July 2023

Financing Local Public Projects*

Abstract

This paper studies the financing of local public projects. The setting is a community with durable housing, undeveloped land available for new homes, and population turnover. The community invests in a public project that may be financed with a mix of a tax on current residents and a debt issue. The paper shows that financing with a debt-tax mix is equivalent to pure tax finance coupled with a tax on future development whose proceeds are shared by future residents. This result has three implications. First, Ricardian Equivalence holds if and only if there would be no future development were the project purely tax financed. Second, when Ricardian Equivalence does not hold, the optimal debt level is such that the associated tax on development appropriately internalizes the negative externalities from this development. Third, when Ricardian Equivalence does not hold, the debt level preferred by current residents will be higher than optimal.

Levon Barseghyan
Department of Economics
Cornell University
Ithaca NY 14853
lb247@cornell.edu

Stephen Coate
Department of Economics
Cornell University
Ithaca NY 14853
sc163@cornell.edu

*We thank Brennan Platt for a suggestion that motivated this paper and two anonymous referees for constructive comments. This paper supersedes our earlier work "On Ricardian Equivalence in Local Public Finance" which explored the Ricardian Equivalence issue in a simpler version of this paper's model.

1 Introduction

This paper studies the financing of local public projects. In particular, the paper seeks to shed light on the implications for localities of the debt-tax mix in project financing and the forces that may shape its choice. The paper is motivated by the fact that debt is routinely utilized by local authorities in the U.S. to finance projects. As of 2021, long term debt accumulated by local governments (excluding pension liabilities and state government debt) stood at $2.1 trillion, which was about 10% of U.S. GDP and 3.4 times bigger than annual property tax revenues.[1] Moreover, there appears to be considerable variation in the amount of outstanding debt across localities, both per capita and relative to public capital stock.[2]

A natural starting point for any discussion of the mix of debt and taxes in financing public spending is the idea of Ricardian Equivalence. If this holds, a shift from tax to debt finance has no impact on the allocation of resources and hence on households' payoffs. Accordingly, the debt-tax mix is irrelevant. The basic logic behind the idea is that government borrowing simply postpones taxes. In terms of a consumer's lifetime budget constraint, being responsible for a given amount of tax today and that same amount with interest in the future makes no difference. As a result, an intertemporal rearrangement of taxes created by government borrowing, should have no impact on consumption paths and consumers' welfare. A large literature has developed understanding the conditions under which Ricardian Equivalence does or does not hold. However, the vast majority of this literature is concerned with the financing of national government spending.

The local setting differs from the national for at least three reasons. The first concerns the mobility of taxpayers. Local taxes are levied on current residents, so that once households leave a locality they are not responsible for its taxes. While the same is typically true for countries, being a resident of a locality is a much more transient state than is being a citizen of a country and this suggests that Ricardian Equivalence may not hold at the local level. Intuitively, a shift from tax to debt finance is a shift from taxing current to future residents and this should transfer the burden of financing public spending from current to future residents. The second reason concerns the nature of becoming a resident of a locality. While being a citizen of a country is usually a

[1] Source: U.S. Census Bureau at https://www.census.gov/data/datasets/2021/econ/local/public-use-datasets.html. Retrieved on July 7, 2023.

[2] In 2017, across 218 largest U.S. cities the average per capita debt was $6,202 with standard deviation of $3,750. Relative to public capital stock, the average was 0.72 with standard deviation of 0.33. (Authors' calculations based on Langley, 2020). Temple (1994) provides earlier evidence of variation from the U.S. states.

matter of birthright, becoming a resident requires the purchase (or rental) of a place to live in the locality, which is a market transaction. This is important because current and future taxes can be capitalized into property prices and such capitalization can result in current residents effectively paying future taxes even if they leave. The third reason is that because of mobility and the market determined nature of residence, the number of residents may be influenced by current and future taxes and the public projects these finance. Indeed, a key motivation of many public projects at the local government level is to spur development.

This paper analyzes the financing of local projects in a framework that incorporates these features that distinguish the local setting. The framework is a dynamic model of a single community that includes resident mobility, durable housing, and the possibility of new construction. The community starts out with a stock of housing and initial residents who own this housing. In each period, there is a pool of potential residents with heterogeneous desires to live in the community, generating a downward-sloping demand curve. There is turnover, with households entering and exiting the pool each period. When current residents leave the pool they sell their homes, which means that the market for housing is always active. Undeveloped land is available on which new housing can be built. This land is owned by landowners who reside outside the community and new construction is supplied by competitive developers. Land has a heterogeneous return in its non-building use, leading to an upward-sloping supply curve of building land.

Project financing is introduced by supposing that, in the initial period, the community invests in a project that provides a durable local public good benefiting future residents. The project is financed by a mix of a tax on those residing in the community in the initial period and an issue of debt. Debt is initially assumed to be in the form of one period bonds that are rolled over in each subsequent period. This means that issuing debt implies an infinite sequence of taxes on future residents to pay the interest costs.

In this framework, the paper establishes a novel equivalence result: *financing the project with a debt-tax mix is equivalent to financing it purely with a tax and then levying a tax on future new construction.* Revenues from this new construction tax are shared among those residing in the community at the time they are raised. The equivalent development tax is equal to the present value of the costs of servicing the debt divided by the number of initial residents. A higher debt-tax mix therefore translates into a larger tax on future development.

The paper then uses this equivalence result to derive three important results about local project financing. The first result clarifies the conditions under which Ricardian Equivalence holds at the local level: *Ricardian Equivalence holds if and only if there would be no development were the project financed purely with a tax.* This conclusion follows immediately from the equivalence result because a tax on development has no impact if and only if there is no development.

The second result suggests a novel normative role for debt in local project financing: namely, *debt should be used to appropriately internalize the negative externalities from development.* Specifically, we show that the socially optimal debt level will be such that the associated tax on development serves as an optimal corrective tax. In the model, the negative externalities from development are created by congestion in the benefits from the local public good.

The third result concerns the political economy of debt choice: *when Ricardian Equivalence does not hold, the debt level preferred by the initial residents will be higher than optimal.* From the equivalence result, choosing the debt level is effectively choosing a tax on development. The choice of such a tax will reflect not only the desire of the initial residents to counter the negative externalities from development but also their wish to raise tax revenues and the value of their homes. The latter two forces will lead them to increase the tax, and hence debt, above the optimal level. Indeed, they could choose a debt level sufficient to deter all development.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature. Section 3 introduces the model. Section 4 describes equilibrium with debt financing and Section 5 explains equilibrium when the project is tax financed but future development is taxed. Section 6 demonstrates the equivalence between these two policy regimes and Section 7 uses this finding to develop our three results about local project financing. Section 8 extends the analysis to the case in which debt takes the form of n period bonds that are not rolled over. Section 9 discusses the results and Section 10 concludes.

2 Related literature

The paper relates to the literature on Ricardian Equivalence, surveys of which are provided by, among others, Bernheim (1987), Seater (1993), and Ricuitti (2003). The literature identifies many limitations to the basic logic underlying Ricardian Equivalence. Perhaps the most fundamental is that if borrowing postpones taxation until after some current consumers have left the tax base, then it will matter. Thus, debt is not neutral in standard models with finite-lived consumers such as the basic overlapping-generations model (Diamond 1965) or the model with uncertain lives

(Blanchard 1985). Importantly, however, Ricardian Equivalence can hold with finite-lived consumers if different cohorts are altruististically linked and bequests are operational (Barro 1974). Another significant limitation is that in economies in which new consumers enter, Ricardian Equivalence will not hold even with infinitely-lived consumers because future taxes are spread over a larger population (Weil 1989). The issues of consumers leaving the tax base and new consumers entering are central to the model of this paper.

The literature on local government debt is much smaller. Two strands of work can be identified. One strand presumes that, because of resident mobility at the local level, debt allows the transfer of the burden of financing public projects from current to future residents. The other challenges this presumption, exploring how capitalization of property values can create Ricardian Equivalence even with resident mobility.

Within the first strand, two distinct normative argument for localities using debt are made. One is that public projects generating significant benefits to future residents should at least be partially financed with debt to ensure an equitable distribution of project costs across beneficiaries (see, for example, Musgrave 1959). The other is that localities should be permitted to use debt to encourage optimal public investment decisions. The idea is that because residents may leave their community, they will underinvest in durable public goods if they must tax finance investment. Debt financing counteracts this distortion because mobility also implies that residents do not bear the full burden of debt issued.[3] This logic underlies the so-called "golden rule" that prescribes that local governments pay for non-durable goods and services with tax revenues and use debt to finance investment in durables.[4]

In the second strand, a leading contribution is Daly (1969). He argues that debt will not transfer the burden of project finance to future residents in a setting where a locality consists of a fixed number of durable houses that are owned by residents. This is because the value of

[3] Several papers explore this argument formally. Schultz and Sjostrom (2001) study the determination of debt and public investment in a two period, two community model in which households choose where to locate in the second period. They assume that households cannot borrow and save which means that government debt plays a key role in smoothing consumption. In the equilibrium they study, they find that, while public investment levels may be efficient, localities overuse debt to deter in-migration. Bassetto and McGranahan (2021) present a quantitative, infinite horizon, multi-community model of public investment. Their model assumes exogenous population growth and migration flows and allows for various specifications for the durability of public investment and rules concerning the use of debt. The model is employed to account for the correlation in U.S. state-level public spending and population dynamics.

[4] The golden rule also applies at the national level with overlapping generations of non altruistic households. For discussion and analysis see Bassetto with Sargent (2006).

future tax payments is fully capitalized into house prices and thus current residents do not escape future taxes when they leave the community. Banzhaf and Oates (2013) present a more formal treatment of this idea as a prelude to their empirical study of U.S. local open space referenda. They also point out that the Ricardian logic implies that the scale of public project residents choose will be independent of the way in which it is financed. Akai (1994) and Hatfield (2010) study Ricardian Equivalence at the local level in multi-community models. They abstract from housing and assume that residence in a locality is determined by land ownership. Agents own land in only one community but the amount of land they own can vary. Importantly, local taxes are uniform across residents independent of the extent of land ownership. This means the tax base of a community can increase if its residents hold smaller amounts of land. The authors employ different assumptions about the utility from land ownership and come to different conclusions about Ricardian Equivalence.

This paper's model is similar to the settings considered by Daly and Banzhaf and Oates in featuring durable housing and a single community. However, it allows new construction, which breaks Ricardian Equivalence. Furthermore, the model features resident turnover that endogenizes current and future housing prices. The model abstracts from the role of debt in facilitating optimal public investment by assuming that the project decision is exogenous and just focusing on financing. The social role of debt in the model differs from that in the literature as will be explained below.

The paper also relates to the literature on the choice of revenue instruments by localities (see, for example, Henderson 1994, Hoyt 1991, and Inman 1989). Localities have many different ways to raise revenue, including taxes on land and property, user fees, income taxes, and sales taxes. This literature therefore seeks to shed light on the forces that shape the mix of policies they employ. The literature employs static models and so does not consider debt as a revenue source. It typically uses general equilibrium models in which multiple communities set their policies simultaneously in contrast to the partial equilibrium approach of this paper. Despite these differences, once we have established that debt is equivalent to a tax on future development, some of the forces driving the choice of debt are familiar from this literature. In particular, the literature tells us that policies (such as a property tax) that tax development are desirable when such development creates congestion. For this literature, one advantage of our dynamic framework is that it permits a clear distinction between current and future homeowners which is difficult to do in a static model (see the discussion in Henderson 1995). This allows us to study how policies redistribute between current and future homeowners.

Another related literature is that on the capitalization of local government amenities into housing prices stemming from Oates (1969) (see Hilber 2017 for a review). This literature recognizes that the extent of capitalization is determined by the elasticity of housing supply. Indeed, this can be illustrated in a standard supply and demand graph in which an improvement in a local government amenity leads to an outward shift in housing demand and the price response to this reflects the slope of the housing supply curve (see, for example, Hilber and Mayer 2009). In this spirit, it is natural to entertain the idea that, in communities with undeveloped land, public projects could create sufficient increase in demand to lead to a supply response in the form of new construction, and, if this happens, we would not expect full debt capitalization. The capitalization literature therefore motivates the importance of incorporating development. Furthermore, as noted by Stadelmann and Eichenberger (2014), the capitalization literature has largely ignored debt despite its obvious significance.[5] This paper draws further attention to debt capitalization by providing a theoretical model in which to study it.

A key theme of this paper is that project financing has implications for community development. Brueckner (1997) also explores this connection. His analysis assumes that development necessitates new infrastructure and studies different methods of financing it. A linear city model is employed in which land differs in proximity to the city center. Land is owned by landowners who choose when to develop it and therafter rent it out. Development is driven by an exogenous increase in the income available in the city. The financing methods studied include two schemes that correspond to tax and debt finance (the "current" and "perpetual" sharing schemes). While both our focus and underlying model are quite different, the spirit of our analysis is similar to Brueckner's work.[6]

[5] This neglect reflects the difficulty of assessing debt capitalization empirically. Local government debt is typically used to finance durable public assets and what should be capitalized is the value of public assets less debt. Good data on the value of public assets is difficult to find. However, Stadelmann and Eichenberger (2014) discover that the Swiss Canton of Zurich provides a setting in which the required data exists. They analyze the empirical relationship between housing prices and net public assets, finding strong support for capitalization. There is also some empirical work looking at the capitalization of unfunded municipal pension liabilities (see Albrecht 2015, Epple and Schipper 1981, Leeds 1985, and MacKay 2014).

[6] Brueckner's main focus is to study how switching infrastructure financing from either a current or a perpetual sharing scheme to an "impact fee" scheme effects the city's development path and land values. He does not directly compare the current and perpetual sharing schemes. Given the results of this paper, a natural question to ask is whether the perpetual scheme is equivalent to the current scheme together with a tax (or sequence of taxes) on future development the proceeds of which are shared among owners of developed land. This is not obvious given

Finally, the paper complements our earlier work in Barseghyan and Coate (2021) and (2022) which employs the same basic model of community development used here. These papers are political economy analyses seeking to endogenize policy choices. Barseghyan and Coate (2021) studies the provision of a continuous durable public good that can be financed with debt and taxes, while Barseghyan and Coate (2022) studies the setting of a development tax when development involves negative externalities. Both papers assume that policies are chosen in each period by forward looking residents and address the analytically challenging problem of characterizing equilibria with sequential policy-making. The focus is on understanding equilibrium development paths. By contrast, this paper seeks to understand the basic economics of how the debt-tax mix used to finance a given public project matters. The key result about the equivalence of debt and a development tax is new to this paper. So too are the results about Ricardian Equivalence and optimal and equilibrium debt levels.[7]

3 The model

Consider a community such as a small town or village. This community is one of a number in a particular geographic area, such as a county. The time horizon is infinite and periods are indexed by $t = 0, ..., \infty$. At the beginning of period 0 and in each subsequent period, there is a pool of potential residents of size 1. These are households who for exogenous reasons (employment opportunities, family ties, etc) need to live in the geographic area in which the community is located and are potentially open to living in the community. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter θ. This desire, for example, may be determined by a household's idiosyncratic reaction to the community's natural amenities. The preference parameter θ is uniformly distributed on $[0, \overline{\theta}]$. Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident will be one in the subsequent period is $\mu \in (0, 1)$. Thus, in each period, a fraction $1 - \mu$ of households leave the pool and are replaced by an equal number of new ones.

the many differences between Brueckner's setting and this paper's.

[7] Regarding Ricardian Equivalence, Barseghyan and Coate (2021) show how increasing public wealth (defined as the value of the public good stock less debt) can increase development and explore how public wealth evolves in light of this. While Ricardian Equivalence is not discussed, the findings make clear that it does not hold.

The community consists of land area L. Units of land can be used for housing or an alternate use. Each house requires one unit of land. In their alternate use, units of land yield a heterogeneous return π, where π is uniformly distributed on $[\underline{\pi}, \overline{\pi}]$. Houses are homogeneous and infinitely durable.[8] A stock of housing H accommodates a fraction H of the pool of potential residents.

At the beginning of period 0, the community has a housing stock H_0, implying that H_0 units of land are already being used for housing. This housing is built on those units of land with the lowest return in their alternate use. Thus, land with return less than $\pi(H_0)$ is being used for housing where the function $\pi(H)$ is defined as

$$\pi(H) \equiv \underline{\pi} + (\overline{\pi} - \underline{\pi}) \frac{H}{L}. \tag{1}$$

New houses can be built on the $L - H_0$ units of land not already used for housing. The cost of constructing a house is C and new construction is supplied by competitive developers. Undeveloped land is owned by landowners who reside outside of the community.

The only way to live in the community is to buy a house. A competitive housing market operates in each period. Demand comes from new households moving in, while supply comes from owners leaving and new construction. The price of a unit of building land in period t is denoted R_t and the price of houses is denoted P_t. The stock of houses at the beginning of period t is denoted H_t. New construction in period t is therefore $H_{t+1} - H_t$. The initial housing stock is assumed to be owned by those in the pool of potential residents at the beginning of period 0 with the highest desire to live in the community; i.e., those with preferences exceeding $(1 - H_0)\overline{\theta}$.

When living in the community, a household with preference θ and consumption x obtains a period payoff of $\theta + x$. Households and landowners discount future payoffs at rate β and can borrow and save at rate $\rho = 1/\beta - 1$. These assumptions mean that agents are indifferent to the intertemporal allocation of their consumption. Each household in the pool receives an exogenous income stream, the present value of which is sufficient to pay taxes and purchase housing in the community.[9] When not living in the community, a household's per-period payoff (net of the consumption benefits from income) is normalized to 0.[10]

[8] The assumption of infinitely durable housing is common in the urban economics literature and is justified by the fact that buildings in developed countries display considerable longevity.

[9] The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

[10] Note that 0 is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

In period 0 the community invests in a project that creates a durable local public good that becomes available starting in period 1. In each period $t \geq 1$, the public good provides a benefit $B(H_{t+1})$ (measured in consumption units) to all residents. The benefit function is weakly decreasing and concave in population (i.e., $B'(H) \leq 0$ and $B''(H) \leq 0$). The rate at which benefits decrease with population reflects the congestibility of the local public good. The cost of the public good is G and it can be financed with a tax on period 0 residents or by issuing debt in the form of one period bonds. Debt is assumed to be rolled over indefinitely and interest payments are financed by a tax on all those residing in the community at the end of each period. Letting D denote the debt issued, the tax on period 0 residents is $(G - D)/H_1$ and the tax on residents in period $t \geq 1$ is $\rho D/H_{t+1}$. The debt level D can range from 0 to G.

Taking the debt level D as given, the equilibrium variables are the quantity of housing and the prices of housing and building land in each period t, $\{H_{t+1}, P_t, R_t\}_{t=0}^{\infty}$. The outcome variables of interest are the payoffs of the different cohorts of residents and the landowners. The *initial potential residents* are all those in the pool at the beginning of period 0. They have mass 1 and are distinguished by their preference θ and whether or not they own homes in the community. Those households who own homes (i.e., those with preferences above $(1 - H_0)\overline{\theta}$) are the *initial residents*. For all periods $t = 0, ..., \infty$, the *period t potential residents* are those who join the pool at time t. These households have mass $1 - \mu$ and are distinguished by their preference θ. The landowners have mass $L - H_0$ and are distinguished by the return of their land π.

Let V_θ denote the lifetime payoff of an initial potential resident of preference θ. Similarly, let $V_{\theta t}$ denote the payoff of a period t potential resident of preference θ from period t onwards and V_π denote the lifetime payoff of a landowner who owns land with return π. We say that *Ricardian Equivalence* holds when these payoffs are independent of D. When it does not hold, we will be interested in the debt level that maximizes a Utilitarian social welfare function that weighs the payoffs of all agents (including landowners) equally and the debt level that maximizes the welfare of the initial residents. We will refer to the former as the *optimal debt level* and the latter as the *equilibrium debt level*. The latter terminology is motivated by the idea that, in a political economy model, the initial residents would determine how the project is financed given that it is undertaken in period 0.

We maintain four assumptions on the parameters throughout the analysis. The first two are designed to ensure that no development takes place before the public good becomes available in period 1. The first is that, without the project, the initial housing stock is sufficient to accommodate all those who would want to live in the community if they had to pay the per-period cost of building a new house. The condition is:

$$(1 - H_0)\overline{\theta} \leq C(1 - \beta) + \pi(H_0). \tag{2}$$

The left hand side is the per-period benefit of living in the community of the marginal household without the project and the right hand side is the per-period cost of building a new house. This condition rules out the community being under-sized before the project is undertaken. The second assumption is that the cost of the project satisfies the condition

$$\frac{G}{H_0} < H_0\overline{\theta} + (\overline{\pi} - \underline{\pi})\frac{H_0}{L}. \tag{3}$$

This guarantees that even if the project is fully debt financed so that period 0 residents pay no tax, the prospect of the project's benefits do not trigger development in period 0.

The final two assumptions are designed to guarantee that, if development does take place, it will not use up all the undeveloped land.[11] The third assumption requires that, with the project, those with the highest preference would want to live in the community even if they had to pay the per-period cost of building a new house, while those with the lowest preference would not; that is,

$$\overline{\theta} + B(H_0) > C(1 - \beta) + \pi(H_0) > B(H_0). \tag{4}$$

The fourth requires that either the community has sufficient land to accomodate all potential residents, or, the marginal household when all undeveloped land is being used for housing would not be willing to pay the per-period cost of building a new house; that is,

$$\textit{either } L > 1 \textit{ or } \overline{\theta}(1 - L) + B(L) < C(1 - \beta) + \overline{\pi}. \tag{5}$$

It should be noted that these two assumptions do not rule out the possibility that the community simply has no undeveloped land available (i.e., $H_0 = L$), a situation that might characterize a mature community. The assumptions only imply that, in this case, the willingness to pay to live in the community of the marginal household is less than $C(1 - \beta) + \overline{\pi}$. If there is no undeveloped land available, $\overline{\pi}$ is a purely fictional concept, so this condition can be assumed to hold with no loss of generality.

[11] This streamlines the presentation by eliminating the need to consider the case where lack of building land constrains the amount of development. It is straightforward to extend the analysis to encompass this case and we point out the highlights in footnotes along the way.

4 Equilibrium with debt

This section explains what happens in the community for a given debt level D. It first describes how the market determines the prices of building land and housing and the size of the community in each period t. It then defines what an equilibrium is and solves for this equilibrium.

4.1 Housing and land markets

At the outset of any period t, households fall into two groups: those who resided in the community in period $t-1$ and those who did not, but could in period t. Households in the first group own homes, while the second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff P_t. The remaining households in the first group and all those in the second must decide whether to live in the community. This decision will depend on their preference θ, current and future housing prices, taxes, and the benefits of the project. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of any period.[12] This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the price of housing in the next period.

To see how this plays out, consider a household with preference θ deciding whether to live in the community in period t. Let B_t denote the benefit provided by the public good in period t, T_t the tax in period t, and P_{t+1} the anticipated period $t+1$ housing price.[13] The household will want to reside in the community if and only if

$$\theta + B_t - T_t \geq P_t - \beta P_{t+1}. \tag{6}$$

The left hand side represents the benefit from locating in the community in period t and the right hand side the cost.[14] The latter assumes that the household buys a house at the beginning of period t and sells it at the beginning of period $t+1$. Given (6) and the fact that household preferences are uniformly distributed over $[0, \overline{\theta}]$, the equilibrium price of housing in period t must

[12] It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community.

[13] Thus, $B_0 = 0$ and $B_t = B(H_{t+1})$ if $t \geq 1$, and $T_0 = (G-D)/H_1$ and $T_t = \rho D/H_{t+1}$ if $t \geq 1$.

[14] The household's income and assets do not enter into the calculus because they impact both the payoff from living in and out of the community and thus cancel.

satisfy the market clearing condition

$$H_{t+1} = 1 - \frac{(P_t - \beta P_{t+1}) - (B_t - T_t)}{\overline{\theta}}. \tag{7}$$

This implies that the equilibrium housing price is

$$P_t = (1 - H_{t+1})\overline{\theta} + B_t - T_t + \beta P_{t+1}. \tag{8}$$

Turning to the supply side of the housing market, the assumption that houses are infinitely durable implies that

$$H_{t+1} \geq H_t. \tag{9}$$

Moreover, because the supply of new construction is perfectly elastic at a price equal to the land price plus the construction cost $C + R_t$, it must also be the case that

$$P_t \leq C + R_t \ (= \text{if } H_{t+1} > H_t). \tag{10}$$

In terms of the land market, the demand for building land in period t is $H_{t+1} - H_t$. The market will allocate to building the land with the lowest return in its alternate use. Thus, at the beginning of time t, land with return π less than $\pi(H_t)$ has already been used for housing. For equilibrium in the land market it must be the case that the owner of land with return $\pi(H_{t+1})$ must be just indifferent between selling it for building land or keeping it. This requires that the price of building land satisfy

$$R_t = \pi(H_{t+1}) + \beta R_{t+1}, \tag{11}$$

where R_{t+1} is the anticipated equilibrium price of building land in period $t + 1$.[15]

4.2 Definition and characterization of equilibrium

In light of the above discussion, given anticipated prices of housing P_{t+1} and building land R_{t+1}, and an initial housing stock H_t, the conditions for (H_{t+1}, P_t, R_t) to be an equilibrium in period t are (8), (9), (10), and (11). Accordingly, we define a sequence of housing levels and prices $\{H_{t+1}, P_t, R_t\}_{t=0}^{\infty}$ to be an *equilibrium with debt level* D if in each period t, (H_{t+1}, P_t, R_t) satisfies the conditions for equilibrium in period t.

[15] To see why (11) must hold, note first that if there is new construction in period $t+1$, so that H_{t+1} is less than H_{t+2}, then the marginal landowner in period t would certainly sell his land in period $t+1$ if he kept it in period t. It follows that the right hand side of (11) describes his payoff from keeping his land in period t. If there is no new construction in period $t + 1$, the marginal owner of land with return $\pi(H_{t+1})$ will be the marginal owner for ever and it must be the case that $R_t = R_{t+1} = \pi(H_{t+1})/(1 - \beta)$, which implies that (11) holds.

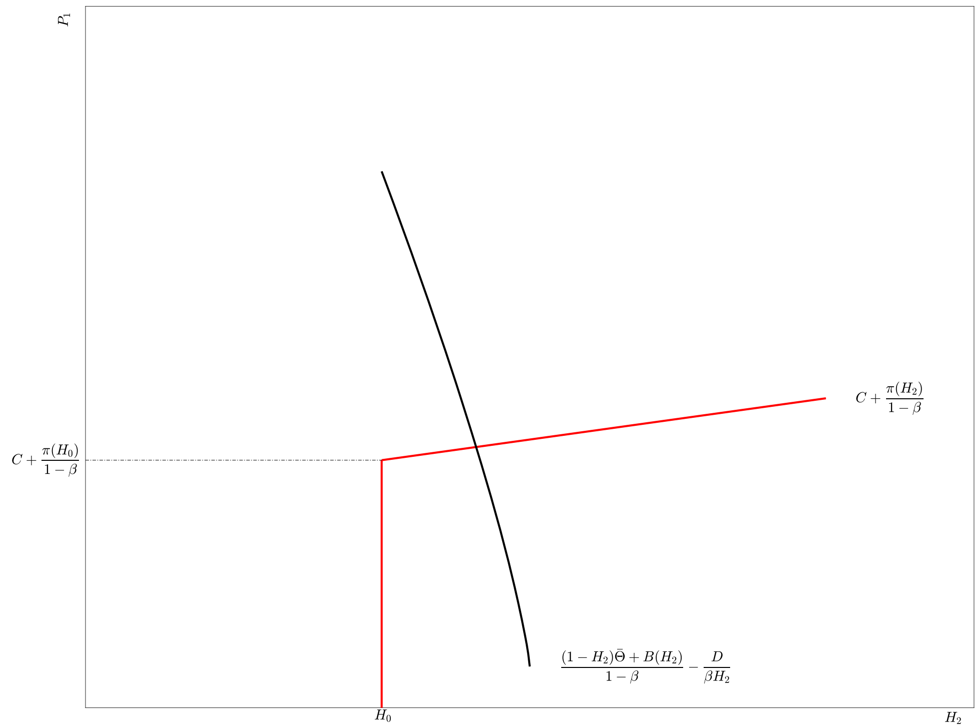

Figure 1: Period 1 equilibrium with debt.

To solve for equilibrium, note first that any development that occurs will take place in period 1 as soon the public good becomes available. After that, the community is in a stationary state. Thus, H_{t+1} will equal H_2 for all periods t beyond 1. Using (8), the price of housing in these periods will be

$$P_t = \frac{(1-H_2)\overline{\theta} + B(H_2)}{1-\beta} - \frac{D}{\beta H_2}. \tag{12}$$

This is just the present value of the surplus flow from living in the community obtained by the marginal household. Note that the present value of taxes necessary to service the debt are reflected in this price. The price of building land in periods 2 and beyond will equal $\pi(H_2)/(1-\beta)$.

It is useful to represent what happens in period 1 diagrammatically. Substituting (12) into (8), the inverse demand curve for housing in period 1 is

$$P_1(H_2) = \frac{(1-H_2)\overline{\theta} + B(H_2)}{1-\beta} - \frac{D}{\beta H_2}. \tag{13}$$

This is the downward sloping curve in Figure 1. On the supply side, since there is no development in period 0, the community has housing stock H_0 at the beginning of period 1. The cost of building a new house is the construction cost plus the cost of a unit of building land; i.e., $C + R_1$. Given that R_2 is equal to $\pi(H_2)/(1-\beta)$, condition (11) implies that R_1 is also equal to $\pi(H_2)/(1-\beta)$. The period 1 supply curve of housing is therefore vertical and equal to H_0 when price is less than $C+\pi(H_0)/(1-\beta)$ and, for housing levels H_2 larger than H_0, is described by $C+\pi(H_2)/(1-\beta)$.[16] In Figure 1, this curve is the upward sloping curve with vertical segment.

If $P_1(H_0)$ is less than $C + \pi(H_0)/(1-\beta)$, the inverse demand curve cuts the supply curve on its vertical portion and equilibrium involves no development. Thus, H_2 equals H_0 and P_1 equals $P_1(H_0)$. If $P_1(H_0)$ exceeds $C+\pi(H_0)/(1-\beta)$, which is the case illustrated in Figure 1, the inverse demand curve cuts the supply curve on its upward sloping portion and the equilibrium involves development. The equilibrium housing level satisfies the market clearing condition that $P_1(H_2)$ is equal to $C + \pi(H_2)/(1-\beta)$.[17]

What happens in period 0 depends on whether or not there is development in period 1. If no development is anticipated, equation (8) implies that the period 0 price of housing is given by

$$P_0 = \frac{(1-H_0)\overline{\theta} + \beta B(H_0)}{1-\beta} - \frac{G}{H_0}. \tag{14}$$

Note that this is independent of the debt level, so debt is fully capitalized into the period 0 housing price. If development is anticipated, the period 0 price of housing is given by:

$$P_0 = (1-H_0)\overline{\theta} - \frac{G-D}{H_0} + \beta\left(C + \frac{\pi(H_2)}{1-\beta}\right). \tag{15}$$

By contrast, this does depend on the debt level. The price of building land in period 0 is $\pi(H_0)/(1-\beta)$ if no development is anticipated and $\pi(H_0) + \beta\pi(H_2)/(1-\beta)$ otherwise.[18]

5 Equilibrium with a development tax

This section considers an alternative policy regime in which the community tax finances the public good in period 0 and taxes development in periods 1 and beyond. Specifically, the community levies a tax G/H_0 on period 0 residents to pay for the public good and then levies a tax τ on each new house constructed in any period $t \geq 1$. This tax is paid by the developers who build

[16] The supply curve becomes vertical again at housing level L, when the community runs out of land. Our assumptions allow us to ignore this part of the supply curve.

[17] If the solution to this equation were to exceed L, the land constraint would bind and H_2 would equal L. The equilibrium price of housing would equal $P_1(L)$ and the equilibrium price of land would equal $P_1(L) - C$.

[18] Assumptions (2) and (3) imply that there will be no development in period 0.

new homes. The revenues from the development tax in period t are $\tau(H_{t+1} - H_t)$ and these are divided equally between all those resident in the community at the end of period t. Thus, at the end of period t, each resident receives a transfer $\tau(H_{t+1} - H_t)/H_{t+1}$. The section describes what happens for a given development tax τ. Having understood this, the goal will be able to compare equilibria with debt and a development tax.

5.1 Housing and land markets

Following the logic of the previous section, the equilibrium housing price in this policy regime continues to satisfy (8), but the period t tax T_t is different.[19] On the supply side, it must also be the case that in periods $t \geq 1$

$$P_t \leq C + R_t + \tau \ (= \text{if } H_{t+1} > H_t). \tag{16}$$

The right hand side reflects the full cost to a developer of providing a new home.

5.2 Definition and characterization of equilibrium

Given anticipated prices of housing P_{t+1} and building land R_{t+1}, and an initial housing stock H_t, the conditions for (H_{t+1}, P_t, R_t) to be an equilibrium in period t are (8), (9), (10) if $t = 0$, (16) if $t \geq 1$, and (11). A sequence of housing levels and prices $\{H_{t+1}, P_t, R_t\}_{t=0}^{\infty}$ is an *equilibrium with development tax* τ if, in each period t, (H_{t+1}, P_t, R_t) satisfies the conditions for equilibrium in period t.

In solving for equilibrium, we restrict consideration to tax rates τ between 0 and $G/\beta H_0$. The reason for this will become clear in the next Section. Under this assumption, any development that occurs will take place in period 1 as soon as the public good becomes available.[20] This means that H_{t+1} will equal H_2 for all periods t beyond 1. There are no development tax revenues beyond period 1, so the price of housing in these periods will just equal

$$P_t = \frac{(1 - H_2)\overline{\theta} + B(H_2)}{1 - \beta}. \tag{17}$$

The price of building land will equal $\pi(H_2)/(1 - \beta)$.

[19] Specifically, T_0 is equal to G/H_0 and T_t is equal to $-\tau(H_{t+1} - H_t)/H_{t+1}$ for $t \geq 1$.

[20] If the development tax exceeds $G/\beta H_0$, it is possible that development will take place in period 0 to avoid the development tax.

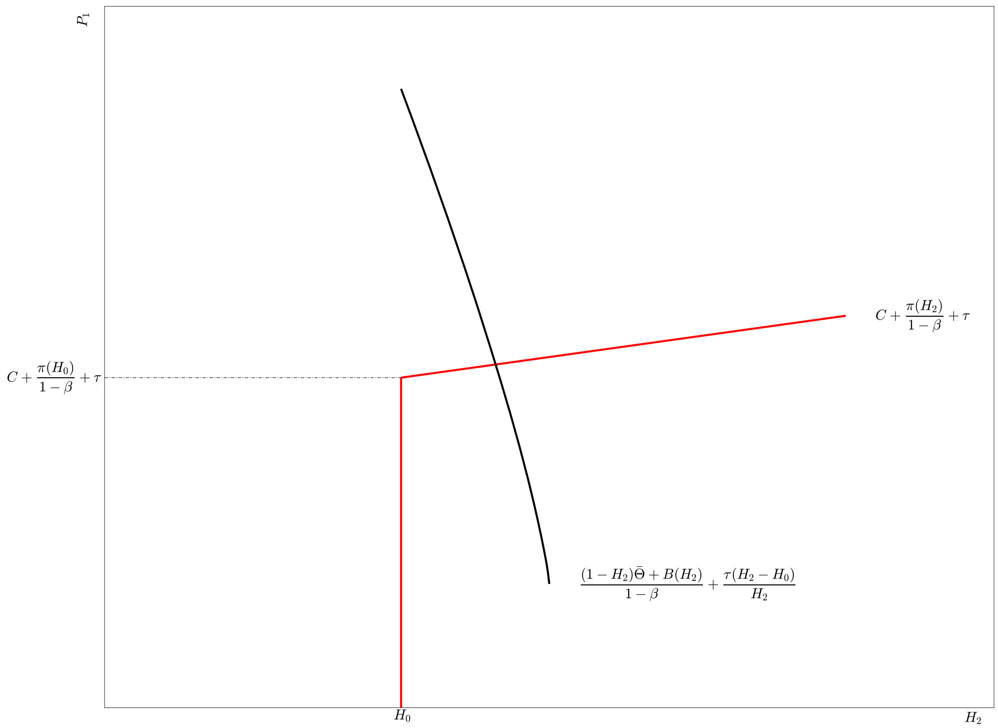

Figure 2: Period 1 equilibrium with a development tax.

Again, it is useful to illustrate what happens in period 1 diagrammatically. Substituting (17) into (8), the inverse demand curve for housing in period 1 is

$$P_1(H_2) = \frac{(1-H_2)\overline{\theta} + B(H_2)}{1-\beta} + \frac{\tau(H_2 - H_0)}{H_2}. \tag{18}$$

This is the downward sloping curve in Figure 2. On the supply side, the development tax means that the cost of building a new house in period 1 is now $C + R_1 + \tau$. The period 1 supply curve of housing is therefore vertical and equal to H_0 when price is less than $C + \pi(H_0)/(1-\beta) + \tau$ and, for housing levels H_2 larger than H_0, is described by $C + \pi(H_2)/(1-\beta) + \tau$. This curve is the upward sloping curve with the vertical segment.

If $P_1(H_0)$ is less than $C+\pi(H_0)/(1-\beta)+\tau$, the inverse demand curve cuts the supply curve on its vertical portion and equilibrium involves no development. Thus, H_2 equals H_0, and P_1 is equal to $P_1(H_0)$. The price of building land is $\pi(H_0)/(1-\beta)$. If $P_1(H_0)$ exceeds $C+\pi(H_0)/(1-\beta)+\tau$, as illustrated in Figure 2, the equilibrium involves development in period 1. The equilibrium housing level satisfies the market clearing condition that $P_1(H_2)$ is equal to $C + \pi(H_2)/(1-\beta) + \tau$. The price of building land is $\pi(H_2)/(1-\beta)$.

In period 0, if no development is anticipated in period 1, the price of housing is given by

$$P_0 = \frac{(1-H_0)\overline{\theta} + \beta B(H_0)}{1-\beta} - \frac{G}{H_0}. \tag{19}$$

If development is anticipated, the price of housing is:

$$P_0 = (1-H_0)\overline{\theta} - \frac{G}{H_0} + \beta\left(C + \frac{\pi(H_2)}{1-\beta} + \tau\right). \tag{20}$$

The price of building land in period 0 is $\pi(H_0)/(1-\beta)$ with no development and $\pi(H_0) + \beta\pi(H_2)/(1-\beta)$ otherwise.

6 An equivalence result

This section explores the relationship between the two policy regimes described above. It establishes that that there is an equivalence between debt financing and tax financing coupled with a development tax. Specifically, the equilibrium that arises when the project is financed with a debt-tax mix is equivalent to the equilibrium that arises with pure tax finance and a tax on future development whose revenues are shared by those residing in the community when they are raised. The equivalent development tax equals the period 1 present value of the costs of servicing the debt divided by the number of period 0 residents. Greater borrowing therefore translates into a higher development tax. The formal statement of the result is as follows:

Proposition 1. *Let $\{H_{t+1}^*, P_t^*, R_t^*\}_{t=0}^{\infty}$ be an equilibrium with debt level D. Then, $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$ is an equilibrium with development tax $\tau^* = D/\beta H_0$, where the price sequence $\{\widetilde{P}_t\}_{t=0}^{\infty}$ is given by $\widetilde{P}_0 = P_0^*$, $\widetilde{P}_1 = P_1^* + \tau^*$, and $\widetilde{P}_t = P_t^* + \tau^* H_0/H_2^*$ for all $t \geq 2$. Furthermore, the payoffs of all agents in the two equilibria are identical.*

It should be emphasized that this result is not saying that Ricardian Equivalence holds. That would require showing that an equilibrium with debt level D was equivalent to an equilibrium with a development tax of 0. Nonetheless, as we will see below, the result is useful in understanding when Ricardian Equivalence does hold.

The equivalence result is striking because the two policy regimes appear quite different. The debt policy reduces the tax on period 0 residents and increases taxes on all future residents. It is thus just an intertemporal shifting of the project's costs. The development tax policy levies the full cost of the project on period 0 residents but taxes future development and, when such

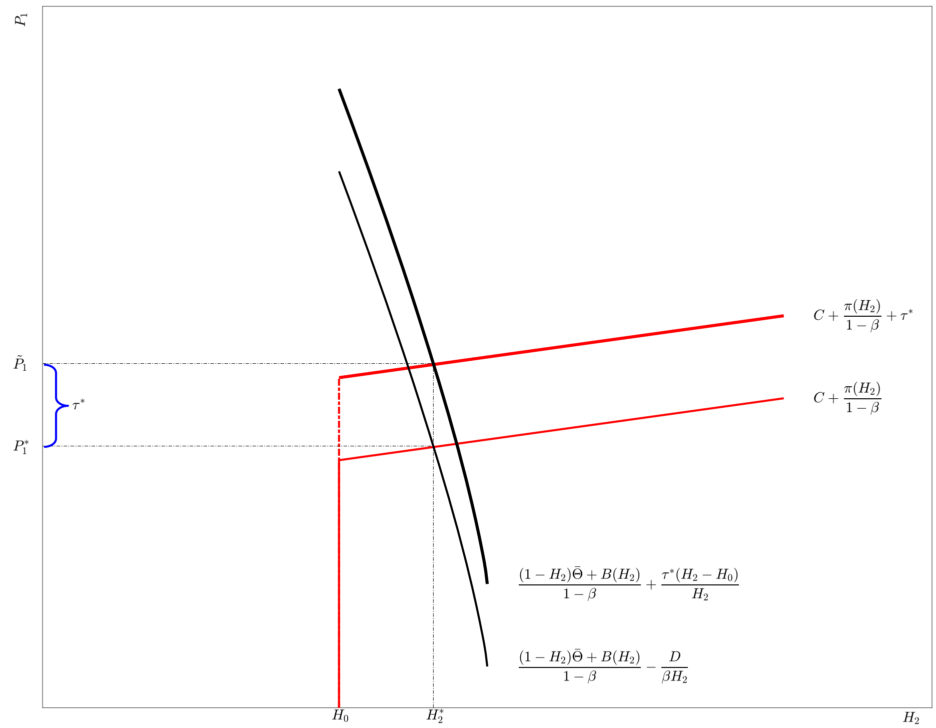

Figure 3: Period 1 equilibrium under the two policies.

development occurs, shares the revenues among the current residents. This development tax would seem to have nothing to do with shifting the costs of the project intertemporally. Why then do the two regimes produce the same outcomes?

The formal proof of Proposition 1, which is provided in the Appendix, is somewhat involved because it is necessary to derive payoff expressions for the various types of agents in the two regimes. Here, we explain the economics underlying the result. Consider first why the level of development with development tax τ^* will be the same as in the equilibrium with debt level D. Figure 3 illustrates period 1 equilibrium in the housing market under the two policies. The lower of the two downward sloping curves represents the inverse demand curve with debt as described in equation (13). The higher curve represents the inverse demand with development tax τ^* as described in equation (18). The development tax inverse demand is higher because residents not only avoid having to pay the tax to service the debt but also share the revenues from the development tax. As is easily verified, the two curves are parallel and the distance between them is τ^*. The lower of the two upward sloping curves represents the inverse supply curve with debt for housing levels above H_0. The higher one is the inverse supply curve with the development tax. The development tax inverse supply is higher because developers have to pay the tax τ^*. Again, the two curves are parallel and the distance between them is τ^*.

As illustrated, the housing level H_2^* that clears the market in the debt regime also clears the market in the development tax regime. This is because both the inverse demand and supply curves in the development tax case are shifted up by the amount of the tax τ^*. The only difference is that the market clearing price of housing is higher by the amount of the tax in the development tax case and this is why $\widetilde{P}_1$ is equal to $P_1^* + \tau^*$.

Next consider why in periods 2 and beyond the housing price with development tax τ^* will equal $P_t^* + \tau^* H_0/H_2^*$. In any such period, the only difference between the situation with debt and that with the development tax is that, in the former, residents have to pay a tax $\rho D/H_2^*$ to service the debt. The discounted present value of these future tax payments is $D/\beta H_2^*$. The benefit of not having to pay these taxes will be capitalized into the price of housing under the development tax policy, so that $\widetilde{P}_t$ should equal $P_t^* + D/\beta H_2^*$. This is the case since $\tau^* H_0/H_2^*$ is equal to $D/\beta H_2^*$.

Finally, consider why in period 0 the price of housing with development tax τ^* will equal P_0^*. There are two differences between the situations with debt and the development tax in period 0. First, with the development tax, residents have to pay a tax G/H_0 as opposed to $(G - D)/H_0$. Second, with the development tax, the period 1 price of housing is higher by the amount $\tau^* = D/\beta H_0$. The price equation (8) implies that the higher tax will be exactly offset by the higher future price and thus the price of housing will be unchanged.

With this as background, we can now understand why the different cohorts of potential residents end up with the same payoffs with the development tax τ^*. Consider first the potential residents for periods 2 and beyond. In the development tax regime, these households do not have to pay taxes to service the debt. With debt, capitalization ensures that households will fully bear these costs even though they might leave the community. Thus, this represents a benefit equal to $D/\beta H_2^*$. However, with the development tax, households face a higher price of housing. The price difference is $\tau^* H_0/H_2^*$ which is exactly equal to $D/\beta H_2^*$, meaning they are no better off than with debt.

The same story holds for the period 1 residents. In the development tax regime, these households not only do not have to service the debt but actually receive a transfer in period 1. However, the period 1 price of housing is higher with the development tax and this offsets both of these benefits. To see this, note that the period 1 price difference is $D/\beta H_0$. The present value of the cost of servicing the debt is $D/\beta H_2^*$ and, after substituting for τ^*, the transfer is $D\left(H_2^* - H_0\right)/\beta H_0 H_2^*$. Together these sum to $D/\beta H_0$.

The most complicated group are the period 0 potential residents. While they face the same price of housing in period 0, with a development tax, households who buy a house in period 0 have to pay a tax G/H_0 in period 0 as opposed to a tax of $(G - D)/H_0$. This means a higher period 0 tax bill by the amount of D/H_0. On the other hand, they do not have to pay the cost of servicing the debt which has present value $\beta D/\beta H_2^*$. Note that the present value of the cost of servicing the debt is strictly smaller than the increase in period 0 taxes if there is development (i.e., if H_2^* exceeds H_0). This is because development creates a larger tax base over which to spread the costs of debt service. However, crucially, if there is development, these households receive a transfer $\tau^*(H_2 - H_0)/H_2$ in period 1 and this is what compensates them for the benefit of spreading the costs of debt service over a larger tax base.

The initial period potential residents just differ from the period 0 potential residents in that the initial residents own homes at the beginning of period 0. The fact that the period 0 housing price is the same with the development tax means that they also end up with the same payoffs in the two regimes. That landowners have the same payoffs follows from the fact that building land prices are the same across regimes.

7 Implications

This section develops three interesting implications of the equivalence result for project financing.

7.1 Ricardian Equivalence

Proposition 1 implies that Ricardian Equivalence holds if and only if there would be no development were the project tax financed. To see this, suppose first that the no development condition is satisfied and consider what would happen if the project were partially financed with debt level D. Proposition 1 tells us that the outcome that would arise would be the same as that arising if the project were tax financed and there were a development tax equal to $D/\beta H_0$. But, given that there would be no development with a zero development tax, the outcome that would arise in this case will involve no development and no development tax revenues and thus be the same as that arising if the project were tax financed. Accordingly, partially financing the project with debt will have no effect.

For the converse, suppose that the no development condition is not satisfied. Consider what would happen if the project were partially financed with debt level D. Again, the Proposition implies that the outcome that would arise would be the same as that arising if the project were tax financed and there were a development tax equal to $D/\beta H_0$. But, given that there would be development with a zero tax, the outcome that would arise in this case will involve *less* development. This is because, when there is a development, a development tax will reduce it.[21] Accordingly, partially financing the project with debt will have an impact on development and Ricardian Equivalence will not hold.

Our next proposition formally states and proves this result. To simplify notation, we let

$$S(H) \equiv (1-H)\overline{\theta} + B(H) - (C(1-\beta) + \pi(H)) . \tag{21}$$

The function $S(H)$ describes the per-period surplus that would be obtained by the marginal household from living in the community when it has population H, the project has been implemented, there is no taxation, and the household pays the per-period cost of building a new house. If tax financed, the project will spur development if and only if $S(H_0)$ is positive.

Proposition 2. *Ricardian Equivalence holds if and only if* $S(H_0) \leq 0$.

This Proposition contributes to the literature by clarifying the conditions under which Ricardian Equivalence holds at the local level. By showing that Ricardian Equivalence holds if there is no development, it reaffirms the argument of Daly (1969). However, it also shows that his argument breaks down if the project being financed can generate development. In this case, the debt level, by increasing future taxes, will reduce future development. While the higher future taxes created by debt are certainly reflected in lower housing prices, the upward sloping supply curve of housing means that part of the adjustment is on the quantity side and this is what has real effects.[22]

[21] This is established in the proof of Proposition 2 and can also be verified by inspection of Figure 2.

[22] Recall that our assumptions rule out the case in which the community starts out with some undeveloped land and development is sufficiently strong that all of it is used up for building. It is worth noting that in this case Ricardian Equivalence is restored in the sense that a marginal decrease in debt will have no real effects. The logic is that once all the community's undeveloped land is used up, the housing supply curve becomes vertical and lower future taxes just leads to an increase in future housing prices with no quantity adjustment.

7.2 Optimal debt

Proposition 1 invites us to think of debt as a tax on future development. To understand the optimal level of debt, therefore, we need to think about the optimal tax on development. Taxing development will be desirable if it imposes negative externalities. In the model, these externalities are created by congestion in the benefits from the local public good. Accordingly, the optimal level of debt is that generating a development tax appropriately internalizing these congestion externalities.

To develop the implications of this line of thinking, note first that the externalities created by congestion in public good benefits are measured by $-HB'(H)/(1-\beta)$. Assuming that some development is optimal, the optimal tax on development is $-(H^o)^2 B'(H^o)/H_0(1-\beta)$, where H^o denotes the optimal community size.[23] The fact that the tax exceeds the size of the externality at the optimal development level reflects the fact that tax revenues are shared by all residents, so the tax needs to be higher to offset this incentive. Proposition 1 then implies that the optimal debt level equals $-(H^o)^2 B'(H^o)/\rho$. If the project is a pure public good, so that there is no congestion, pure tax financing will be optimal. More generally, the greater the congestibility of the public good, the higher the appropriate debt-tax mix.

The following proposition develops this argument more formally and accounts for the complications that no development could be optimal or that full debt financing might not generate a large enough development tax to achieve the optimal level of development.

Proposition 3. *Suppose that Ricardian Equivalence does not hold. Then, if* $S(H_0) \leq -H_0B'(H_0)$*, no development is optimal and any debt level at least as big as* $\min\{G, H_0S(H_0)/\rho\}$ *is optimal. If* $S(H_0) > -H_0B'(H_0)$*, the optimal amount of development is* $H^o - H_0$*, where* H^o *satisfies* $S(H^o) = -H^oB'(H^o)$*, and the optimal debt level is* $\min\{G, -(H^o)^2 B'(H^o)/\rho\}$*.*

The normative role for debt in local project financing suggested by this Proposition is to appropriately regulate any development the project creates. Debt financing can fulfill this role because it is effectively a tax on development. This role differs from those identified in the literature; namely, to fairly allocate project costs across different cohorts of residents and to provide residents with the correct incentives to invest in projects. While these roles are obviously important in practice, our framework abstracts from them to highlight what is new. The first

[23] The optimal community size satisfies the condition that the per period marginal social benefit of an additional house (which is $S(H)$) equals the marginal social cost (which is $-HB'(H)$).

role does not apply in our setting because our social welfare function is utilitarian and we have assumed that utility is linear in consumption. The second role does not apply because we have assumed that the project decision is exogenous.[24]

7.3 Equilibrium debt

The equilibrium debt level is that preferred by the initial residents. Again, Proposition 1 tells us that debt is effectively a tax on future development. To understand the equilibrium debt-tax mix, therefore, we need to think through how the initial residents would want to tax development. Certainly, they will want to internalize the congestion impacts of development. Beyond this, they will want to raise tax revenues and the value of their homes. These two forces will lead the initial residents to increase the tax, and hence debt, above the optimal level. Indeed, it could be that they will choose to issue sufficient debt to completely choke off development.[25]

The following proposition characterizes the equilibrium level of debt.

Proposition 4. *Suppose that Ricardian Equivalence does not hold. Then, if*

$$S(H_0) \leq -H_0 B'(H_0) + \left(\frac{\mu(1-\beta)+1}{1-\mu\beta} \right) (1-\mu)\overline{\theta} H_0, \tag{22}$$

the initial residents prefer no development and any debt level at least as big as $\min\{G, H_0 S(H_0)/\rho\}$ *is an equilibrium debt level. If condition (22) does not hold, the initial residents prefer a level of development* $H^e - H_0$*, where* H^e *satisfies*

$$S(H^e) = -H^e B'(H^e) + \left(\overline{\theta} + \pi'(H^e)\right)(H^e - H_0) + \left(\frac{\mu(1-\beta)+1}{1-\mu\beta} \right) (1-\mu)\overline{\theta} H_0, \tag{23}$$

and the equilibrium debt level is $\min\{G, H^e S(H^e)/\rho\}$*. The equilibrium debt level is at least as high as the optimal level and strictly higher when the optimal debt level equals* $-(H^o)^2 B'(H^o)/\rho$*.*

The concern that current residents will rely too much on debt finance when debt allows the transfer of the burden of financing public projects to future residents is a standard one in the literature. This Proposition describes how this concern plays out in the model of this paper.

[24] Obviously, it would be possible to use the model to analyze the conditions under which debt would facilitate the project being provided.

[25] As noted in Section 2, Barseghyan and Coate (2022) analyze the political determination of a development tax in a similar model of community development in which growth is assumed to create negative externalities. However, they allow the development tax to vary over time. Their analysis reveals how residents' choice of development taxes is distorted by their desire to raise tax revenues and the value of their homes. It also shows that, when policies are chosen sequentially, multiple equilibrium development paths can emerge. In addition, it compares development taxes with other regulatory policies such as zoning and impact fees.

Relative to past work, a distinctive feature of this setting is that it incorporates housing ownership, capitalization, and development. Furthermore, recognizing that debt is effectively a tax on development permits an easier way of understanding the incentives at work than focusing on debt directly. The equivalent development tax is simple to understand because it only raises revenue in one period. By contrast, understanding the impact of a debt increase requires thinking through the implications of an intertemporal shifting of taxation that impacts all periods.

Raising the development tax will (typically) increase the housing price in period 1 and reduce the level of development.[26] The increased future value of housing will boost the period 0 price which will benefit those initial residents who have to leave the community in period 0. The reduced level of development will increase the future benefits coming from the project by decreasing congestion. The only downside is the possible impact on tax revenues. The optimal tax from the perspective of the initial residents will therefore balance the benefits of raising the period 0 housing price and reducing congestion with the costs of reduced tax revenues. The relative importance of these three factors will depend on the probability that the initial residents will remain in the community, which is μ. When μ equals 1, the initial residents do not care about the value of their homes. They only care about the congestion externality and tax revenues. In this case, they will always choose a tax level that allows some development (this can be seen from the fact that condition (22) can only hold when μ is less than 1). This relects the fact that the revenue gains from setting the tax to allow a small amount of development offset the congestion costs. At the other extreme, when μ equals 0, the initial residents only care about the value of their homes as they will be leaving the community for sure. Accordingly, they will (typically) increase the tax to the point at which development is choked off.

Interestingly, U.S. states have not generally provided their localities with the authority to tax development (Altshuler and Gomez-Ibanez 1993). Given that borrowing effectively creates a development tax, the same considerations that motivate this should also justify restrictions on local debt finance. Such restrictions are indeed commonplace but typically take the form of

[26] It is possible that a lower development tax could both increase the period 1 housing price and increase development. This can arise if the period 1 inverse demand curve is upward sloping over the first part of its range. This is possible because the transfer $\tau(H - H_0)/H$ is increasing in H. Our assumptions only guarantee the period 1 inverse demand curve has a slope less than the inverse supply curve. Starting from a level in which the tax chokes off development, the condition under which a small tax reduction will increase the period 1 housing price is that $S(H_0)$ exceeds $-H_0 B'(H_0) + \overline{\theta} H_0$. It is hard to find parameter values satisfying our Assumptions under which this inequality holds. Indeed, if (2) holds with equality so the community is optimally sized before the project is undertaken, this condition cannot hold if the project satisfies the Samuelson condition for desirability that the present value of the aggregate public good benefits $\beta H_0 B(H_0)/(1-\beta)$ exceeds its cost G.

requirements that a locality's aggregate debt should not exceed a certain fraction of its assessed property values. This is obviously distinct from restrictions based on the congestion externalities generated by future development.

8 Bonds of different maturities

We have thus far assumed that debt is in the form of one period bonds and that these bonds are rolled over, so that each cohort of residents after period 0 pays the same total amount for debt service. This is equivalent to assuming that debt is in the form of perpetual bonds that pay out a fixed coupon for ever, so-called consols. This is both the simplest assumption to make for the purposes of the analysis and squares with the assumption that the public project yields a perpetual benefit stream. Nonetheless, it is important to understand how alternative methods of debt financing can be incorporated into the analysis. Accordingly, this section extends the analysis to the case in which debt is in the form of bonds that have an n period maturity.

We now assume that borrowing an amount D to finance the project requires that the local government pay a coupon ρD in periods 1 through n and repays the principal D in period n. We further assume that the local government deals with this obligation by raising a constant amount of revenue in periods 1 through n sufficient both to pay the coupon and to accumulate enough reserves to repay the principal after n periods. This amount of revenue is

$$R(D,n) \equiv \frac{D}{\sum_{t=1}^{n} \beta^t}. \tag{24}$$

The case already analyzed corresponds to the limit when $n \to \infty$.

An equilibrium with debt is defined as before. Different maturities just alter the profile of tax payments $\{T_t\}_{t=0}^{\infty}$. Solving for equilibrium is more complicated because there is no taxation after period n and this provides an additional boost to development. Equilibrium either involves no development, development only in period $n+1$, or development in periods 1 and $n+1$. In each case, the price of housing increases through period $n+1$ to reflect the diminished future tax liability. Since there is nothing new conceptually, we relegate the details of describing the equilibrium to the On-line Appendix.

The development tax regime with which a debt regime is compared is now defined in a slightly different way: in particular, the development tax only applies in periods 1 through n. Thus, the community levies a tax G/H_0 on period 0 residents and then imposes a tax τ on each new house constructed in period $t \in \{1, ..., n\}$. As before, the revenues from the development tax in period t are divided equally between all those resident in the community at the end of period t. An equilibrium with n period development tax is defined as in Section 4 except that condition (16) applies only in periods 1 through n. Condition (10) applies in the later periods. Solving for equilibrium is again more complicated because the removal of the development tax in period $n+1$ provides an additional boost to development. Equilibrium either involves no development, development only in period $n+1$, or development in periods 1 and $n+1$. When there is no development, the price of housing remains constant through time. When there is development, housing prices decrease over time before becoming constant after period $n+1$. Again, we relegate the details to the On-line Appendix.

The generalization of Proposition 1 to the case of bonds with different maturities is as follows:

Proposition 5. *Suppose that debt takes the form of bonds that have an n period maturity. Let $\{H^*_{t+1}, P^*_t, R^*_t\}^\infty_{t=0}$ be an equilibrium with debt level D. Then, $\{H^*_{t+1}, \widetilde{P}_t, R^*_t\}^\infty_{t=0}$ is an equilibrium with n period development tax $\tau^* = D/\beta H_0$, where the price sequence $\{\widetilde{P}_t\}^\infty_{t=0}$ is given by $\widetilde{P}_0 = P^*_0$, $\widetilde{P}_1 = P^*_1 + \tau^*$, $\widetilde{P}_t = P^*_t + \tau^* H_0 \sum_{z=1}^{n+1-t} \beta^z / H^*_2 \sum_{z=1}^{n} \beta^z$ for $t = 2, ...n$, and $\widetilde{P}_t = P^*_t$ for all $t \geq n+1$. Furthermore, the payoffs of all agents in the two equilibria are identical.*

Thus, the equilibrium that arises when the community finances the project with a combination of a tax on period 0 residents and a debt issue in the form of n period bonds is equivalent to the equilibrium that arises with pure tax finance and a development tax that is levied in periods 1 through n. As in Proposition 1, the equivalent development tax equals the period 1 present value of the costs of servicing the debt divided by the number of initial period residents. Interestingly, therefore, the maturity of the bonds used only impacts the time period for which the equivalent development tax applies, not the size of this tax.

Like Proposition 1, this Proposition has immediate implications for Ricardian Equivalence and optimal and equilibrium debt levels when debt takes the form of n period bonds. Since these implications are similar to those discussed in Section 6, they are developed in the On-line Appendix. In addition, the Proposition has implications for another issue: optimal and equilibrium maturities. Since the community will be able to choose the maturity of the bonds it issues, it is natural to ask what would be the socially optimal maturity and what maturity would we expect residents to choose. The answers to these questions are immediate from Proposition 5: optimal and equilibrium maturities are both infinite. As noted, Proposition 5 tells us that the maturity of bonds determines the length of time the development tax is levied, but not the magnitude of that tax. Thus, the only issue is how long should the tax be applied. The socially optimal length of time for the development tax is infinite because the externality from development persists through time. This reflects the assumption that the project is infinitely durable in the sense that it yields a perpetual stream of congestible benefits. Similarly, the length of time for the development tax preferred by the initial residents will be infinite. A finite length limits the control they have over development. As noted above, an infinite maturity is equivalent to the arrangement considered in the base model in which one period bonds are continually rolled over. Thus, the results from the base model concerning optimal and equilibrium debt levels can be interpreted as applying whenever maturity is endogenous.

9 Discussion

This section discusses the positive implications of the analysis and the implications of changing some of the key assumptions underlying it. These assumptions concern the community's tax system, whether residents own or rent their housing, and community competition.

9.1 Positive implications

The fundamental positive implication of the basic model (i.e., the model with an exogenous debt-tax mix) is that a higher debt-tax mix in the financing of local projects should reduce development. This is because a higher debt-tax mix corresponds to a higher development tax and a higher development tax leads to lower development. Investigating whether such a causal relation exists is a task beyond the scope of this paper. However, one can readily measure the correlation between these objects in the data, conditional on several controls. We use data for 218 U.S. cities in the Fiscally Standardized Cities Database, Langley (2020), to run the following panel regression with time and city fixed effects:

$$\Delta R_{i,t+k} = \alpha_i + \eta_t + \beta_t D_{i,t} + \gamma \Delta R_{i,t+k-1} + \delta R_{i,t+k} + \varepsilon_{i,t+k},$$

where $R_{i,t+k}(\Delta R_{i,t+k})$ is the population (population growth) in city i in year $t+k$, and $D_{i,t}$ is the debt-to-public capital ratio held by city i in year t.[27] We find a statistically significant negative

[27] To construct the public capital stock we take the discounted (at a rate of 10%) sum of city capital spending in all previous years.

correlation: a 1 percentage point increase in the debt-to-public capital ratio is associated with more than 0.5 percentage point lower population growth, contemporaneously ($k = 0$) as well as in the future (for $k = 1, ..., 5$).

The model can also be used to make predictions on the debt-tax mix as explained in Section 7.3. It is then possible to study how changes in the models' key parameters impact the equilibrium debt-tax mix. Such parameters include the strength of demand to live in the community, as measured by $\overline{\theta}$, the permanence of residents, as measured by μ, and the cost of constructing new houses, as measured by C.

To illustrate, consider how to assess the impact of a change in $\overline{\theta}$ on the debt-tax mix. Assuming the parameters are such that the initial residents prefer a positive level of development, the equilibrium debt level is $H^e S(H^e)/\rho$ (Proposition 3).[28] Accordingly, the comparative static effect of an increase in $\overline{\theta}$ is

$$\frac{1}{\rho}\left[\frac{d\left(H^e S(H^e)\right)}{dH}\frac{\partial H^e}{\partial\overline{\theta}} + H^e\frac{\partial S(H^e)}{\partial\overline{\theta}}\right]. \tag{25}$$

The $\partial S(H^e)/\partial\overline{\theta}$ term is the derivative of the marginal household's per-period surplus as defined in (21) and is clearly positive. Thus, the second term in the square brackets is positive. The sign of the first term is ambiguous. The $\partial H^e/\partial\overline{\theta}$ term is the derivative of the equilibrium level of development with respect to $\overline{\theta}$ and can be computed from (23). It has an ambigous sign in general, but will be positive if H^e is less than 1/2. The term multiplying it, $d\left(H^e S(H^e)\right)/dH$, is negative, as shown in the proof of Proposition 4. This implies that the first term in the square brackets will be negative if H^e is less than 1/2. Nonetheless, computing all the terms and comparing them, we can show that the second term necessarily outweighs the first, making the whole expression positive.[29]

Proceeding in a similiar fashion, we can assess the impact of changing μ and C on the debt-tax mix. Summarizing the findings, the model predicts that an increase in the strength of demand to live in the community raises the debt-tax mix, while increasing the permanence of residents or the cost of construction reduces it. Testing such predictions would require finding convincing proxy measures for the model's parameters, which is something we leave for future research.

[28] This assumes that the upper bound constraint on debt ($D < G$) is not binding.

[29] The details are contained in the On-line Appendix.

9.2 Alternative assumptions

9.2.1 Taxes

Our model assumes that taxes are levied on the residents of the community. This is key to the finding that Ricardian Equivalence does not hold when the project stimulates development. The increase in future taxes created by an increase in debt reduces the demand for housing from potential residents. The supply curve of housing is unaffected and the result is a reduction in development (see Figure 1). This reduction in development has real effects.

An alternative assumption that would restore Ricardian Equivalence is that taxes are levied on land. While not common in practice, a pure land tax is a useful theoretical benchmark that is often studied in state and local public finance. The way it would work in our model is that each unit of land in period t would be assessed a tax ς_t. Recalling that the community consists of L units of land, revenues in period t would therefore equal $\varsigma_t L$.

Under such a tax system, the burden of financing the project would be bourne by the initial residents and the owners of undeveloped land no matter how it is financed. An increase in debt would just reduce future land and housing prices and have no impact on the level of development. While higher future land taxes would reduce the demand for housing from potential residents who would have to pay these taxes, they would also shift down the supply of housing as owners of undeveloped land seek to unload their land to avoid the future tax burden. The result is no change in development and a reduction in housing and land prices. This reduction in future land and home values would reduce prices in the initial period, fully offsetting the decrease in initial period taxes.

In the U.S., local governments finance their expenditures primarily with property taxes, sales taxes, user fees, and intergovernmental grants. Sales taxes and user fees will be paid by residents. Property taxes are also primarily paid by residents, although some may also be paid by owners of undeveloped land, depending on how undeveloped land is treated by the community's property tax system (agricultural land, for example, may be untaxed or taxed at a lower rate). In light of all this, we think our assumption is natural.

An alternative set-up that blends features of the tax system considered in our model and a land tax would be that the community relies on a property tax that taxes all property (houses and undeveloped land) at a flat proportionate rate. This case differs from the land tax because a home is worth more than the marginal unit of undeveloped land and thus residents pay more in taxes than do landowners. Under this assumption, debt would continue to reduce development and Ricardian Equivalence would not hold when the project stimulates development. We conjecture that debt financing would remain equivalent to pure tax finance coupled with a tax on development, although the details would differ because of the necessity of accounting for the taxes contributed by the owners of undeveloped land.

9.2.2 Home ownership

The model assumes that all residents own their homes. This contrasts with much of the literature in urban economics that assumes that residents rent from absentee landlords. It is straightforward to change the model to adopt this more standard assumption. We just need to assume that the initial housing stock is owned by absentee landlords and further that the owners of undeveloped land will build houses on their units and rent them out if and when they find it profitable to do so. Potential residents wishing to live in the community in any period t pay a per-period rental rate, which we denote r_t, that ensures that all available homes are rented and all potential residents wanting to rent are housed. Taxes continue to be levied only on residents.

In this set-up, an equilibrium will be summarized by a sequence of housing levels and rental rates $\{H_{t+1}, r_t\}_{t=0}^{\infty}$. The equilibrium path of development (i.e., the sequence $\{H_{t+1}\}_{t=0}^{\infty}$) will be the same as in the model with owners. An equivalence result analogous to Proposition 1 holds, although the relationship between the rental rates in an equilibrium with debt and an equilibrium with an equivalent development tax differs somewhat.[30] Proposition 2 remains true: Ricardian Equivalence holds if and only if there would be no development if the project is tax financed. Proposition 3 also remains true.

The main change concerns the political economy implications of the model (i.e., Proposition 4). If we assume that the financing decision is made by those renting houses in period 0, *they will choose to fully tax finance.*[31] The reason is that rental rates adjust to ensure that all available

[30] This equivalence result is as follows: Let $\{H_{t+1}^*, r_t^*\}_{t=0}^{\infty}$ be an equilibrium with debt level D. Then, $\{H_{t+1}^*, \widetilde{r}_t\}_{t=0}^{\infty}$ is an equilibrium with development tax $\tau^* = D/\beta H_0$, where the rental rate sequence $\{\widetilde{r}_t\}_{t=0}^{\infty}$ is given by $\widetilde{r}_0 = r_0^* - D/H_0$, $\widetilde{r}_1 = r_1^* + \tau^*$, and $\widetilde{r}_t = r_t^* + \tau^* H_0(1-\beta)/H_2^*$ for all $t \geq 2$. Furthermore, the payoffs of all agents in the two equilibria are identical.

[31] This prediction differs from that proposed by Stadelmann and Eichenberger (2012). They argue that renters will prefer more debt than property owners because the latter suffer more from a higher debt burden because of the impact on the value of their properties. Consistent with this, they find a strong and robust negative effect of higher property ownership on the debt-to-tax ratio using data from a cross section of the 171 communities in the Swiss Canton of Zurich in the year 2000.

homes are rented. Holding fixed the available homes, if a tax is going to be imposed on residents, the equilibrium rental rate will just fall by the amount of this tax. Similarly, if residents expect to enjoy the benefits of a public project, the equilibrium rental rate will rise by the amount of this benefit. Thus, given that there is no development in period 0, the period 0 rental rate will just fall to compensate period 0 renters for the period 0 tax. Similarly, future rental rates will be adjusted to compensate for future tax payments necessary to finance debt and for the benefits of the project. Ultimately what matters for renters is therefore the size of the stock of available homes: the larger it is, the higher the surplus they obtain. Intuitively, potential residents must be offered more surplus to rent a larger stock of homes. Renters therefore want to encourage the maximum possible development. They do this by maximizing the discounted present value of rental rates that owners of undeveloped land can earn if they build homes on their units in period 1. This requires making sure that residents do not have any debt payments to finance in periods 1 and beyond.

9.2.3 Competing communities

This paper takes a partial equilibrium approach, focusing on the decisions of one community in isolation. The community faces a pool of potential residents with heterogenous preferences for the community's amenities, creating a downward-sloping demand curve. When choosing policies, the initial residents are thus in the position of a monopolist. In practice, communities must compete for residents and it is natural to wonder how such competition would change the conclusions about equilibrium debt.

Extending this type of model of community development to incorporate multiple communities would be a challenging task. Nonetheless, we would expect the main conclusions to hold provided that the communities retain some market power. It is the market power that leads residents to increase debt above the optimal level. Market power seems very plausible even with multiple communities because natural amenities make every community is unique. Thus, potential residents will have heterogeneous preferences that make them favor particular communities. Obviously, the degree of distortion will be attenuated by community competition, but the nature of it should not change.

10 Conclusion

Given the amount of borrowing done by local governments in practice, understanding the role of debt in the financing of local public projects seems an important problem. This paper has explored the issue in a model that incorporates some of the features that seem particularly salient for the local setting. These are i) population turnover, so current residents need not be future residents; ii) durable housing, so future taxes can be capitalized into housing prices; and iii) the possibility of development.

In this model, debt financing is effectively a tax on development. Specifically, financing with a debt-tax mix is equivalent to pure tax finance coupled with a tax on development whose revenues are shared by those residing in the community when they are raised. The logic underlying this novel equivalence result exploits two key observations. First, because of capitalization, debt does not matter for agents' payoffs if there is no development. However, when there is development, by transfering tax payments to periods in which there is a larger tax base, debt has the potential to benefit those residing in the community when the project is undertaken. Second, levying a development tax and sharing the revenues among the current residents can create exactly the same benefits for the earlier residents in a present value sense as they obtain from spreading the costs of the project among a larger group of residents.

In the model, the equivalence result has a number of interesting implications for project financing. First, if the project does not create development, then the way it is financed does not matter. Second, if the project does create development, the appropriate normative role for debt is to counter any negative externalities coming from the development created by the project. If there are no such externalities, then the project should be tax financed. Third, if localities are given free rein in choosing project financing, they will over-rely on debt.

The base model assumes that debt takes the form of one period bonds that are rolled over indefinitely creating a constant stream of tax obligations for future residents. However, the equivalence result generalizes to the case in which debt takes the form of bonds with an n period maturity that are not rolled over. Specifically, debt financing with n period bonds is equivalent to pure tax finance coupled with an n period tax on development. This observation generates the additional implication that both the socially optimal maturity and the maturity preferred by those resident in the community when the project is undertaken is infinite.

The model employed in this paper is undeniably simple. This has permitted the development of what we hope are sharp and interesting results. While we have discussed briefly some extensions of the model, there are many other features of local economies that could usefully be incorporated. These include decay of the housing stock, depreciation of the project, heterogenous housing stock, and exogenous population growth. Also potentially important are institutional features of the municipal debt market like the fact that communities may enjoy lower borrowing costs than their residents (Banzhaf and Oates 2013). We hope that this paper will stimulate further research on this important topic.

References

Akai, N., (1994), "Ricardian Equivalence for Local Government Bonds: Budget Constraint Approach," *Economics Letters*, 44(1-2), 191-195.

Albrecht, K., (2015), "Capitalization of Unfunded Public Pension Liabilities," Chapter 2 of PhD Dissertation, Cornell University.

Bailey, M., (1993), "Note on Ricardian Equivalence," *Journal of Public Economics*, 51(3), 437-446.

Banzhaf, S. and W. Oates, (2013), "On Fiscal Illusion in Local Public Finance: Re-Examining Ricardian Equivalence and the Renter Effect," *National Tax Journal*, 66(3), 511-540.

Barro, R., (1974), "Are Government Bonds Net Wealth?" *Journal of Political Economy*, 82(6), 1095-1117.

Barseghyan, L. and S. Coate, (2021), "Community Development by Public Wealth Accumulation," *Journal of Urban Economics,* 121, 103297.

Barseghyan, L. and S. Coate, (2022), "Community Development with Externalities and Corrective Taxation," *Journal of Economic Geography*, 22(3), 499-545.

Bassetto, M. and L. McGranahan, (2021), "Mobility, Population Growth, and Public Capital Spending in the United States," *Review of Economic Dynamics*, 41, 255-277.

Bassetto, M. with T. Sargent, (2006), "Politics and Efficiency of Separating Capital and Ordinary Government Budgets," *Quarterly Journal of Economics*, 121(4), 1167-1210.

Bernheim, B.D., (1987), "Ricardian Equivalence: An Evaluation of Theory and Evidence," *NBER Macroeconomics Annual*, 2, 263-304.

Blanchard, O., (1985), "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223-247.

Brinkman, J., D. Coen-Pirani and H. Sieg, (2018), "The Political Economy of Municipal Pension Funding," *American Economic Journal: Macroeconomics*, 10(3), 215-246.

Brueckner, J., (1997), "Infrastructure Financing and Urban Development: The Economics of Impact Fees," *Journal of Public Economics*, 66, 383-407.

Buiter, W., (1989), "Debt Neutrality, Professor Vickrey and Henry George's 'Single Tax'," *Economics Letters* 29(1), 43-47.

Daly, G., (1969), "The Burden of the Debt and Future Generations in Local Finance," *Southern Economic Journal*, 36(1), 44-51.

Diamond, P., (1965), "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55(5), 1126-1150.

Epple, D. and K. Schipper, (1981), "Municipal Pension Funding: A Theory and Some Evidence," *Public Choice*, 37(1), 179-187.

Glaeser, E. and G. Ponzetto, (2014), "Shrouded Costs of Government: The Political Economy of State and Local Public Pensions," *Journal of Public Economics*, 116, 89-105.

Hatfield, J., (2010), "Ricardian Equivalence for Local Government Bonds: A Utility Maximization Approach," *Economics Letters*, 107(2), 148-151.

Henderson, J.V., (1994), "Community Choice of Revenue Instruments," *Regional Science and Urban Economics*, 24, 59-83.

Henderson, J.V., (1995), "Will Homeowners Impose Property Taxes?" *Regional Science and Urban Economics*, 25, 153-181.

Hilber, C., (2017), "The Economic Implications of House Price Capitalization: A Synthesis," *Real Estate Economics*, 45(2), 301-339.

Hilber, C. and C. Mayer, (2009), "Why Do Households Without Children Support Local Public Schools? Linking House Price Capitalization to School Spending," *Journal of Urban Economics*, 65(1), 74-90.

Hoyt, W., (1991), "Competitive Jurisdictions, Congestion, and the Henry George Theorem: When Should Property be Taxed Instead of Land," *Regional Science and Urban Economics*, 21, 351–370.

Inman, R., (1989), "The Local Decision to Tax: Evidence from Large U.S. Cities," *Regional Science and Urban Economics*, 19, 455–491.

Langley, A., (2020),"Methodology Used to Create the Fiscally Standardized Cities Database," Lincoln Institute of Land Policy, https://www.lincolninst.edu/research-data/data-toolkits/fiscally-standardized-cities.

Leeds, M., (1985), "Property Values and Pension Underfunding in the Local Public Sector," *Journal of Urban Economics*, 18(1), 34-46.

MacKay, R., (2014), "Implicit Debt Capitalization in Local Housing Prices: An Example of Unfunded Pension Liabilities," *National Tax Journal*, 67(1), 77-112.

Musgrave, R., (1959), *The Theory of Public Finance*, McGraw Hill.

Oates, W., (1969), "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis," *Journal of Political Economy*, 77, 57-71.

Ricciuti, R., (2003), "Assessing Ricardian Equivalence," *Journal of Economic Surveys*, 17(1), 55-78.

Schultz, C. and T. Sjostrom, (2001), "Local Public Goods, Debt, and Migration," *Journal of Public Economics*, 80, 313-337.

Seater, J., (1993), "Ricardian Equivalence," *Journal of Economic Literature*, 31(1), 142-190.

Stadelmann, D. and R. Eichenberger, (2012), "Consequences of Debt Capitalization: Property Ownership and Debt versus Tax Choice," *Southern Economic Journal*, 78(3), 976-998.

Stadelmann, D. and R. Eichenberger, (2014), "Public Debts Capitalize into Property Prices: Empirical Evidence for a New Perspective on Debt Incidence," *International Tax and Public Finance*, 21(3), 498-529.

Temple, J., (1994), “The Debt/Tax Choice in the Financing of State and Local Capital Expenditures,” *Journal of Regional Science*, 34(4), 529-547.

Weil, P., (1989), “Overlapping Families of Infinitely-lived Agents,” *Journal of Public Economics*, 38(2), 183-198.

11 Appendix

11.1 Payoffs in the equilibrium with debt

Consider first the period t potential residents for $t \geq 1$. If θ exceeds $(1 - H_2)\overline{\theta}$, such a household purchases a home in the community in period t. In period $t + 1$, if they remain in the pool, they enjoy a continuation payoff which is the same as in period t, except that they avoid the cost of buying a house. Given that $P_{t+1} = P_t = P_1$, this implies that

$$V_{\theta t} = \theta + B(H_2) - \frac{\rho D}{H_2} - P_1 + \beta \left[\mu(V_{\theta t} + P_1) + (1 - \mu)P_1\right]. \tag{26}$$

Solving this equation for $V_{\theta t}$, we have that for all $t \geq 1$

$$V_{\theta t} = \frac{\theta + B(H_2) - \frac{\rho D}{H_2}}{1 - \mu\beta} - \frac{(1 - \beta)P_1}{1 - \mu\beta}. \tag{27}$$

This is just the present discounted value of the flow of payoffs less the per-period cost of the house. If θ is less than $(1 - H_2)\overline{\theta}$, a period t potential resident never purchases a home in the community and thus their payoff is zero.

Next consider the period 0 potential residents. If θ exceeds $(1 - H_0)\overline{\theta}$, such a household purchases a home in the community in period 0. In period 1, if they remain in the pool, they enjoy a continuation payoff which is equal to $V_{\theta 1}$ except that they avoid the cost of buying a house. Thus,

$$V_{\theta 0} = \theta - \frac{G - D}{H_0} - P_0 + \beta \left[\mu(V_{\theta 1} + P_1) + (1 - \mu)P_1\right]. \tag{28}$$

Using (27), we can write this as:

$$V_{\theta 0} = \frac{\theta + \mu\beta \left(B(H_2) - \frac{\rho D}{H_2}\right)}{1 - \mu\beta} - \frac{G - D}{H_0} - P_0 + \frac{(1 - \mu)\beta P_1}{1 - \mu\beta}. \tag{29}$$

This expression reveals how the resident's future payoff depends on both the taxes to service the debt and the value of housing. If θ is between $(1-H_2)\overline{\theta}$ and $(1-H_0)\overline{\theta}$ a period 0 potential resident purchases a home in the community in period 1 if they remain in the pool at that time and enjoys a continuation payoff equal to $V_{\theta 1}$. Thus, $V_{\theta 0}$ is equal to $\beta\mu V_{\theta 1}$. If θ is less than $(1 - H_2)\overline{\theta}$, a period 0 potential resident obtains a zero payoff.

The final cohort of residents are the initial potential residents. If they remain in the pool in period 0, the initial residents; i.e., those for whom θ exceeds $(1 - H_0)\overline{\theta}$, enjoy a payoff equal to $V_{\theta 0}$ except that they avoid the cost of buying a house. Thus,

$$V_\theta = \mu \left(V_{\theta 0} + P_0\right) + (1 - \mu)P_0 = \mu V_{\theta 0} + P_0. \tag{30}$$

The initial potential residents for whom θ is between $(1 - H_2)\overline{\theta}$ and $(1 - H_0)\overline{\theta}$ do not own a home but purchase a home in the community in period 1 if they remain in the pool and enjoy a continuation payoff equal to $V_{\theta 1}$. Thus, $V_\theta = \mu\beta\mu V_{\theta 1}$. If θ is less than $(1 - H_2)\overline{\theta}$, an initial potential resident obtains a payoff of zero.

Finally, consider the landowners. Those owning land with return π between $\pi(H_0)$ and $\pi(H_2)$ sell their land in period 1 and obtain a payoff of $\pi + \beta R_1$. Those owning land with higher return never sell their land for building and just obtain a payoff $\pi/(1 - \beta)$.

11.2 Payoffs in the equilibrium with development tax

Consider first the period $t \geq 2$ potential residents. Given that $P_{t+1} = P_t = P_2$, if θ exceeds $(1 - H_2)\overline{\theta}$, the same logic used to derive (27) can be used to show that

$$V_{\theta t} = \frac{\theta + B(H_2)}{1 - \mu\beta} - \frac{(1 - \beta)P_2}{1 - \mu\beta}. \tag{31}$$

The remaining period t potential residents obtain a payoff of zero.

Turning to period 1 potential residents, the logic used to derive (28) implies that if θ exceeds $(1 - H_2)\overline{\theta}$, it is the case that

$$V_{\theta 1} = \theta + B(H_2) + \frac{\tau(H_2 - H_0)}{H_2} - P_1 + \beta\left[\mu(V_{\theta 2} + P_2) + (1 - \mu)P_2\right]. \tag{32}$$

Using (31), this implies that

$$V_{\theta 1} = \frac{\theta + B(H_2)}{1 - \mu\beta} + \frac{\tau(H_2 - H_0)}{H_2} - P_1 + \frac{(1 - \mu)\,\beta}{1 - \mu\beta}P_2. \tag{33}$$

The remaining period 1 potential residents obtain a payoff of zero.

Continuing backwards in time, consider the period 0 potential residents. If θ exceeds $(1 - H_0)\overline{\theta}$, by now familiar logic implies that

$$V_{\theta 0} = \theta - \frac{G}{H_0} - P_0 + \beta\left[\mu(V_{\theta 1} + P_1) + (1 - \mu)P_1\right]. \tag{34}$$

Using (33), we can write this as:

$$V_{\theta 0} = \frac{\theta + \mu\beta B(H_2)}{1 - \mu\beta} - \frac{G}{H_0} - P_0 + \mu\beta\frac{\tau(H_2 - H_0)}{H_2} + (1 - \mu)\beta P_1 + \mu\beta\frac{(1 - \mu)\,\beta}{1 - \mu\beta}P_2. \tag{35}$$

For the initial potential residents, the same logic and formulas apply as in the policy regime with debt. Thus, the initial residents enjoy a payoff of $\mu V_{\theta 0} + P_0$, the initial potential residents for whom θ is between $(1-H_2)\overline{\theta}$ and $(1-H_0)\overline{\theta}$ obtain a payoff $\mu\beta\mu V_{\theta 1}$, and the remainder obtain a payoff of zero. The same is true for the landowners. Thus, owning land with return π between $\pi(H_0)$ and $\pi(H_2)$ obtain a payoff of $\pi+\beta R_1$ and those owning land with higher return obtain $\pi/(1-\beta)$.

11.3 Proof of Proposition 1

Let $\{H_{t+1}^*, P_t^*, R_t^*\}_{t=0}^{\infty}$ be an equilibrium with debt level D. We first show that $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$ is an equilibrium with development tax τ^* where the tax τ^* and price sequence $\{\widetilde{P}_t\}_{t=0}^{\infty}$ are as defined in the statement of the Proposition. There are two possibilities to consider: the equilibrium with debt involves development ($H_2^* > H_0$) or no development ($H_2^* = H_0$). We consider each in turn.

Development In the development case, we know from the discussion in Section 4 that

$$\frac{(1-H_2^*)\overline{\theta}+B(H_2^*)}{1-\beta}-\frac{D}{\beta H_2^*}=C+\frac{\pi(H_2^*)}{1-\beta}, \tag{36}$$

and that, for all $t \geq 1$, (P_t^*, R_t^*) is equal to $(C+\pi(H_2^*)/(1-\beta), \pi(H_2^*)/(1-\beta))$. Furthermore, in period 0, there is no development so H_1^* equals H_0. The price of building land is $\pi(H_0)+\beta\pi(H_2^*)/(1-\beta)$ and the price of housing is

$$P_0^* = (1-H_0)\overline{\theta}-\frac{G-D}{H_0}+\beta\left(C+\frac{\pi(H_2^*)}{1-\beta}\right).$$

We now show that $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$ satisfies the conditions to be an equilibrium with development tax τ^* that were described in Section 5. Note from (36) that

$$\frac{H_2^*\left[(1-H_2^*)\overline{\theta}+B(H_2^*)-C(1-\beta)-\pi(H_2^*)\right]}{H_0}=\frac{H_2^*\left[\frac{(1-\beta)D}{\beta H_2^*}\right]}{H_0}=\tau^*(1-\beta).$$

Rearranging this equation reveals that

$$\frac{(1-H_2^*)\overline{\theta}+B(H_2^*)}{1-\beta}+\frac{\tau^*(H_2^*-H_0)}{H_2^*}=C+\frac{\pi(H_2^*)}{1-\beta}+\tau^*. \tag{37}$$

Thus, H_2^* satisfies the market clearing condition for the equilibrium with development tax τ^* described in Section 5.

Turning to prices, we have that

$$\widetilde{P}_0 \quad = \quad P_0^* = (1-H_0)\overline{\theta}-\frac{G-D}{H_0}+\beta\left(C+\frac{\pi(H_2^*)}{1-\beta}\right)$$

$$
\begin{aligned}
&= (1-H_0)\overline{\theta} - \frac{G}{H_0} + \beta\left(C + \frac{\pi(H_2^*)}{1-\beta} + \frac{D}{\beta H_0}\right) \\
&= (1-H_0)\overline{\theta} - \frac{G}{H_0} + \beta\left(C + \frac{\pi(H_2^*)}{1-\beta} + \tau^*\right).
\end{aligned}
$$

This is the equation for the period 0 housing price in the equilibrium with development tax τ^* given in (20). Furthermore,

$$
\widetilde{P}_1 = P_1^* + \tau^* = C + \frac{\pi(H_2^*)}{1-\beta} + \tau^*,
$$

and for all $t \geq 2$

$$
\begin{aligned}
\widetilde{P}_t &= P_t^* + \tau^* \frac{H_0}{H_2^*} \\
&= C + \frac{\pi(H_2^*)}{1-\beta} + \tau^* \frac{H_0}{H_2^*} \\
&= C + \frac{\pi(H_2^*)}{1-\beta} + \tau^* - \tau^* \left(\frac{H_2^* - H_0}{H_2^*}\right) \\
&= \frac{(1-H_2^*)\overline{\theta} + B(H_2^*)}{1-\beta},
\end{aligned}
$$

where the last line follows from (37). These are the housing price rules described in the description of equilibrium in Section 5 and (17). The building land prices also line up with those described in Section 5.

Finally, we verify that there is no incentive for development in period 0. This requires showing that

$$
\widetilde{P}_0 < C + \pi(H_0) + \beta \frac{\pi(H_2^*)}{1-\beta}.
$$

This is equivalent to

$$
\widetilde{P}_0 = (1-H_0)\overline{\theta} - \frac{G-D}{H_0} + \beta\left(C + \frac{\pi(H_2^*)}{1-\beta}\right) < C + \pi(H_0) + \beta\frac{\pi(H_2^*)}{1-\beta},
$$

which amounts to

$$
(1-H_0)\overline{\theta} - \frac{G-D}{H_0} < C(1-\beta) + \pi(H_0).
$$

This is implied by assumption (2). We conclude that $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$ is an equilibrium with development tax τ^*.

No development In the no development case, we know from the discussion in Section 4 that

$$
\frac{(1-H_0)\overline{\theta} + B(H_0)}{1-\beta} - \frac{D}{\beta H_0} \leq C + \frac{\pi(H_0)}{1-\beta}. \tag{38}
$$

The price of building land in each period equals $\pi(H_0)/(1-\beta)$. In period 1 and beyond the price of housing is equal to

$$P_t^* = \frac{(1-H_0)\overline{\theta} + B(H_0)}{1-\beta} - \frac{D}{\beta H_0},$$

and the period 0 price of housing equals

$$P_0^* = \frac{(1-H_0)\overline{\theta} + \beta B(H_0)}{1-\beta} - \frac{G}{H_0}.$$

We now show that $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$ satisfies the conditions to be an equilibrium with development tax τ^* that were described in Section 5. It follows from (38) that

$$\frac{(1-H_0)\overline{\theta} + B(H_0)}{1-\beta} \leq C + \frac{\pi(H_0)}{1-\beta} + \tau^*,$$

which is the condition under which there would be no development in the equilibrium with development tax τ^*. In addition,

$$\widetilde{P}_0 = P_0^* = \frac{(1-H_0)\overline{\theta} + \beta B(H_0)}{1-\beta} - \frac{G}{H_0}$$

and, given that $H_2^* = H_0$, for all $t \geq 1$

$$\begin{aligned} \widetilde{P}_t &= P_t^* + \tau^* = \frac{(1-H_0)\overline{\theta} + B(H_0)}{1-\beta} - \frac{D}{\beta H_0} + \tau^* \\ &= \frac{(1-H_0)\overline{\theta} + B(H_0)}{1-\beta}. \end{aligned}$$

Thus, housing prices are as described in (17) and (19). The building land prices also line up with those described in Section 5 in the no development case. Thus, $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$ is an equilibrium with development tax τ^*.

Payoffs It remains to show that agents have the same payoffs in the two equilibria. We begin with the period $t \geq 2$ potential residents. As described in (27), in the equilibrium with debt $\{H_{t+1}^*, P_t^*, R_t^*\}_{t=0}^{\infty}$, the period $t \geq 2$ potential residents who purchase housing in the community get a payoff of

$$V_{\theta t}^* = \frac{\theta + B(H_2^*) - \frac{\rho D}{H_2^*} - (1-\beta)P_1^*}{1-\mu\beta}.$$

As described in (31), in the equilibrium with development tax τ^* $\{H_{t+1}^*, \widetilde{P}_t, R_t^*\}_{t=0}^{\infty}$, the period $t \geq 2$ potential residents who purchase housing in the community get a payoff of

$$\widetilde{V}_{\theta t} = \frac{\theta + B(H_2^*) - (1-\beta)\widetilde{P}_2}{1-\mu\beta}$$

$$
\begin{aligned}
&= \frac{\theta + B(H_2^*) - (1-\beta)\left(P_2^* + \tau^* \frac{H_0}{H_2^*}\right)}{1-\mu\beta} \\
&= \frac{\theta + B(H_2^*) - (1-\beta)\left(P_1^* + \tau^* \frac{H_0}{H_2^*}\right)}{1-\mu\beta}.
\end{aligned}
$$

Thus, for $V_{\theta t}^* = \widetilde{V}_{\theta t}$ we need that

$$
(1-\beta)\tau^* \frac{H_0}{H_2^*} = \frac{\rho D}{H_2^*},
$$

which follows from the definition of τ^*.

Next consider the period 1 potential residents who purchase housing in the community. As described in (27), in the equilibrium with debt they get

$$
V_{\theta 1}^* = \frac{\theta + B(H_2^*) - \frac{\rho D}{H_2^*} - (1-\beta)P_1^*}{1-\mu\beta}.
$$

As described in (33), in the equilibrium with development tax, they get

$$
\begin{aligned}
\widetilde{V}_{\theta 1} &= \frac{\theta + B(H_2^*)}{1-\mu\beta} + \frac{\tau^*(H_2^* - H_0)}{H_2^*} - \widetilde{P}_1 + \frac{(1-\mu)\beta}{1-\mu\beta}\widetilde{P}_2 \\
&= \frac{\theta + B(H_2^*)}{1-\mu\beta} + \frac{\tau^*(H_2^* - H_0)}{H_2^*} - P_1^* - \tau^* + \frac{(1-\mu)\beta}{1-\mu\beta}\left(P_1^* + \tau^* \frac{H_0}{H_2^*}\right) \\
&= \frac{\theta + B(H_2^*)}{1-\mu\beta} - P_1^* \left(\frac{1-\beta}{1-\mu\beta}\right) - \tau^* \frac{H_0}{H_2^*}\left(\frac{1-\beta}{1-\mu\beta}\right) \\
&= \frac{\theta + B(H_2^*) - \tau^* \frac{H_0}{H_2^*}(1-\beta) - (1-\beta)P_1^*}{1-\mu\beta} \\
&= \frac{\theta + B(H_2^*) - \frac{\rho D}{H_2^*} - (1-\beta)P_1^*}{1-\mu\beta} = V_{\theta 1}^*.
\end{aligned}
$$

Continuing backwards, consider the period 0 potential residents. As described in (29), in the equilibrium with debt, if they purchase a house in the community in period 0, they get

$$
V_{\theta 0}^* = \frac{\theta + \mu\beta\left(B(H_2^*) - \frac{\rho D}{H_2^*}\right)}{1-\mu\beta} - \frac{G-D}{H_0} - P_0^* + \frac{(1-\mu)\beta P_1^*}{1-\mu\beta}.
$$

As described in (35), in the equilibrium with a development tax, they get

$$
\begin{aligned}
\widetilde{V}_{\theta 0} &= \frac{\theta + \mu\beta B(H_2^*)}{1-\mu\beta} - \frac{G}{H_0} - \widetilde{P}_0 + \beta\mu \frac{\tau^*(H_2^* - H_0)}{H_2^*} + \beta(1-\mu)\widetilde{P}_1 + \beta\mu \frac{(1-\mu)\beta}{1-\mu\beta}\widetilde{P}_2 \\
&= \frac{\theta + \mu\beta B(H_2^*)}{1-\mu\beta} - \frac{G}{H_0} - P_0^* + \beta\mu \frac{\tau^*(H_2^* - H_0)}{H_2^*} + \beta(1-\mu)\left(P_1^* + \tau^*\right) + \beta\mu \frac{(1-\mu)\beta}{1-\mu\beta}\left(P_1^* + \tau^* \frac{H_0}{H_2^*}\right) \\
&= \frac{\theta + \mu\beta B(H_2^*)}{1-\mu\beta} - \frac{G}{H_0} - P_0^* + \frac{\beta(1-\mu)}{1-\mu\beta}P_1^* + \beta\tau^* + \beta\mu\left(\frac{(1-\mu)\beta}{1-\mu\beta} - 1\right)\tau^* \frac{H_0}{H_2^*} \\
&= \frac{\theta + \mu\beta B(H_2^*)}{1-\mu\beta} - \frac{G}{H_0} - P_0^* + \frac{(1-\mu)\beta}{1-\mu\beta}P_1^* + \beta\tau^* - \beta\mu\left(\frac{1-\beta}{1-\mu\beta}\right)\tau^* \frac{H_0}{H_2^*}.
\end{aligned}
$$

Thus, we need to show that

$$\frac{D}{H_0} - \frac{\mu\beta\frac{\rho D}{H_2^*}}{1-\mu\beta} = \beta\tau^* - \beta\mu\left(\frac{1-\beta}{1-\mu\beta}\right)\tau^*\frac{H_0}{H_2^*}.$$

We have

$$\begin{aligned}
\beta\tau^* - \beta\mu\left(\frac{1-\beta}{1-\mu\beta}\right)\tau^*\frac{H_0}{H_2^*} &= \beta\frac{\rho D}{H_0(1-\beta)} - \beta\mu\left(\frac{1-\beta}{1-\mu\beta}\right)\frac{\rho D}{H_0(1-\beta)}\frac{H_0}{H_2^*} \\
&= \beta\frac{\rho D}{H_0(1-\beta)} - \left(\frac{\beta\mu}{1-\mu\beta}\right)\frac{\rho D}{H_2^*} \\
&= \frac{D}{H_0} - \frac{\beta\mu\frac{\rho D}{H_2^*}}{1-\mu\beta}.
\end{aligned}$$

Finally, consider the initial potential residents. In the equilibrium with debt, the initial residents get $V_\theta^* = \mu V_{\theta 0}^* + P_0^*$, while in the equilibrium with a development tax they get $\widetilde{V}_\theta = \mu\widetilde{V}_{\theta 0} + \widetilde{P}_0$. We know that $\widetilde{P}_0 = P_0^*$ and we have shown that $\widetilde{V}_{\theta 0} = V_{\theta 0}^*$. Thus, these payoffs are the same. The same is true for the remaining initial potential residents. Landowners have the same payoffs in the two equilibria because the prices of building land are the same across the two equilibria.

11.4 Proof of Proposition 2

Suppose first that $S(H_0) \leq 0$. By definition, Ricardian Equivalence holds when the payoffs of all agents in the equilibrium with debt level D do not depend on D. Consider a particular level of $D > 0$. By Proposition 1, there exists an equilibrium with development tax $\tau = D/\beta H_0$ in which the payoffs of all agents are the same as in the equilibrium with debt level D. Note that in this equilibrium there will be no development since

$$\frac{(1-H_0)\overline{\theta} + B(H_0)}{1-\beta} < C + \frac{\pi(H_0)}{1-\beta} + \tau.$$

Thus, the equilibrium will be exactly the same as that which would arise if the project were tax financed and there were no development tax; i.e., the equilibrium with development tax 0. This equilibrium is obviously the same as the equilibrium with debt level 0. Accordingly, all agents' payoffs in the equilibrium with debt level D are the same as those in the equilibrium with debt level 0 and Ricardian Equivalence holds.

For the converse, suppose that $S(H_0) > 0$. Note that in this case, the level of development that would arise in the equilibrium with development tax 0, which we denote $H_2(0)$, satisfies $S(H_2) = 0$. Now consider a particular level of debt $D > 0$. By Proposition 1, there exists an equilibrium with development tax $\tau = D/\beta H_0$ in which the payoffs of all agents are the same as in the equilibrium with debt level D. The level of development in this equilibrium, which we denote $H_2(\tau)$, satisfies

$$\frac{(1-H_2)\overline{\theta}+B(H_2)}{1-\beta}+\frac{\tau(H_2-H_0)}{H_2}=C+\frac{\pi(H_2)}{1-\beta}+\tau,$$

or equivalently $S(H_2) = \tau H_0(1-\beta)/H_2$. Since $S(H)$ is decreasing, $H_2(\tau) < H_2(0)$. This implies that the payoffs of agents in the equilibrium with development tax τ are not the same as those in the equilibrium with development tax 0. Consider, for example, the payoffs of period t potential residents for $t \geq 2$. From (31), they get a payoff of

$$\frac{\theta + B(H_2) - (1-\beta)P_2}{1-\mu\beta}.$$

Substituting in for P_2, this becomes

$$\frac{\theta + B(H_2) - (1-\beta)\left(\frac{(1-H_2)\overline{\theta}+B(H_2)}{1-\beta}\right)}{1-\mu\beta} = \frac{\theta-(1-H_2)\overline{\theta}}{1-\mu\beta},$$

which is increasing in H_2. It follows that the payoffs of all agents in the equilibrium with debt level D are not the same as those in the equilibrium with debt level 0 and thus Ricardian Equivalence does not hold.

11.5 Proof of Proposition 3

The planner's objective function is

$$W = \int_0^{\overline{\theta}} V_\theta \frac{d\theta}{\overline{\theta}} + (1-\mu)\sum_{t=0}^{\infty}\beta^t \int_0^{\overline{\theta}} V_{\theta t}\frac{d\theta}{\overline{\theta}} + L\int_{\pi(H_0)}^{\overline{\pi}} V_\pi \frac{d\pi}{\overline{\pi}-\underline{\pi}}.$$

Substituting the payoff expressions derived in Section 10.1 into this expression, we can show that welfare in an equilibrium with debt level D, can be written very simply as

$$\begin{aligned} W = \beta C H_0 - G + \textstyle\int_{\overline{\theta}(1-H_0)}^{\overline{\theta}} \theta \frac{d\theta}{\overline{\theta}} + L\int_{\pi(H_0)}^{\overline{\pi}} \pi \frac{d\pi}{\overline{\pi}-\underline{\pi}} \\ +\tfrac{\beta}{1-\beta}\left(\int_{\overline{\theta}(1-H_2)}^{\overline{\theta}} (\theta + B(H_2))\frac{d\theta}{\overline{\theta}} + L\int_{\pi(H_2)}^{\overline{\pi}} \pi \frac{d\pi}{\overline{\pi}-\underline{\pi}} - C(1-\beta)H_2\right). \end{aligned} \tag{39}$$

Here H_2 is the equilibrium size of the community which is reached in period 1. This should make sense intuitively because, given linear consumption utility and utilitarian welfare weights, the planner is indifferent to the allocation of surplus among agents. All that matters is aggregate surplus and the only thing impacting this is the level of development.

Observe that the first derivative of welfare with respect to H_2 is

$$\frac{\beta}{1-\beta}\left[(1-H_2)\overline{\theta}+B(H_2)+H_2B'(H_2)-C(1-\beta)-\pi(H_2)\right],$$

while the second derivative is

$$\frac{\beta}{1-\beta}\left(-\overline{\theta}+2B'(H_2)+H_2B''(H_2)-\pi'(H_2)\right)<0.$$

It follows that welfare is concave in H_2. Accordingly, if

$$(1-H_0)\overline{\theta}+B(H_0)+H_0B'(H_0)\leq C(1-\beta)+\pi(H_0),$$

the optimal level of H_2 from the planner's perspective is H_0, meaning that no development is optimal. Otherwise, some development is optimal and the optimal community size satisfies the first order condition

$$(1-H_2)\overline{\theta}+B(H_2)+H_2B'(H_2)=C(1-\beta)+\pi(H_2).$$

This condition says that the per-period social marginal benefit of an additional resident is equal to the marginal cost. Let H^o denote this optimal level and note that the first order condition can be rewritten as $S(H^o)=-H^oB'(H^o)$.

Suppose first that no development is optimal. In an equilibrium with debt level D, there will be no development if

$$\frac{(1-H_0)\overline{\theta}+B(H_0)}{1-\beta}-\frac{D}{\beta H_0}\leq C+\frac{\pi(H_0)}{1-\beta}.$$

Thus, any debt level such that

$$\frac{\rho D}{H_0}\geq(1-H_0)\overline{\theta}+B(H_0)-C(1-\beta)-\pi(H_0),$$

will result in no development. If

$$\frac{\rho G}{H_0}<(1-H_0)\overline{\theta}+B(H_0)-C(1-\beta)-\pi(H_0),$$

then the planner cannot eliminate development through debt financing. Nonetheless, the best strategy in this case will be to fully debt finance, since this will minimize the amount of development. The optimal debt level in this case is therefore any level at least as big as

$$\min\{G,\frac{H_0\left[(1-H_0)\overline{\theta}+B(H_0)-C(1-\beta)-\pi(H_0)\right]}{\rho}\}=\min\{G,\frac{H_0S(H_0)}{\rho}\}.$$

Now suppose that some development is optimal. In an equilibrium with debt level D, if there is development, the level will satisfy

$$\frac{(1-H_2)\overline{\theta}+B(H_2)}{1-\beta}-\frac{D}{\beta H_2}=C+\frac{\pi(H_2)}{1-\beta}.$$

Thus, the optimal debt level is such that

$$\frac{\rho D}{H^o}=-H^o B'(H^o).$$

If

$$\frac{\rho G}{H^o}<-H^o B'(H^o),$$

then the planner cannot achieve the optimal level of development through debt financing. Again, the best strategy in this case will be to fully debt finance since this will minimize the amount of excess development. The optimal debt level in this case is therefore equal to $\min\{G,-(H^o)^2 B'(H^o)/\rho\}$.

11.6 Proof of Proposition 4

We solve for the initial residents' preferred development tax and then use Proposition 1 to infer the equilibrium debt level. Given that $V_\theta=\mu V_{\theta 0}+P_0$ and the expression for $V_{\theta 0}$ in (35), the initial residents' optimal taxation problem can be posed as choosing a development tax τ to maximize the objective function

$$\max_{\tau}\mu\left(\frac{\theta+\mu\beta B(H_2)}{1-\mu\beta}-\frac{G}{H_0}-P_0+\beta\mu\frac{\tau(H_2-H_0)}{H_2}+\beta(1-\mu)P_1+\beta\mu\frac{(1-\mu)\beta}{1-\mu\beta}P_2\right)+P_0, \quad (40)$$

where the housing level H_2 and the prices P_0, P_1, and P_2 are those arising in the equilibrium with development tax τ.

Let

$$\frac{(1-H_0)\overline{\theta}+B(H_0)}{1-\beta}=C+\frac{\pi(H_0)}{1-\beta}+\widehat{\tau}.$$

From the discussion in Section 5, there will be no development if $\tau\geq\widehat{\tau}$. All tax rates higher than $\widehat{\tau}$ will yield the same payoff as $\widehat{\tau}$ since there will be no development and no revenues raised. Thus, these tax rates can be ignored.

If $\tau<\widehat{\tau}$, there will be development. From the discussion in Section 5, the level of development will be such that

$$\frac{(1-H_2)\overline{\theta}+B(H_2)}{1-\beta}+\frac{\tau(H_2-H_0)}{H_2}=C+\frac{\pi(H_2)}{1-\beta}+\tau.$$

This implies that

$$\tau = \frac{H_2\left[(1-H_2)\overline{\theta} + B(H_2) - C(1-\beta) - \pi(H_2)\right]}{(1-\beta)H_0}. \tag{41}$$

Note that

$$\widehat{\tau} = \frac{(1-H_0)\overline{\theta} + B(H_0) - C(1-\beta) - \pi(H_0)}{1-\beta}, \tag{42}$$

so equation (41) also holds for $\tau = \widehat{\tau}$. From (17), the price of housing in periods 2 and beyond is

$$P_t = \frac{(1-H_2)\overline{\theta} + B(H_2)}{1-\beta}.$$

As discussed in Section 5, the price of housing in period 1 is

$$P_1 = C + \frac{\pi(H_2)}{1-\beta} + \tau = \frac{(1-H_2)\overline{\theta} + B(H_2)}{1-\beta} + \frac{\tau(H_2 - H_0)}{H_2}.$$

and the price of housing in period 0 is

$$\begin{aligned} P_0 &= (1-H_0)\overline{\theta} - \frac{G}{H_0} + \beta\left(C + \frac{\pi(H_2)}{1-\beta} + \tau\right) \\ &= (1-H_0)\overline{\theta} - \frac{G}{H_0} + \beta P_1. \end{aligned}$$

Substituting these prices into the objective function in (40), allows us to pose the initial residents' optimal tax problem as:

$$\max_{(\tau,H_2)} \left\{ \begin{array}{c} \frac{\mu\theta}{1-\mu\beta} - \frac{G}{H_0} + \beta\frac{\tau(H_2-H_0)}{H_2} + \frac{\beta B(H_2)}{1-\beta} + (1-\mu)(1-H_0)\overline{\theta} + \left(\frac{\mu(1-\beta)+1}{1-\mu\beta}\right)\frac{\beta(1-\mu)}{1-\beta}(1-H_2)\overline{\theta} \\ s.t.\ \tau = \frac{H_2\left[(1-H_2)\overline{\theta}+B(H_2)-C(1-\beta)-\pi(H_2)\right]}{(1-\beta)H_0} \\ H_2 \geq H_0 \end{array} \right\}. \tag{43}$$

The constraints embody the requirement that H_2 be an equilibrium housing level given τ. If the solution involves $H_2 = H_0$, then it involves no development and a tax of $\widehat{\tau}$. If $H_2 > H_0$, then the solution involves development and a tax less than $\widehat{\tau}$.

To solve problem (43), we substitute the expression for the tax into the objective function to produce a problem involving only the choice of H_2. After the substitution, the objective function is (ignoring constants)

$$\begin{aligned} &\beta\frac{\left[(1-H_2)\overline{\theta}+B(H_2)-C(1-\beta)-\pi(H_2)\right](H_2-H_0)}{(1-\beta)H_0} \\ &+\frac{\beta B(H_2)}{1-\beta} + \left(\frac{\mu(1-\beta)+1}{1-\mu\beta}\right)\frac{\beta(1-\mu)}{1-\beta}(1-H_2)\overline{\theta}. \end{aligned}$$

The first derivative of this objective function is

$$\beta\tfrac{\left[(1-H_2)\overline{\theta}+B(H_2)-C(1-\beta)-\pi(H_2)\right]}{(1-\beta)H_0}+\beta\tfrac{\left[-\overline{\theta}+B'(H_2)-\pi'(H_2)\right](H_2-H_0)}{(1-\beta)H_0}$$
$$+\tfrac{\beta B'(H_2)}{1-\beta}-\left(\tfrac{\mu(1-\beta)+1}{1-\mu\beta}\right)\tfrac{\beta(1-\mu)}{1-\beta}\overline{\theta},$$

and the second derivative is

$$2\beta\frac{\left[-\overline{\theta}+B'(H_2)-\pi'(H_2)\right]}{(1-\beta)H_0}+\beta\frac{\left[B''(H_2)-\pi''(H_2)\right](H_2-H_0)}{(1-\beta)H_0}+\frac{\beta B''(H_2)}{1-\beta}<0.$$

The objective function is therefore strictly concave.

If the first derivative of the objective function is negative at H_0, the initial residents will want to prevent development. This condition boils down to

$$(1-H_0)\overline{\theta}+B(H_0)-C(1-\beta)-\pi(H_0)\leq -H_0B'(H_0)+\left(\frac{\mu(1-\beta)+1}{1-\mu\beta}\right)(1-\mu)\overline{\theta}H_0,$$

which is (22). Any tax at least as big as $\widehat{\tau}$ will choke off development. Given (42), the corresponding debt level is therefore anything bigger than

$$\frac{D}{\beta H_0}=\frac{(1-H_0)\overline{\theta}+B(H_0)-C(1-\beta)-\pi(H_0)}{1-\beta},$$

or, equivalently, bigger than $D=H_0S(H_0)/\rho$. If $H_0S(H_0)/\rho$ exceeds G, then the initial residents cannot completely choke off development with debt financing. Their best strategy is nonetheless to choose the highest level of debt possible. We conclude that the equilibrium level of debt if (22) holds is any debt level at least as big as $\min\{G,H_0S(H_0)/\rho\}$.

If (22) does not hold, the initial residents will want the level of development that equates the first derivative of the objective function to zero. This housing level satisfies

$$(1-H_2)\overline{\theta}+B(H_2)-C(1-\beta)-\pi(H_2)$$
$$=-H_2B'(H_2)+\left[\overline{\theta}+\pi'(H_2)\right](H_2-H_0)+\left(\tfrac{\mu(1-\beta)+1}{1-\mu\beta}\right)(1-\mu)\overline{\theta}H_0.$$

This housing level corresponds to H^e as defined in (23). The associated tax is

$$\tau=\frac{H^e\left[(1-H^e)\overline{\theta}+B(H^e)-C(1-\beta)-\pi(H^e)\right]}{(1-\beta)H_0}.$$

The corresponding debt level therefore satisfies

$$\frac{D}{\beta H_0}=\frac{H^e\left[(1-H^e)\overline{\theta}+B(H^e)-C(1-\beta)-\pi(H^e)\right]}{(1-\beta)H_0},$$

or, equivalently, $D = H^e S(H^e)/\rho$. If $H^e S(H^e)/\rho$ exceeds G, then the initial residents cannot completely choke off development with debt financing. Their best strategy is nonetheless to choose the highest level of debt possible. We conclude that the equilibrium level of debt if (22) does not hold is $\min\{G, H^e S(H^e)/\rho\}$.

It remains to establish the claim that the equilibrium debt level is at least as high as the optimal level and strictly higher when the optimal debt level equals $-\left(H^o\right)^2 B'(H^o)/\rho$. Proposition 3 tells us that if $S(H_0) \leq -H_0 B'(H_0)$, any debt level at least as big as $\min\{G, H_0 S(H_0)/\rho\}$ is optimal, while if $S(H_0) > -H_0 B'(H_0)$, the optimal debt level is $\min\{G, -\left(H^o\right)^2 B'(H^o)/\rho\}$ where H^o satisfies $S(H^o) = -H^o B'(H^o)$. Note that since $S(H^o) = -H^o B'(H^o)$, we have that

$$\min\{G, \frac{-\left(H^o\right)^2 B'(H^o)}{\rho}\} = \min\{G, \frac{H^o S(H^o)}{\rho}\}.$$

To prove the claim, suppose first that (22) does not hold. Then we know that $S(H_0) > -H_0 B'(H_0)$. Thus, we need to show that $H^e S(H^e) > H^o S(H^o)$. It is clear that $H^e < H^o$. Thus, it suffices to show that $HS(H)$ is decreasing for $H \in [H^e, H^o]$. Observe that

$$\frac{dHS(H)}{dH} = (1-H)\overline{\theta} + B(H) + HB'(H) - C(1-\beta) - \pi(H) - H\left(\overline{\theta} + \pi'(H)\right),$$

and

$$\frac{d^2 HS(H)}{dH^2} = 2B'(H) + HB''(H) - 2\left(\overline{\theta} + \pi'(H)\right) < 0.$$

Thus, $HS(H)$ is concave. Now note that

$$\begin{aligned}
\frac{dH^e S(H^e)}{dH} &= (1-H^e)\overline{\theta} + B(H^e) + H^e B'(H^e) - C(1-\beta) - \pi(H^e) - \left(\overline{\theta} + \pi'(H^e)\right) H^e \\
&= \left(\frac{\mu(1-\beta)+1}{1-\mu\beta}\right)(1-\mu)\overline{\theta} H_0 - \left(\overline{\theta} + \pi'(H^e)\right) H_0 \\
&= -\overline{\theta} H_0 \left(1 - \frac{\left(\mu(1-\beta)+1\right)(1-\mu)}{1-\mu\beta}\right) - \pi'(H^e) H_0 \\
&= -\overline{\theta} H_0 \left(\frac{\mu^2(1-\beta)}{1-\mu\beta}\right) - \pi'(H^e) H_0 < 0.
\end{aligned}$$

Thus, on the interval $[H^e, H^o]$, $HS(H)$ is decreasing as required.

Now suppose that (22) holds. There are two possibilities: $S(H_0) \leq -H_0 B'(H_0)$ and $S(H_0) > -H_0 B'(H_0)$. In the first possibility, the equilibrium and optimal debt levels coincide and so the claim holds. In the second possibility, to prove the claim requires showing that $H_0 S(H_0) > H^o S(H^o)$. Following the argument just made, it suffices to show that $dH_0 S(H_0)/dH \leq 0$. We have that

$$\frac{dH_0S(H_0)}{dH} = (1-H_0)\overline{\theta} + B(H_0) + H_0B'(H_0) - C(1-\beta) - \pi(H_0) - H_0\left(\overline{\theta} + \pi'(H_0)\right).$$

Given that (22) holds, we know that

$$\begin{aligned}
& (1-H_0)\overline{\theta} + B(H_0) + H_0B'(H_0) - C(1-\beta) - \pi(H_0) - H_0\left(\overline{\theta} + \pi'(H_0)\right) \\
\leq\ & C(1-\beta) + \pi(H_0) + \left(\frac{\mu(1-\beta)+1}{1-\mu\beta}\right)(1-\mu)\overline{\theta}H_0 - C(1-\beta) - \pi(H_0) - H_0\left(\overline{\theta} + \pi'(H_0)\right) \\
=\ & \left(\frac{\mu(1-\beta)+1}{1-\mu\beta}\right)(1-\mu)\overline{\theta}H_0 - H_0\left(\overline{\theta} + \pi'(H_0)\right) < 0.
\end{aligned}$$