

This requires showing that for any $H \in [H_0, 1]$, it is the case that $H'(H) \geq H$ and that there does not exist an alternative housing level $\hat{H}$ satisfying the constraint that $\hat{H} \geq H$ which generates a higher value of the objective function.

First, let $H \in [\widetilde{H}, 1]$. Then we have that $H'(H) = H$. Moreover, for any alternative housing level $\hat{H} > H$, the equilibrium play of future residents would be to simply keep housing at $\hat{H}$. Thus, the problem faced by the residents is identical to that in the commitment case and, since $\widetilde{H} \geq H^*$, we know that the optimal strategy is just to maintain the current housing stock.

Second, let $H \in [H_0, \widetilde{H})$. Then we have that $H'(H) = H_u(H)$. Note that the assumptions on $H_u(H)$ together with the fact that $H \in [H_0, \widetilde{H})$ imply that $H_u(H) > H$. Deviation to some housing level $\hat{H} \in [H, \widetilde{H}]$ cannot increase the value of the objective function because in this region the objective function arising if future housing choices are determined by $H_u(H)$ is the same as that arising if future housing choices are determined by $H'(H)$. Deviation to some $\hat{H} \in [\widetilde{H}, 1]$ cannot be profitable either. To understand why, note that once such a deviation occurs, the equilibrium play of future residents would be to simply keep housing at $\hat{H}$. The problem of optimally choosing such a deviation amounts to

$$\max_{\hat{H}} \left\{ \begin{array}{c} \frac{(1-\hat{H})\overline{\theta} + S(\hat{H})}{1-\beta} + \frac{\left(\hat{H} - H\right)\mathcal{B}(\hat{H})}{H} - \frac{\mu(1-\hat{H})\overline{\theta}}{1-\mu\beta} \\ s.t.\ \hat{H} \geq \widetilde{H} \end{array} \right\}$$

This problem has the same objective function as in problem (35). In the proof of Proposition 3, we established this objective function is concave. Moreover, for $H < H^*$ it has a maximum at $\mathcal{H}(H) < \widetilde{H}$ and for $H \geq H^*$ it has a maximum at $H < \widetilde{H}$. It follows that the objective function will have the highest value at the corner: $\widetilde{H}$. Hence such a deviation cannot be profitable because, as we have just shown, $H_u(H)$ provides a higher payoff than $\widetilde{H}$.

Proof of (43)

We need to show that the first order condition

$$\sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + (H_t(H') - H)\, \mathcal{B}'(H_t(H'))}{H} H_t'(H') = \tag{A59}$$

$$(\overline{\theta} + s) \left(\sum_{t=0}^{\infty} \beta^t H_t'(H') \right) - \mu\overline{\theta} \sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H'),$$

implies that the solution housing rule $H_u(H)$ satisfies the condition

$$\begin{gathered}(1 - H')\overline{\theta} + S - 2sH' - C(1-\beta) - \underline{\pi} - (\tfrac{\overline{\pi}-\underline{\pi}}{L})H' \\ = \overline{\theta}(H' - \mu H) \left[1 + \sum_{t=1}^{\infty} (\mu\beta)^t\, H_t'(H') \right] + (\tfrac{\overline{\pi}-\underline{\pi}}{L})(H' - H)\, [1 + \textstyle\sum_{t=1}^{\infty} \beta^t H_t'(H')]\end{gathered} \; . \tag{A60}$$

Note first that the first order condition (A59) can be rewritten as

$$\sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + H_t(H')\mathcal{B}'(H_t(H'))}{H} H_t'(H') =$$

$$\sum_{t=0}^{\infty} \beta^t \mathcal{B}'(H_t(H'))H_t'(H') + (\overline{\theta} + s) \left(\sum_{t=0}^{\infty} \beta^t H_t'(H') \right) - \mu\overline{\theta} \sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H').$$

Using the fact that

$$\mathcal{B}'(H_t(H')) = -\left(\overline{\theta} + s + \frac{\overline{\pi} - \underline{\pi}}{L} \right),$$

this implies that

$$\sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + H_t(H')\mathcal{B}'(H_t(H'))}{H} H_t'(H') = -\sum_{t=0}^{\infty} \beta^t \left(\frac{\overline{\pi} - \underline{\pi}}{L} \right) H_t'(H') - \mu\overline{\theta} \sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H'). \tag{A61}$$

Next we claim that

$$\sum_{t=1}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + H_t(H')\mathcal{B}'(H_t(H'))}{H} H_t'(H') = \tag{A62}$$

$$-(\frac{\overline{\pi} - \underline{\pi}}{L}) \left(\frac{H' \sum_{t=1}^{\infty} \beta^t H_t'(H')}{H} \right) - \frac{\mu\beta\overline{\theta}H' \sum_{t=1}^{\infty} (\mu\beta)^{t-1}\, H_t'(H')}{H}.$$

To see this, observe that the first order condition for $H_1(H')$ implies that

$$\sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H_1(H'))) + (H_t(H_1(H')) - H')\,\mathcal{B}'(H_t(H_1(H')))}{H'} H_t'(H_1(H')) =$$
$$(\overline{\theta} + s)\left(\sum_{t=0}^{\infty} \beta^t H_t'(H_1(H'))\right) - \mu\overline{\theta}\sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H_1(H')).$$

Multiplying this through by $\beta H_1'(H')$ we obtain

$$\sum_{t=0}^{\infty} \beta^{t+1} \frac{\mathcal{B}(H_t(H_1(H'))) + (H_t(H_1(H')) - H')\,\mathcal{B}'(H_t(H_1(H')))}{H'} H_t'(H_1(H'))H_1'(H') =$$
$$(\overline{\theta} + s)\left(\sum_{t=0}^{\infty} \beta^{t+1} H_t'(H_1(H'))H_1'(H')\right) - \mu\beta\overline{\theta}\sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H_1(H'))H_1'(H'),$$

which implies that

$$\sum_{t=1}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + (H_t(H') - H')\,\mathcal{B}'(H_t(H'))}{H'} H_t'(H') = (\overline{\theta}+s)\left(\sum_{t=1}^{\infty} \beta^t H_t'(H')\right) - \mu\beta\overline{\theta}\sum_{t=1}^{\infty} (\mu\beta)^{t-1}\, H_t'(H').$$

This follows from the fact that for all $t \geq 2$

$$H_t'(H_1(H'))H_1'(H') = H_t'(H').$$

This can be rewritten as

$$\sum_{t=1}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + H_t(H')\mathcal{B}'(H_t(H'))}{H'} H_t'(H') =$$
$$\sum_{t=1}^{\infty} \beta^t \mathcal{B}'(H_t(H'))H_t'(H') + (\overline{\theta} + s)\left(\sum_{t=1}^{\infty} \beta^t H_t'(H')\right) - \mu\beta\overline{\theta}\sum_{t=1}^{\infty} (\mu\beta)^{t-1}\, H_t'(H').$$

Using again the fact that

$$\mathcal{B}'(H_t(H')) = -\left(\overline{\theta} + s + \frac{\overline{\pi} - \underline{\pi}}{L}\right),$$

this implies that

$$\sum_{t=0}^{\infty} \beta^t \frac{\mathcal{B}(H_t(H')) + H_t(H')\mathcal{B}'(H_t(H'))}{H} H_t'(H') = -\sum_{t=0}^{\infty} \beta^t \left(\frac{\overline{\pi} - \underline{\pi}}{L}\right) H_t'(H') - \mu\overline{\theta}\sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H').$$

Multiplying through by H'/H yields (A62).

Using (A62), we can rewrite (A61) as follows:

$$\frac{\mathcal{B}(H') + H'\mathcal{B}'(H')}{H} - \frac{\overline{\pi} - \underline{\pi}}{L}\left(\frac{H'\sum_{t=1}^{\infty}\beta^t H_t'(H')}{H}\right) - \frac{\mu\beta\overline{\theta}H'\displaystyle\sum_{t=1}^{\infty}(\mu\beta)^{t-1}\, H_t'(H')}{H} =$$
$$-\sum_{t=0}^{\infty} \beta^t \left(\frac{\overline{\pi} - \underline{\pi}}{L}\right) H_t'(H') - \mu\overline{\theta}\sum_{t=0}^{\infty} (\mu\beta)^t\, H_t'(H').$$

Multiplying through by H yields

$$\mathcal{B}(H') + H'\mathcal{B}'(H') - (\frac{\overline{\pi} - \underline{\pi}}{L})H' \sum_{t=1}^{\infty} \beta^t H_t'(H') - \overline{\theta} H' \sum_{t=1}^{\infty} (\mu\beta)^t H_t'(H') =$$

$$-\sum_{t=0}^{\infty} \beta^t \left(\frac{\overline{\pi} - \underline{\pi}}{L}\right) H H_t'(H') - H\mu\overline{\theta} \sum_{t=0}^{\infty} (\mu\beta)^t H_t'(H') =$$

$$-\left(\frac{\overline{\pi} - \underline{\pi}}{L}\right) H - \sum_{t=1}^{\infty} \beta^t \left(\frac{\overline{\pi} - \underline{\pi}}{L}\right) H H_t'(H') - H\mu\overline{\theta} - H\mu\overline{\theta} \sum_{t=1}^{\infty} (\mu\beta)^t H_t'(H').$$

Thus, (A61) implies that

$$\mathcal{B}(H') + H'\mathcal{B}'(H') + (\frac{\overline{\pi} - \underline{\pi}}{L})H' + \overline{\theta} H' = \tag{A63}$$

$$(H' - H) \left(\frac{\overline{\pi} - \underline{\pi}}{L}\right) (1 + \sum_{t=1}^{\infty} \beta^t H_t'(H')) + (H' - \mu H)\, \overline{\theta} (1 + \sum_{t=1}^{\infty} (\mu\beta)^t H_t'(H')).$$

Since

$$\mathcal{B}(H) = (1 - H)\overline{\theta} + S(H) - (\pi(H) + C(1 - \beta)),$$

we have that

$$\mathcal{B}(H') + H'\mathcal{B}'(H') = (1 - H')\overline{\theta} + S(H') - (\pi(H') + C(1 - \beta)) - H' \left(\overline{\theta} + s + \frac{\overline{\pi} - \underline{\pi}}{L}\right).$$

Substituting this into (A63), reveals that

$$\begin{aligned} & (1 - H')\overline{\theta} + S(H') - (\pi(H') + C(1 - \beta)) - H' \left(\overline{\theta} + s + \frac{\overline{\pi} - \underline{\pi}}{L}\right) + (\frac{\overline{\pi} - \underline{\pi}}{L})H' + \overline{\theta} H' \\ = \; & (H' - H) \left(\frac{\overline{\pi} - \underline{\pi}}{L - H_0}\right) (1 + \sum_{t=1}^{\infty} \beta^t H_t'(H')) + (H' - \mu H)\, \overline{\theta} (1 + \sum_{t=1}^{\infty} (\mu\beta)^t H_t'(H')). \end{aligned}$$

This means that

$$(1 - H')\overline{\theta} + S - 2sH' - C(1 - \beta) - \underline{\pi} - (\tfrac{\overline{\pi} - \underline{\pi}}{L})H'$$

$$= \overline{\theta}(H' - \mu H) \left[1 + \sum_{t=1}^{\infty} (\mu\beta)^t H_t'(H')\right] + (\tfrac{\overline{\pi} - \underline{\pi}}{L})(H' - H) \left[1 + \textstyle\sum_{t=1}^{\infty} \beta^t H_t'(H')\right],$$

which is (A60).

Omitted details of the Proof of Proposition 7

Deviations to a housing level $H' > H^{**}$

Given the equilibrium play following this deviation, the payoff from it can be written as

$$\frac{(1 - H')\overline{\theta} + S(H')}{1 - \beta} + \frac{(H' - H)\, \mathcal{B}(H')}{H(1 - \beta)} - \frac{\mu}{1 - \mu\beta} (1 - H')\, \overline{\theta}.$$

Thus, to show that the deviation is not profitable, we need to show that

$$\frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}+\frac{(\widetilde{H}-H)\mathcal{B}(\widetilde{H})}{H(1-\beta)}-\frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta}\geq$$
$$\frac{(1-H')\overline{\theta}+S(H')}{1-\beta}+\frac{(H'-H)\,\mathcal{B}(H')}{H(1-\beta)}-\frac{\mu}{1-\mu\beta}\,(1-H')\,\overline{\theta},$$

or, equivalently, that

$$\frac{1}{H}\left[\frac{(\widetilde{H}-H)\mathcal{B}(\widetilde{H})}{1-\beta}-\frac{(H'-H)\,\mathcal{B}(H')}{1-\beta}\right]\geq\frac{(1-H')\overline{\theta}+S(H')-\Big((1-\widetilde{H})\overline{\theta}+S(\widetilde{H})\Big)}{1-\beta}+\frac{\mu\overline{\theta}(H'-\widetilde{H})}{1-\mu\beta}.$$

This is equivalent to

$$\frac{1}{H}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H})-\frac{H'}{1-\beta}\mathcal{B}(H')\right]\geq$$
$$\mu\overline{\theta}\left[\frac{H'}{1-\mu\beta}-\frac{\widetilde{H}}{1-\mu\beta}\right]+\frac{(1-H')\overline{\theta}+S(H')}{1-\beta}-\frac{\mathcal{B}(H')}{1-\beta}-\left(\frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}-\frac{\mathcal{B}(\widetilde{H})}{1-\beta}\right).$$

Substituting in the expressions for $\mathcal{B}(H')$ and $\mathcal{B}(\widetilde{H})$ this is equivalent to

$$\frac{1}{H}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H})-\frac{H'}{1-\beta}\mathcal{B}(H')\right]\geq\mu\overline{\theta}\left[\frac{H'}{1-\mu\beta}-\frac{\widetilde{H}}{1-\mu\beta}\right]+\frac{\pi(H')-\pi(\widetilde{H})}{1-\beta}.$$

Condition (55) implies that

$$\frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}-P(\widetilde{H})\geq\mu\overline{\theta}\left[\sum_{t=1}^{\infty}(\mu\beta)^{t-1}\,H_t(\widetilde{H})-\frac{\widetilde{H}}{1-\mu\beta}\right]$$

which is equivalent to

$$\frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}-\frac{\mu}{1-\mu\beta}(1-\widetilde{H})\overline{\theta}\geq P(\widetilde{H})-\mu\sum_{t=1}^{\infty}(\mu\beta)^{t-1}\Big(1-H_t(\widetilde{H})\Big)\,\overline{\theta}=V(\widetilde{H})$$

where $V(H)$ is the value function in (25). Moreover, since $V(H)$ is the value function for problem (39), we must have that

$$V(\widetilde{H})\geq\frac{(1-H')\overline{\theta}+S(H')}{1-\beta}+\frac{\Big(H'-\widetilde{H}\Big)\,\mathcal{B}(H')}{\widetilde{H}(1-\beta)}-\frac{\mu\,(1-H')\,\overline{\theta}}{1-\mu\beta}.$$

Thus, we have that

$$\frac{(1-\widetilde{H})\overline{\theta}+S(\widetilde{H})}{1-\beta}-\frac{\mu(1-\widetilde{H})\overline{\theta}}{1-\mu\beta}\geq\frac{(1-H')\overline{\theta}+S(H')}{1-\beta}+\frac{\Big(H'-\widetilde{H}\Big)\,\mathcal{B}(H')}{\widetilde{H}(1-\beta)}-\frac{\mu\,(1-H')\,\overline{\theta}}{1-\mu\beta},$$

which implies that

$$\frac{\widetilde{H}\mathcal{B}(\widetilde{H})}{\widetilde{H}(1-\beta)} - \frac{H'\mathcal{B}(H')}{\widetilde{H}(1-\beta)} + \frac{(1-\widetilde{H})\overline{\theta} + S(\widetilde{H})}{1-\beta} - \frac{\mathcal{B}(\widetilde{H})}{1-\beta} - \frac{\mu(1-\widetilde{H})\overline{\theta}}{1-\mu\beta} \geq$$
$$\frac{(1-H')\overline{\theta} + S(H')}{1-\beta} - \frac{\mathcal{B}(H')}{(1-\beta)} - \frac{\mu\,(1-H')\,\overline{\theta}}{1-\mu\beta},$$

or

$$\frac{1}{\widetilde{H}}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - \frac{H'}{1-\beta}\mathcal{B}(H')\right] \geq \mu\overline{\theta}\left[\frac{H'}{1-\mu\beta} - \frac{\widetilde{H}}{1-\mu\beta}\right] + \frac{\pi(H') - \pi(\widetilde{H})}{1-\beta}.$$

Since H' exceeds $\widetilde{H}$, it follows that

$$\frac{1}{\widetilde{H}}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - \frac{H'}{1-\beta}\mathcal{B}(H')\right] \geq \mu\overline{\theta}\left[\frac{H'}{1-\mu\beta} - \frac{\widetilde{H}}{1-\mu\beta}\right] + \frac{\pi(H') - \pi(\widetilde{H})}{1-\beta} > 0.$$

Since $H \leq \widetilde{H}$, it therefore follows that

$$\frac{1}{H}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - \frac{H'}{1-\beta}\mathcal{B}(H')\right] \geq \frac{1}{\widetilde{H}}\left[\frac{\widetilde{H}}{1-\beta}\mathcal{B}(\widetilde{H}) - \frac{H'}{1-\beta}\mathcal{B}(H')\right]$$
$$\geq \quad \mu\overline{\theta}\left[\frac{H'}{1-\mu\beta} - \frac{\widetilde{H}}{1-\mu\beta}\right] + \frac{\pi(H') - \pi(\widetilde{H})}{1-\beta},$$

as required.

Properties of $\varphi(H)$

It only remains to show that $\varphi(H)$ is concave and that $\lim_{H\searrow 0}\varphi(H) = -\infty$. For the former, note that $\sum_{t=1}^{\infty}(\mu\beta)^{t-1}H_t(H)$ is linear in H because $H'(H)$ and, and hence, all $H_t(H)$ are linear in H. Furthermore, $\frac{(1-H)\overline{\theta}+S(H)}{1-\beta} + \mu\overline{\theta}\frac{H}{1-\mu\beta}$ is also linear in H. From (50), we know that

$$P(H) = \sum_{t=0}^{\infty}\beta^t\left[(1 - H_t(H'(H)))\overline{\theta} + S(H_t(H'(H)))\right] + \sum_{t=0}^{\infty}\beta^t\frac{(H_t(H'(H)) - H)\,\mathcal{B}(H_t(H'(H)))}{H}$$

and from (53), we know that

$$\begin{aligned} P'(H) &= -\mu\overline{\theta}\sum_{t=0}^{\infty}(\mu\beta)^t\,H_t'(H'(H))\frac{dH'(H)}{dH} - \sum_{t=0}^{\infty}\beta^t\frac{H_t(H'(H))}{H^2} \\ &= -\frac{\mu\gamma\overline{\theta}}{1-\mu\beta\gamma} - \sum_{t=0}^{\infty}\beta^t\frac{H_t(H'(H))}{H^2}. \end{aligned}$$

It follows that

$$\begin{aligned} P''(H) &= \sum_{t=0}^{\infty}\beta^t\frac{2H_t(H'(H)) - H H_t'(H'(H))\frac{dH'(H)}{dH}}{H^2} \\ &= \sum_{t=0}^{\infty}\beta^t\frac{2H_t(H'(H)) - \gamma^{t+1}H}{H^2} > 0. \end{aligned}$$

Thus, $P(H)$ is convex, implying that $\varphi(H)$ is concave.

To show the limit result, note that .

$$\lim_{H \searrow 0} \varphi(H) \equiv - \lim_{H \searrow 0} P(H)$$

and, since $\lim_{H \searrow 0} H'(H) = \xi > 0$,

$$\lim_{H \searrow 0} \sum_{t=0}^{\infty} \beta^t \left[(1 - H_t(H'(H)))\overline{\theta} + S(H_t(H'(H))) \right] + \sum_{t=0}^{\infty} \beta^t \frac{(H_t(H'(H)) - H)\, \mathcal{B}(H_t(H'(H)))}{H} = \infty.$$