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Community Development by Public Wealth Accumulation*

Abstract

This paper presents a dynamic political economy model of community development. The model highlights the role of public wealth accumulation in development. A community's wealth is the difference between the value of its publicly-owned assets and liabilities. Wealth accumulation arises when residents tax-finance investment in durable public goods or pay down public debt. It stimulates future development because of the fiscal externality created by the collective ownership of community wealth. Moreover, when this development occurs, future residents have an incentive to engage in further accumulation because the cost can be spread over a larger group. In this way, public wealth accumulation can fuel gradual community development.

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1 Introduction

Community development is an important topic in public and urban economics. In order for development to occur, it is necessary for housing to be built for new residents to live in and investments to be made in local public goods (schools, roads, public safety, parks, libraries, shopping and entertainment areas, etc) for their use. How this occurs and the distortions that arise in different institutional settings is a question of significant interest.

This paper presents and analyzes a novel political economy model of community development. It is a dynamic partial equilibrium model of a single community. The community starts out with a stock of housing and initial residents who own this housing. The community can grow by building new housing and new construction is supplied by competitive developers. There is a pool of potential residents with heterogeneous desires to live in the community, generating a downward sloping demand curve. There is turnover, with households entering and exiting the pool each period, so that the market for housing is always active. The possibility that current residents may leave the pool gives them an incentive to care about the value of their homes. The community provides a durable local public good, investment in which can be financed by debt or a uniform tax on all residents. Public investment and financing decisions are made in each period by the current residents and residents are fully forward-looking.

The main insight from the analysis concerns the potential role of public wealth accumulation in community development. The model's structure implies that the community has public assets and liabilities in the form of a stock of public good and public debt. Ownership of these assets and liabilities is collective by the residents of the community. This collective ownership creates what is known as a *fiscal externality* (see, for example, Henderson 1988 p.166). Specifically, building an additional house in the community creates effectively a transfer (positive or negative) from the new resident to the existing residents. If the community's wealth (i.e., the value of its public good stock less its debt) is positive, adding a new resident will reduce wealth per capita and the fiscal externality is negative. If wealth is negative, adding a new resident will increase wealth per capita and the fiscal externality will be positive. This externality is analogous to that arising in a common pool problem (although it can be positive) and will potentially distort the decisions of potential residents to build in the community.

Of course, the community's wealth is endogenous and depends on its past fiscal decisions. These decisions shape the sign and strength of the externality and make it conceptually distinct from other fiscal externalities or common pool problems. When making fiscal decisions, forward-looking residents take into account their future impact on the externality. If the community has negative wealth, residents may want to increase it to benefit from the positive externality. For example, if they can attract more residents by reducing the community's debt partially, this will spread the burden of the remaining debt over a larger group of residents. More importantly, even when the community has positive wealth, residents may want to further increase it to attract potential residents to benefit from a second fiscal externality - that created by new residents sharing the costs of public good provision.

To improve the community's wealth requires residents to either tax-finance investment in the public good or raise taxes to pay down debt. In doing so, they are effectively subsidizing future residents. They will not always want to engage in this behavior, but, if they do, they create development for the community beyond the level that could be supported with its initial wealth. Moreover, if such development occurs, future residents may have an incentive to engage in more accumulation. This is because the cost of further accumulation can be spread over a larger group of residents. In this way, public wealth accumulation can fuel the gradual development of the community.

The paper's formal analysis begins by finding an equilibrium of the model. The interaction described by the model gives rise to a dynamic game between the different cohorts of residents. Following the usual approach in dynamic political economy models, we look for a Markov equilibrium in which residents' policy decisions and housing market outcomes just depend on the state variables - the public assets and housing stock. The novelty of the model makes finding an equilibrium a significant challenge. Our strategy is "guess and verify". To develop intuition for what to guess, we characterize how development would proceed if residents, rather than just choose policies for one period, could commit future residents to following a complete development plan. These development plans are relatively simple and reveal the basic economics underlying the strategy of public wealth accumulation. We then consider whether future residents would choose to follow these plans. This sheds light on when the plans could describe equilibrium policies and, when they do not, how residents will need to modify the policies they control. We find that if the community's wealth is below a critical level, future residents will want to deviate from earlier residents' plans. They will desire additional public wealth accumulation because its cost is lower with a larger population. On the basis of this understanding of the incentives of the different cohorts of residents, we formulate a guess for an equilibrium and verify analytically that this is indeed an equilibrium.

We then discuss existence, verifying by numerical methods that our equilibrium exists for a broad range of parameterizations of our model.

Once we have this equilibrium, we describe how the community develops for any initial conditions (i.e., initial public assets and housing stock). We then clarify the circumstances under which equilibrium involves community development by public wealth accumulation. This is defined as arising whenever the community grows beyond the size that could be supported by its initial wealth. We show that this occurs if and only if the community's initial wealth falls below a critical level. We further show that this development comes in one of two forms. In the first, development begins in the initial period and the size of the community increases gradually and continually over time, converging asymptotically to a steady state level. In each period, the residents build the community's wealth by financing some of the additional public good provided for the larger population with taxation. This accumulation paves the way for more development in the next period. The logic behind this gradual development is subtle because it is the off-equilibrium path behavior of future residents that provides incentives to current residents to engage in wealth accumulation. In the second form, there is no development in the first period and taxes are raised to build wealth. This accumulation allows development to begin in the next period. This development can either be gradual or rapid depending on the extent of accumulation in the initial period. Either way, the community converges to the same steady state.

Finally, we turn to the normative potential of community development by public wealth accumulation. We show that it does not allow the community to reach its efficient size. This reflects the fact that the future benefits of accumulation necessarily accrue partially to future residents via the fiscal externality. Thus current residents do not fully appropriate these benefits. These forces mean that accumulation will be under-supplied by residents. Nonetheless, the difference between the long run size of the community and the efficient size converges to zero as the probability of residents leaving the pool of potential residents converges to zero. We also show that, irrespective of the eventual size of the community, development by public wealth accumulation involves inefficient delay.

The organization of the remainder of the paper is as follows. Section 2 discusses related literature. Section 3 introduces the model and Section 4 establishes a normative benchmark by characterizing socially optimal development. Section 5 explains our strategy for finding equilibrium and describes the equilibrium that we uncover. Section 6 analyzes the conditions under which this equilibrium involves community development by public wealth accumulation and the nature of this development. Section 7 investigates how such development compares with optimal development and the reasons for the discrepancies. Section 8 discusses the empirical plausibility of development by wealth accumulation and identifies some other lessons from the analysis. Section 9 concludes with a brief summary and suggestions for future work.

2 Related literature

This paper relates to a variety of literatures in public economics, urban economics, and political economy. A long tradition in public economics sees the role of communities as facilitating the joint consumption of local public goods. Accordingly, a large literature studies the formation and development of communities which independently finance and provide their local public goods (see Ross and Yinger 1999 and Wildasin 1986 for reviews). The most prominent strand of this literature analyzes how households who differ in incomes or public good preferences or both, locate across different communities under the assumption that, once located, households collectively choose taxes and public good levels for their communities and optimize their housing consumption (see, for example, Epple, Filimon, and Romer 1984, Fernandez and Rogerson 1998, and Rose-Ackerman 1979). The number of communities and the housing supplies in each community are exogenous and the focus is on the existence of equilibrium and on whether the population is segmented across communities in the way envisioned by Tiebout (1956). Our model shares with this literature the view that the role of communities is to facilitate the joint consumption of local public goods. It also has in common the assumption that fiscal decisions are made by residents. It differs in that its focus is on the dynamic development of a single community rather than the allocation of households across multiple communities.

Another strand of this literature takes a different approach to the community formation process by assuming that communities are formed by monopoly developers (see, for example, Henderson 1985, Henderson and Thisse 2001, and Sonstelie and Portney 1978). These developers acquire land, build housing, and provide public goods with the aim of attracting residents and making profits. They face a pool of potential residents who must be induced to locate in their communities. Developers' profits are the revenue from selling property less the costs of construction and public good provision. This literature studies the efficiency of public good provision, housing levels, and the allocation of households across communities. Our model shares with this literature the idea that community decisions are made strategically with an eye on how they will attract potential residents.

It differs in the sense that decisions are made by residents, rather than monopoly developers. Moreover, these residents have only imperfect control over the level of new construction, which is supplied by competitive developers.

While there is recognition of its importance, the dynamic development of communities has received limited attention in this literature. Henderson (1980) studies the developer's problem in a two period, single community setting. There are two groups of potential residents: those around in the first period and those arriving in the second. The developer sells homes in the initial period to the period one potential residents, sells further homes in period two to the second group, and provides public goods in both periods. Period one residents remain in the community for both periods and public good provision is financed by a property tax in each period. Henderson shows that if the developer can commit to policies at the beginning of period one, the home sizes and public good levels provided in both periods will be efficient.[1] In our model, the residents are the decision-makers and decision-making is sequential, so there is no commitment. Moreover, homes come in one variety and, while the set of potential residents is changing, its size is constant across periods. Finally, and most significantly, because the public good is durable and the community can borrow, the community has assets and liabilities. These create the fiscal externality that drives development.

The urban economics literature explores reasons for agglomeration other than the collective consumption of local public goods. The famous monocentric city model originating in the works of Alonso (1964), Mills (1967), and Muth (1969), assumes that households agglomerate to be near their place of work in the city center. In the influential work of Krugman (1991), producers of differentiated products and worker/consumers with tastes for variety agglomerate to be able to produce, work, and trade with lower transactions costs. Some work in this literature develops dynamic models of city development. For example, a number of authors have studied development in the monocentric city model under the assumption that population is growing exogenously (for a review of this work, see Brueckner 2000). Of particular interest is the work of Henderson and Venables (2009) who consider a model in which new population arrives continuously and cities form sequentially. Each city has the structure of the monocentric model, but the model allows for

[1] Henderson notes a tension between the interests of the developer and the period one residents. At the beginning of period two, the developer would like to shift more of the second period tax burden to initial residents and hence extract higher sales revenues from period two residents. He points out that this tension creates a potential time inconsistency problem in the sense that the developer would like to change policies in period two. This theme is discussed further in Henderson (1985) and suggests support for considering the implications of sequential decision-making.

other forms of agglomeration economies. As in our model, housing, once constructed, is infinitely durable. The planning solution involves one city after another being created and filled to optimal size with arriving citizens. A decentralized equilibrium is studied in which construction, as in our model, is chosen by competitive builders and there are no local city governments to coordinate development. In this equilibrium, cities could be too small or too large. The paper then studies an equilibrium in which profit maximizing city developers offer subsidies to residents to live in their cities. Developers commit to these subsidies before their cities are formed and subsidies vary over the course of city development. This equilibrium replicates the planning solution.

This paper differs from this literature in its focus on the development of a small community in which people live, rather than a city where people also produce and trade. More importantly, what is distinctive about this paper is its modelling of community policy-making (specifically, sequential decision-making by home-owning residents) and its incorporation of community assets and liabilities (a durable public good and debt). This set-up leads to community development by public wealth accumulation. This completely novel type of development emerges organically rather than being driven by exogenous changes in the environment, such as population growth.

Turning back to public economics, the paper relates to the literature on the capitalization of local government amenities (school quality, taxes, crime, etc) into housing prices stemming from Oates (1969) (see Hilber 2017 for a review). The structure of the model implies that such capitalization is operational, but incomplete. In any period, the supply curve of housing looks like an inverted L. It is vertical up to a price equal to the construction cost with a quantity equal to the current housing stock, and then becomes horizontal, as new construction is added. This implies that an improvement in the community's amenities will not be capitalized into housing prices if demand prior to the improvement is already sufficient to bring forth new construction. The increase in demand will just be met by an increase in new construction. However, if even after the improvement, demand is insufficient to spur new construction, then capitalization will be just as in a model with fixed housing supply. This incomplete capitalization has interesting implications for residents' incentives to invest and borrow.

Another related public economics literature, is that on the political economy of public good provision. This literature explores how public good provision is determined in different political settings (see, for example, Baron 1996, Bergstrom 1979, Lizzeri and Persico 2001, and Romer and Rosenthal 1979). Most of the work, including the state and local literature just discussed, focuses on the provision of static public goods, which must be provided anew each period. In practice, many important public goods are durable, lasting for many years and depreciating relatively slowly. Understanding the political provision of such goods is more challenging, because of their durable nature. Recent work has studied the provision of such goods in a variety of settings.[2] This paper adds to this strand of the literature by studying the provision of a local durable public good by residents in a growing community with population turnover. In the equilibrium we find, despite population turnover and incomplete capitalization, investment is always efficient given the size of the community.[3] This reflects the assumption that the government can finance investment with debt.

Also related, is the literature on the political economy of public debt. A large literature studies the accumulation of debt at the national government level in various political settings (for a review see Alesina and Passalacqua 2016). The key focus has been on understanding why debt accumulation may be excessive. Less attention has been paid to the debt of local governments. A notable feature of such debt is that, once a resident leaves the locality, he/she ceases to have any responsibility for it. One theme in the literature is that this may result in the costs of debt not being fully bourne by the issuing residents. On the other hand, in a world in which the supply of housing is fixed, local government debt should be capitalized into the price of housing, putting the full burden of debt on the issuing residents even if they leave (see, for example, Daly 1969). As just discussed, in the model of this paper, capitalization is incomplete. Nonetheless, the fact that higher debt levels are capitalized into housing prices if new construction is not undertaken, prevents debt from being abused. In particular, residents do not increase debt and use it to finance tax cuts for themselves because this would deter development and lead to a fall in the value of their homes. In equilibrium, the wealth of the community either stays constant or grows over time and debt plays a key role in both allowing the community to develop and to provide public goods efficiently.[4]

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[2] Battaglini and Coate (2007), Battaglini, Nunnari, and Palfrey (2012), and LeBlanc, Snyder, and Tripathi (2000) study legislative provision, Barseghyan and Coate (2014) and Coate and Ma (2017) study bureaucratic provision, and Conley, Driskill, and Wang (2013) and Schultz and Sjostrom (2001) study provision in a multi-community setting.

[3] In a community with a fixed housing supply and population turnover, capitalization provides incentives for residents to make efficient public good investment decisions despite the fact that they will not be around to enjoy all the benefits. See Brueckner and Joo (1991), Conley, Driskill, and Wang (2013) and Hilber (2017) for further discussion of the incentives provided by capitalization for public good provision.

[4] The role played by debt in the model differs from the role commonly discussed in the literature. The usual argument is that because residents may leave their community, they will underinvest in durable public goods if they must finance investment with taxes. Debt financing counteracts this distortion because mobility also implies that residents do not bear the full burden of debt issued (for a formal analysis see Schultz and Sjostrom 2001). This logic underlies the so-called “golden rule” that prescribes that local governments pay for non-durable goods and services with tax revenues and use debt to finance investment in durables (for discussion and analysis see Bassetto

Finally, the paper contributes to the fast growing literature developing and analyzing infinite horizon political economy models of policy-making with rational, forward-looking decision makers.[5] It is well recognized that many interesting issues arise from recognizing the dynamic linkage of policies across periods. Such linkages arise directly, as with public investment or debt, or indirectly because current policy choices impact citizens' private investment decisions. The model studied in this paper is distinctive in featuring both state variables directly controlled by the voters (debt and the stock of public good) and a state variable determined by the market (the housing stock). It also features a changing group of decision-makers, as the size of the community is growing. Despite these complications, we are able to provide something very close to a closed form solution of the model.

3 The model

Consider a community such as a small town or village. This community can be thought of as one of a number in a particular geographic area. The time horizon is infinite and periods are indexed by $t = 0, ..., \infty$. There is a pool of potential residents of the community of size 1. These can be thought of as households who for exogenous reasons (employment opportunities, family ties, etc) need to live in the geographic area in which the community is located and are potentially open to living in the community. Potential residents are characterized by their desire to live in the community (as opposed to an alternative community in the area) which is measured by the preference parameter θ. This desire, for example, may be determined by a household's idiosyncratic reaction to the community's natural amenities. The preference parameter θ is uniformly distributed on $[0, \overline{\theta}]$. Reflecting the fact that households' circumstances change over time, in each period new households join the pool of potential residents and old ones leave. The probability that a household currently a potential resident will be one in the subsequent period is $\mu \in (0, 1)$. Thus, in each period, a fraction $1 - \mu$ of households leave the pool and are replaced by an equal number of new ones.

The only way to live in the community is to own a house. The community has sufficient land to accommodate housing for all the potential residents. Moreover, the only use for land is building

with Sargent 2006). In our model, no golden rule is imposed and residents finance investment with a mix of debt and taxes. Debt allows the community to accumulate wealth in a way that does not require distorting public good investment. Wealth accumulation is necessary to attract new residents.

[5] Examples of this style of work are Azzimonti (2011), Battaglini and Coate (2008), Bowen, Chen, and Eraslan (2015), Coate and Morris (1999), Hassler, Rodriguez Mora, Storesletten, and Zilibotti (2003), and Krusell and Rios-Rull (1999). Examples which share the focus of this paper on state and local public finance are Barseghyan and Coate (2016), Brinkman, Coen-Pirani, and Sieg (2018), and Glaeser and Ponzetto (2014).

houses.[6] Houses are infinitely durable and the cost of building a new one is C.[7] Housing is supplied by competitive developers. The stock of houses at the beginning of period t is denoted by H_t. New construction in period t is therefore $H_{t+1} - H_t$. New houses built in period t are available to be used in that period. A stock of housing H can accommodate a fraction H of the pool of potential residents. The initial housing stock, H_0, is exogenous and positive, implying that the community has some residents at the beginning of period 0.[8]

The community provides a durable local public good which depreciates at rate $\delta \in (0, 1)$. The good costs c per unit. The stock at the beginning of period t is denoted by g_t. Investment in the public good in period t is available to be used in that period and is subject to depreciation. Accordingly, the amount of public good available to be used in period t is $g_{t+1}/(1-\delta)$ and period t investment is $g_{t+1}/(1-\delta) - g_t$.[9] The initial stock of the public good, g_0, is exogenous.

When living in the community, households have preferences defined over the public good and consumption. In period t, a household with preference parameter θ and consumption x_t obtains a payoff of $\theta + x_t + B([g_{t+1}/(1-\delta)] \,/\, (H_{t+1})^{\alpha})$ from living in the community.[10] The public good benefit function B is increasing, smooth, strictly concave, and satisfies the limit conditions that $lim_{z \searrow 0} B'(z) = \infty$ and $lim_{z \nearrow \infty} B'(z) = 0$. The parameter α measures the congestibility of the public good and belongs to the interval $[0, 1]$. The smaller is α, the closer the good is to a pure public good. The higher is α, the closer the good is to a publicly provided private good.

Households discount future payoffs at rate β and can borrow and save at rate $\rho = 1/\beta - 1$. This assumption means that households are indifferent to the intertemporal allocation of their consumption. Each household in the pool receives an exogenous income stream, the present value

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[6] We could alternatively assume that land not used for housing has a constant productivity in agricultural use.

[7] The assumption of infinitely durable housing is common in the urban economics literature and is justified by the fact that buildings in developed countries display considerable longevity. See Brueckner (2000) for more discussion of the different modelling assumptions used in the literature.

[8] If this were not the case, there would be nobody to choose the period 0 policies. This assumption contrasts with Henderson (1980) who considers the development of an empty community by a developer. In future work, it would be interesting to add an additional initial development stage managed by developers to the model of this paper. One could then try to model the transition of political control from developers to residents. See Chapter 7 of Fischel (2015) for discussion of this process and Knapp (1991) for a model focusing on the timing of the transfer of control of the maintenance of a durable public good from a developer to residents in a condominium setting.

[9] If investment in period t is I_t, then the stock of the public good at the beginning of period $t+1$ is $g_{t+1} = (1-\delta)(g_t + I_t)$. Thus, the amount of public good available in period t, $g_t + I_t$, is equal to $g_{t+1}/(1-\delta)$. Note that we are assuming disinvestment is possible, so that I_t can be negative. This assumption is made for the purposes of tractability. While unrealistic for durable public goods like roads, the reader can be assured that disinvestment does not occur in the equilibrium we study.

[10] We do not distinguish between the size of the housing stock and the number of residents since these will be the same in equilibrium.

of which is sufficient to pay taxes and purchase housing in the community.[11] When not living in the community, a household's per period payoff (net of the consumption benefits from income) is $\underline{u}$.[12]

A competitive housing market operates in each period. Demand comes from new households moving into the community, while supply comes from owners leaving the community and new construction. The price of houses in period t is denoted P_t. This price can fall below the construction cost C when demand at C is less than the stock at the beginning of the period, H_t.

The community can also borrow and save at rate ρ. The community's debt level at the beginning of period t is denoted by b_t. The community levies a tax T_t which is paid by all households who reside in the community at the end of period t. The community's budget constraint in period t is therefore

$$(1+\rho)b_t + c\left(\frac{g_{t+1}}{1-\delta} - g_t\right) = b_{t+1} + H_{t+1}T_t. \tag{1}$$

The left hand side is government spending and consists of debt repayment and public good investment. The right hand side is government revenues and consists of new borrowing and tax revenues. The community's initial debt level, b_0, is exogenous.

The timing of the model is as follows. At the beginning of any period t, the community starts with a public good level g_t, a debt level b_t, and a stock of houses H_t. The triple (g_t, b_t, H_t) describes the state of the community at the beginning of period t. The action begins with the existing residents choosing how much to invest in the public good, how to adjust the community's debt position, and how much tax to levy. This determines g_{t+1}, b_{t+1}, and T_t. Then, households who were in the pool of potential residents in the previous period learn whether they will be remaining and new households join. Those in the pool decide whether to live in the community and existing residents no longer in the pool prepare to leave it. The housing market opens and the equilibrium price P_t is determined along with new construction or, equivalently, next period's housing stock H_{t+1}. New residents buy houses and move into the community and old ones sell up and leave. Residents enjoy public good benefits $B([g_{t+1}/(1-\delta)]\,/\,(H_{t+1})^{\alpha})$ and pay taxes T_t. The policies must satisfy the community's budget constraint (1). Period $t+1$ begins with the state $(g_{t+1}, b_{t+1}, H_{t+1})$ and the process repeats.

[11] The assumption that utility is linear in consumption means that there are no income effects, so it is not necessary to be specific about the income distribution.

[12] Note that $\underline{u}$ is both the per period payoff of living in one of the other communities in the geographic area if a household is in the pool and the payoff from living outside the area when a household leaves the pool.

3.1 Housing market

We now explain how the housing market determines price and new construction in each period. At the beginning of any period t, households fall into two groups: those who resided in the community in the previous period and those who did not, but could in the current period. Households in the first group own homes, while the second group do not. Households in the first group who leave the pool sell their houses and obtain a continuation payoff of

$$P_t + \frac{\underline{u}}{1-\beta}. \tag{2}$$

The remaining households in the first group and all those in the second must decide whether to live in the community in period t. This decision will depend on their preference parameter θ, current and future housing prices, and public good provision and taxes. Since selling a house and moving is costless, there is no loss of generality in assuming that all households sell their property at the beginning of each period.[13] This makes each household's location decision independent of its property ownership state. It also means that the only future consequences of the current location choice is through the price of housing in the next period.

To see how this plays out, consider a household of type θ deciding whether to live in the community in period t. Let $P(g_{t+1}, b_{t+1}, H_{t+1})$ denote the anticipated equilibrium price of housing in period $t+1$ when the state is $(g_{t+1}, b_{t+1}, H_{t+1})$. Then, if the period t state is (g_t, b_t, H_t), fiscal policies are (g_{t+1}, b_{t+1}, T_t), the price of housing is P_t, and households anticipate H_{t+1} households living in the community, the household will choose to reside in the community if

$$\theta + B\left(\frac{g_{t+1}/(1-\delta)}{(H_{t+1})^{\alpha}}\right) - T_t - P_t + \beta P(g_{t+1}, b_{t+1}, H_{t+1}) \geq \underline{u}. \tag{3}$$

The left hand side of this inequality represents the per-period payoff from locating in the community (net of the consumption benefits from income) in period t. Per the discussion above, it assumes that the household buys a house at the beginning of period t and sells it back at the beginning of period $t+1$. The right hand side represents the per-period payoff from living elsewhere.[14]

[13] It should be stressed that this is just a convenient way of understanding the household decision problem. The equilibrium we study is perfectly consistent with the assumption that the only households selling their homes are those who plan to leave the community. This is because a household that does not sell its house is economically equivalent to one who sells its house and buys one back.

[14] The household's income and assets do not enter into the calculus because they impact both sides of (3) and thus cancel. To illustrate, suppose the household earns y each period and has non-housing net assets A_t. Recall that our assumptions imply that the household is indifferent to its intertemporal allocation of consumption, so assume that it simply keeps its assets constant. Thus income to be used in period t is $y + \rho A_t$. If the household does not live in the community, its period t payoff is $y + \rho A_t + \underline{u}$. If the household lives in the community in period t, then it pays taxes T_t. It also buys a house at cost P_t. We can think of the household as borrowing $\beta P(g_{t+1}, b_{t+1}, H_{t+1})$ of this cost to be repaid in period $t+1$ when it sells the house. The household's period t payoff is then equal to $y + \rho A_t$ plus the left hand side of (3).

Given (3) and the fact that household preferences are uniformly distributed over $[0, \overline{\theta}]$, the equilibrium price of housing P_t in period t must satisfy the market clearing condition

$$H_{t+1} = 1 - \frac{\underline{u} - \left(B\left(\frac{g_{t+1}/(1-\delta)}{(H_{t+1})^{\alpha}}\right) - T_t - P_t + \beta P(g_{t+1}, b_{t+1}, H_{t+1})\right)}{\overline{\theta}}. \tag{4}$$

This implies that the equilibrium price in period t is

$$P_t = (1 - H_{t+1})\overline{\theta} + B\left(\frac{g_{t+1}/(1-\delta)}{(H_{t+1})^{\alpha}}\right) - T_t + \beta P(g_{t+1}, b_{t+1}, H_{t+1}) - \underline{u}. \tag{5}$$

On the supply side, the assumption that houses are infinitely durable implies that

$$H_{t+1} \geq H_t. \tag{6}$$

Moreover, because the supply of new construction is perfectly elastic at a price equal to the construction cost C, it must also be the case that

$$P_t \leq C \ (= \text{if } H_{t+1} > H_t). \tag{7}$$

Given that period $t+1$'s housing price is described by the function $P(g_{t+1}, b_{t+1}, H_{t+1})$, any tuple $(g_{t+1}, b_{t+1}, T_t, H_{t+1}, P_t)$ satisfying the budget constraint (1), is consistent with housing market equilibrium in period t if and only if (5), (6), and (7) are satisfied.[15]

3.2 Policy choice

Next we turn to residents' choice of policies in each period. As explained above, the timing of the model is first that the existing residents choose fiscal policies, and then the housing market determines new construction and the price of housing. Obviously, when residents choose fiscal policies they will anticipate how these policies impact the housing market. Rather than deriving the relationship between the housing market equilibrium and the fiscal policies, and then analyzing the optimal policies, it is easier to think of residents as directly choosing the housing price and new construction along with the fiscal policies, but subject to the constraint that their choice be consistent with housing market equilibrium. Thus, in period t, given the state (g_t, b_t, H_t), we assume that residents choose $(g_{t+1}, b_{t+1}, T_t, H_{t+1}, P_t)$ but subject to the market equilibrium constraints (5), (6), and (7) along with the community budget constraint (1).

[15] This formulation implicitly assumes that P_t can, in principle, be negative. This is unrealistic because, in reality, residents can simply abandon their houses and leave the community if they so choose. However, imposing the constraint that the equilibrium housing price must be non-negative creates some additional complications that are inessential. In particular, it requires that we introduce a community debt limit. Such a limit is needed to prevent current residents from borrowing a large amount, using it to finance transfers to residents, and then abandoning the community and its debt the next period.

While period t residents differ in their desires to live in the community θ, they will have identical preferences over policies and hence there is no collective choice problem to resolve. To understand this, note first that given the housing stock at the beginning of period t, H_t, existing residents will have preferences in the interval $[(1-H_t)\overline{\theta}, \overline{\theta}]$.[16] We know that the market will allocate housing to those in the pool of potential residents with the highest θ and that the supply of housing can only expand. It follows that these residents will all anticipate living in the community as long as they stay in the pool. Accordingly, residents have no difference in policy preferences resulting from different time horizons. Second, the preference θ enters as an additive term and does not impact the benefits from public goods or the marginal value of consumption. Thus, all residents enjoy the same public goods surplus.

In light of this, the period t residents' policy problem can be written as

$$\max_{(g_{t+1}, b_{t+1}, T_t, H_{t+1}, P_t)} \left\{ \begin{array}{c} (1-\mu)\left[P_t + \frac{u}{1-\beta}\right] + \mu\left[B\left(\frac{g_{t+1}/(1-\delta)}{(H_{t+1})^{\alpha}}\right) - T_t + \beta V(g_{t+1}, b_{t+1}, H_{t+1})\right] \\ s.t.\ (1),\ (5),\ (6),\ \&\ (7) \end{array} \right\}. \tag{8}$$

This objective function reflects the fact that, with probability $1-\mu$, a resident leaves the pool and sells its house, and, with probability μ, it remains in the pool and continues to live in the community. The value function $V(g_{t+1}, b_{t+1}, H_{t+1})$ measures the continuation payoff of a household who is residing in the community at the beginning of period $t+1$ net of the future benefits coming from the additive preference θ. As discussed below, it is determined by the policies that future residents are expected to choose. Next period's price, $P(g_{t+1}, b_{t+1}, H_{t+1})$, which enters in constraint (5), is also determined by next period's residents.

3.3 Equilibrium defined

The model gives rise to a dynamic game between the different cohorts of residents. The equilibrium concept we use to solve this game is Markov perfect equilibrium. In such an equilibrium, strategies in any period just depend on the state at the beginning of that period. As discussed, the state is the public good level, debt level, and housing stock. Because all that matters is the state, when defining equilibrium it is not necessary to index strategies by the time period t. All we need to do is to distinguish the state at the beginning of a period, which we denote (g, b, H), and the state

[16] In periods $t = 1, ..., \infty$ this follows from the fact that, in equilibrium, the households with the highest preference for living in the community purchase houses in the community in the previous period. We assume that this condition also characterizes the initial distribution of residents in period 0.

at the beginning of the next period, which we denote (g', b', H'). The latter will be determined by the policy choices made during the period.

In a Markov perfect equilibrium, the residents in a period in which the state is (g, b, H) choose policies (g', b', T, H', P) to maximize their payoff under the assumption that future policies will be chosen according to policy rules that depend only on whatever the state is at that time. In equilibrium, the policies residents choose must be consistent with the policy rules that govern future policies. Formally, an *equilibrium* consists of an investment rule $g'(g, b, H)$, a debt rule $b'(g, b, H)$, a tax rule $T(g, b, H)$, a housing rule $H'(g, b, H)$, a price rule $P(g, b, H)$, and a value function $V(g, b, H)$ satisfying two conditions. The first condition is that, for all states (g, b, H), the policies prescribed by the equilibrium policy rules solve problem (8). The second condition is that the value function is consistent with the equilibrium policy rules. Thus, for all states (g, b, H), the value function satisfies the equality

$$V(g, b, H) = (1-\mu)\left[P(\cdot) + \frac{\underline{u}}{1-\beta}\right] + \mu\left[B\left(\frac{g'(\cdot)/(1-\delta)}{H'(\cdot)^{\alpha}}\right) - T(\cdot) + \beta V(g'(\cdot), b'(\cdot), H'(\cdot))\right], \tag{9}$$

where $g'(\cdot)$ denotes the policy $g'(g, b, H)$, etc.

With an equilibrium, we can trace out the dynamic evolution of the community. In period 0, the public good level will be $g_1/(1-\delta)$ where $g_1 = g'(g_0, b_0, H_0)$. Borrowing will be $b_1 = b'(g_0, b_0, H_0)$ and taxes will be $T(g_0, b_0, H_0)$. The number of residents will be $H_1 = H'(g_0, b_0, H_0)$ and the price of housing will be $P(g_0, b_0, H_0)$. Similarly, in period 1, the public good level will be $g_2/(1-\delta)$ where $g_2 = g'(g_1, b_1, H_1)$. Borrowing will be $b_2 = b'(g_1, b_1, H_1)$ and taxes will be $T(g_1, b_1, H_1)$. The number of residents will be $H_2 = H'(g_1, b_1, H_1)$ and the price of housing will be $P(g_1, b_1, H_1)$. Continuing on in this way, we can see exactly how the community develops both with respect to its housing stock and its local public goods and debt. In this way, an equilibrium implies a positive theory of community development.

4 Optimal community development

To evaluate the normative performance of community development by public wealth accumulation, we need a benchmark for comparison. This section characterizes the development plan that would be optimal for a Utilitarian planner. Such a planner maximizes the discounted sum of the aggregate payoffs of the different pools of potential residents. The assumption that utility is linear in consumption, implies that the planner is indifferent between transfers of consumption both between households in the same pool and across different pools. Accordingly, there is no loss of generality in simply assuming that, in any period, the cost of new construction and investment is financed by lump-sum taxation of the pool of potential residents.

Given the community's initial stocks of public good and housing (g_0, H_0), the planner chooses a time path for investment and new construction or, equivalently, a sequence $\{g_{t+1}, H_{t+1}\}_{t=0}^{\infty}$. In any period t, the planner will allocate the households in the pool with the highest θ to the H_{t+1} houses. Given that θ is uniformly distributed on $[0, \overline{\theta}]$, this implies that households in the interval $[(1 - H_{t+1})\,\overline{\theta}, \overline{\theta}]$ will be assigned to live in the community. Accordingly, the planner's objective function can be written as

$$\sum_{t=0}^{\infty} \beta^t \left[\int_{(1-H_{t+1})\overline{\theta}}^{\overline{\theta}} \theta \frac{d\theta}{\overline{\theta}} + H_{t+1} B \left(\frac{g_{t+1}/(1-\delta)}{(H_{t+1})^{\alpha}} \right) + \underline{u}\,(1 - H_{t+1}) - C(H_{t+1} - H_t) - c \left(\frac{g_{t+1}}{1-\delta} - g_t \right) \right]. \tag{10}$$

The first two terms represent the benefits received by the households assigned to the community, while the third term represents the benefits to those not so assigned. The fourth and fifth terms represent, respectively, the costs of new construction and investment. The planner's problem is to maximize objective function (10) subject to the durability constraint (6).

It is straightforward to verify that g_{t+1} must equal $(1 - \delta)g^o(H_{t+1})$ where $g^o(H)$ satisfies the dynamic Samuelson rule

$$H^{1-\alpha} B' \left(\frac{g^o}{H^{\alpha}} \right) = c(1 - \beta(1 - \delta)). \tag{11}$$

The left hand side measures the per-period social benefit of an additional unit of public good and the right hand side the per-period cost. The latter reflects the fact that a fraction $1 - \delta$ of a unit purchased this period will be available for use next period.

To characterize the optimal level of housing in a way that makes it comparable with what happens in equilibrium, it is convenient to introduce the function $S(H)$ which represents per-resident *optimized public good surplus*, defined as

$$S(H) \equiv B \left(\frac{g^o(H)}{H^{\alpha}} \right) - \frac{c(1 - \beta(1 - \delta))g^o(H)}{H}. \tag{12}$$

This surplus is the difference between the public good benefits enjoyed by each resident at the optimal level and the per-resident cost of this level computed using the per-period marginal cost from (11). Then, it is again routine to show that for all t, H_{t+1} must equal H^o where[17]

$$(1 - H^o)\,\overline{\theta} + S(H^o) + H^o S'(H^o) - C(1 - \beta) = \underline{u}. \tag{13}$$

The left hand side represents the net social benefit from assigning an additional household to the community. The first term is the preference of the marginal household for living in the community, the second is the optimized public good surplus accruing to the marginal household, the third is the impact of adding the household on the public good surpluses of the other residents, and the fourth is the per-period cost of an additional house. The housing level H^o is such that this net social benefit is just equal to the benefit the household receives when not residing in the community, $\underline{u}$. Note that

$$HS'(H) = \frac{(1-\alpha)c(1-\beta(1-\delta))g^o(H)}{H}, \tag{14}$$

so that the impact on other residents' public good surpluses of adding a household is always positive provided that α is less than 1. This reflects the benefits of sharing the costs of the public good.

We impose the following assumption to make sure that the planner's problem is well-behaved.

Assumption 1 (i) *For all* $H \in [H_0, 1]$

$$-\overline{\theta} + 2S'(H) + HS''(H) < 0.$$

(ii)

$$(1 - H_0)\,\overline{\theta} + S(H_0) + H_0 S'(H_0) - C(1 - \beta) > \underline{u} > S(1) + S'(1) - C(1 - \beta).$$

Part (i) of the assumption implies that net social benefit from assigning an additional household to the community is decreasing in the number of households. Part (ii) implies that this net social benefit exceeds $\underline{u}$ at the initial population H_0 but falls below it when the entire pool lives in the community. Together, the two parts imply that there exists a unique solution to the first order condition (13) that lies between H_0 and 1. This solution unambiguously defines the optimal housing level. We may therefore conclude:

Proposition 1 *Under Assumption 1, the optimal community development plan is to construct $H^o - H_0$ new houses in period 0 and invest in $g^o(H^o) - g_0$ units of the public good. Thereafter, no more housing should be constructed and the public good level should be maintained at $g^o(H^o)$.*

There are three main points to take away about the optimal plan. First, development occurs immediately. Second, the public good satisfies the dynamic Samuelson rule. Third, the size of the

[17] This presumes that $H^o > H_0$ which is implied by Assumption 1 below.

community balances the net social benefit of an additional household to the payoff households get from living elsewhere.

5 Finding equilibrium

This section is devoted to finding an equilibrium of the model. As discussed in the introduction, the strategy for finding equilibrium in this type of model is "guess and verify". To develop intuition for what to guess, we start by characterizing the development plans that would be optimal for residents if they could commit future residents to following their plans. Understanding these plans is useful in its own right as it reveals the incentives underlying public wealth accumulation. We then investigate the circumstances under which future residents would indeed follow these plans. This provides insights into when commitment plans could be outcomes of equilibrium play and, when this is not the case, how residents will need to modify the policies they control. Using these insights, we make a guess of what equilibrium will look like and verify that this guess is correct.

5.1 Residents' optimal plans with commitment

In equilibrium, the residents in any period simply get to choose the policies for that period. Here we ask what would they choose if they could commit the community to following a complete development plan? To be concrete, consider the period t residents. For them, a complete development plan is described by $\{g_{\tau+1}, b_{\tau+1}, T_\tau, H_{\tau+1}, P_\tau\}_{\tau=t}^{\infty}$. Their optimal plan maximizes the objective function

$$\sum_{\tau=t}^{\infty} (\mu\beta)^{\tau-t} \left\{ (1-\mu) \left[P_\tau + \frac{u}{1-\beta} \right] + \mu \left[B\left(\frac{g_{\tau+1}/(1-\delta)}{(H_{\tau+1})^{\alpha}} \right) - T_\tau \right] \right\}, \tag{15}$$

subject to in each period τ satisfying the budget constraint (1) and the constraints of market equilibrium (5), (6), and (7).[18]

Before we can describe the solution, we need some additional notation and assumptions. First, let W denote the community's wealth (i.e., $W = cg - (1+\rho)b$). Wealth at the beginning of period t is denoted W_t. The community's wealth is a key determinant of the public good surplus it can offer residents. In particular, the public good surplus a household would enjoy in a period in which the community starts with wealth W, has H residents, supplies the efficient public good level $g^o(H)$, and finances provision so as to keep wealth constant, is $S(H) + (1-\beta)W/H$. To

[18] We also need to add the standard transversality condition that $\lim_{\tau\to\infty} \beta^\tau b_\tau = 0$ to rule out the period t residents operating a Ponzi scheme.

see this, note that public good surplus in this scenario is $B\left(g^o(H)/H^{\alpha}\right) - T$ and the tax T must equal $\left[cg^o(H)(1-\beta(1-\delta)) - W(1-\beta)\right]/H$.[19] The claim then follows from the definition of $S(H)$ in (12). This nicely illustrates the fiscal externality created by the collective ownership of community assets and liabilities discussed in the introduction. If W is positive, increasing H reduces $(1-\beta)W/H$, while if W is negative, increasing H increases it.

Next, for all wealth levels for which it is well defined, let $\mathcal{H}(W)$ be the largest housing level in the interval $[0,1]$ that satisfies the equality

$$(1-H)\,\overline{\theta} + S(H) + \frac{(1-\beta)W}{H} - C(1-\beta) = \underline{u}. \tag{16}$$

Given the interpretation of $S(H) + (1-\beta)W/H$ just discussed, the expression on the left hand side represents the per-period benefit that the marginal home buyer would obtain from living in the community if the price of housing is constant at C and the community had H residents, a wealth level W, and provided the efficient public good level, financing provision so as to keep wealth constant. It follows that $\mathcal{H}(W)$ is the largest population the community can attract when it has wealth W, assuming it provides the efficient public good level and keeps wealth constant.

Note that $\mathcal{H}(W)$ will not be well-defined if W is so large that all types of households would get a positive benefit from living in the community or if W is so small (i.e., sufficiently negative) that there is no population size at which residents would be willing to pay a price of housing C to live in the community. On the interval of wealth levels on which it is well-defined, $\mathcal{H}(W)$ will be increasing under Assumption 1. Intuitively, the community will be more attractive to potential residents with higher wealth because it offers more public good surplus. It is also the case that $\mathcal{H}(W)$ will be concave under Assumption 1.[20] Finally, note that because $\mathcal{H}(W)$ is increasing on the interval of wealth levels on which it is well-defined, it has an inverse. This function, which we denote $\mathcal{W}(H)$, tells us the wealth the community needs to attract a population of size H when it provides the efficient public good level, financing provision so as to keep wealth constant, and the price of housing is constant at C. In light of the properties of $\mathcal{H}(W)$, $\mathcal{W}(H)$ will be increasing and convex.

Our additional assumptions concern the community's initial state (g_0, b_0, H_0). The first is that the community's initial housing stock H_0 satisfies:

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[19] This can be verified from (1). Set $H_{t+1} = H$, $T_t = T$, and assume that $cg_t - (1+\rho)b_t$ is equal to W. Then (1) implies that T must equal $\left[cg^o(H) - W - b_{t+1}\right]/H$ if $g_{t+1}/(1-\delta) = g^o(H)$. If the community's wealth is constant at W, this means that $c(1-\delta)g^o(H) + (1+\rho)b_{t+1}$ is equal to W. Solving this equation for b_{t+1} and using the fact that $1+\rho$ is equal to $1/\beta$, we see that T must equal $\left[cg^o(H)(1-\beta(1-\delta)) - W(1-\beta)\right]/H$.

[20] The properties of $\mathcal{H}(W)$ are established in Appendix 0.

Assumption 2

$$\underline{u} + H_0\overline{\theta} > (1 - H_0)\,\overline{\theta} + S(H_0) + H_0 S'(H_0) - C(1-\beta) > \underline{u} + H_0\overline{\theta}\left(1 - \frac{\mu^2(1-\beta)}{1-\mu\beta}\right).$$

The first inequality in this Assumption guarantees that the interval on which the function $\mathcal{W}(H)$ is well-defined includes all the housing levels that could in principle arise in equilibrium (i.e., those in the interval $[H_0, 1]$). The role of the second inequality will be explained below. The second assumption concerns the community's initial wealth W_0:

Assumption 3 *The community's initial public good level g_0 and debt level b_0 are such that*

$$W_0 < \mathcal{W}(1).$$

This assumption ensures that the community does not start out with so much wealth that all potential residents would want to live in it.

As a final preliminary, we introduce the housing level H^s which is implicitly defined by the equation

$$(1 - H^s)\,\overline{\theta} + S(H^s) + H^s S'(H^s) - C(1-\beta) = \underline{u} + H^s\overline{\theta}\left(1 - \frac{\mu^2(1-\beta)}{1-\mu\beta}\right). \tag{17}$$

This equation is quite similar to that which defines the optimal housing level H^o in (13) but contains an additional term on the right hand side. This term is positive provided that μ is less than 1. Assumption 1 and the second inequality in Assumption 2 imply that H^s is well-defined and lies between H_0 and H^o.[21] The significance of H^s is that the period t residents' optimal plan depends crucially on how H_t compares with it.

We now describe the period t residents' optimal plan when H_t is less than H^s.

Proposition 2 *If $H_t \in [H_0, H^s)$, Assumptions 1 and 2 are satisfied, and $W_t < \mathcal{W}(1)$, there exist wealth levels $W^*(H_t)$ and $W_n(H_t)$, satisfying $\mathcal{W}(H_t) < W^*(H_t) < W_n(H_t)$, such that under the period t residents' optimal plan:*

(i) If $W_t \geq W^*(H_t)$, the community invests in $g^o(\mathcal{H}(W_t)) - g_t$ units of the public good in period t and the market provides $\mathcal{H}(W_t) - H_t$ new houses. The community finances investment so as to keep its wealth constant, meaning that all but $c\delta g^o(\mathcal{H}(W_t))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_t))$ and no more housing is provided. Taxes are set so that wealth remains at W_t. Throughout, the price of housing is C.

[21] The second inequality in Assumption 2 implies that the left hand side of equation (17) exceeds the right hand side at housing level H_0. The right hand side is increasing in H and Assumption 1(i) implies that the left hand side is decreasing in H. Moreover, the left hand side is less than the right hand side at housing level H^o.

(ii) If $W_t < W^*(H_t)$, the community invests in $g^o(H_t) - g_t$ units of the public good in period t and the market provides no new houses. The community chooses debt and taxes so that its wealth increases to $W_n(H_t)$. In period $t+1$, the community invests in $g^o(\mathcal{H}(W_n(H_t))) - (1-\delta)g^o(H_t)$ units of the public good and the market provides $\mathcal{H}(W_n(H_0)) - H_0$ new houses. Investment is financed so as to keep wealth constant, implying that all but $c\delta g^o(\mathcal{H}(W_n(H_0)))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_n(H_0)))$ and no more housing is provided. Taxes are set so that wealth remains at $W_n(H_0)$. The price of housing is less than C in period 0 and C thereafter.

Furthermore, the functions $W^*(H_t)$ and $W_n(H_t)$ are differentiable and increasing on $[H_0, H^s)$, and satisfy the limit condition that $\lim_{H_t \nearrow H^s} W^*(H_t) = \lim_{H_t \nearrow H^s} W_n(H_t) = \mathcal{W}(H^s)$.

Thus, when H_t is smaller than H^s, the period t residents' optimal plan takes one of two forms. Under the first, anticipating an increase in population, residents invest in the public good in period t, and the market provides new construction. The residents finance the increase in the stock of the public good entirely with debt, keeping the community's wealth constant. Thereafter, there is no further development. This means that the size of the community grows to $\mathcal{H}(W_t)$ which is the largest population that it can attract with its initial wealth if it provides the public good efficiently. Under the second form, the community accumulates wealth in period t and no development takes place. The motivation for accumulating wealth is to spur development in the next period. Thereafter, things follow the pattern of the first form. However, the size of the community grows to $\mathcal{H}(W_n(H_t))$ which is the largest population it can attract with its new higher wealth level.

The optimal plan involves wealth accumulation when the community's wealth W_t is low. Here, low is defined relative to the endogenous threshold wealth level $W^*(H_t)$. Notice that the proposition tells us that this threshold wealth level $W^*(H_t)$ exceeds $\mathcal{W}(H_t)$. This means that when the community's wealth W_t lies between $\mathcal{W}(H_t)$ and $W^*(H_t)$, the period t residents choose to accumulate wealth even though some development is possible without accumulation (since W_t being larger than $\mathcal{W}(H_t)$ implies that $\mathcal{H}(W_t)$ exceeds H_t). They do this, because the sacrifice in period t in terms of higher taxes and a smaller population, is compensated by the benefits of a larger population in the future. Effectively, the residents find it profitable to leverage the fiscal externality created by the collective ownership of the community's assets to increase the extent of overall development.

The proposition also notes that both $W^*(H_t)$ and $W_n(H_t)$ are increasing in H_t. This implies that, for housing levels in the interval $[H_0, H^s)$, the residents of a larger community are more willing to choose public wealth accumulation and do more accumulation (in an absolute sense) when they do so. This reflects the fact that the costs of such accumulation are spread over a larger population.

The next result covers the case in which H_t is larger than H^s.

Proposition 3 *If $H_t \in [H^s, 1]$, Assumptions 1 and 2 are satisfied, and $W_t < \mathcal{W}(1)$, then under the period t residents' optimal plan:*

(i) If $W_t > \mathcal{W}(H_t)$, the community invests in $g^o(\mathcal{H}(W_t)) - g_t$ units of the public good in period t and the market provides $\mathcal{H}(W_t) - H_t$ new houses. The community finances investment so as to keep its wealth constant, meaning that all but $c\delta g^o(\mathcal{H}(W_t))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_t))$ and no more housing is provided. Taxes are set so that wealth remains at W_t. Throughout, the price of housing is C.

(ii) If $W_t \leq \mathcal{W}(H_t)$, the market provides no new houses and the size of the community remains at H_t. The community invests in $g^o(H_t) - g_t$ units of the public good in period t and, thereafter, the public good is maintained at $g^o(H_t)$. The way in which the community finances investment is not tied down, but wealth remains no greater than $\mathcal{W}(H_t)$. One possibility is that wealth increases to $\mathcal{W}(H_t)$ and then remains there. In this case, the price of housing is less than C in the initial period and equal to C thereafter.

The key point to note is that when H_t is larger than H^s, the period t residents do not engage in public wealth accumulation to increase future development. Development will occur if the community's wealth is sufficient to attract in more residents but not otherwise. This reflects the concavity of the function $\mathcal{H}(W)$. To see this, suppose that W_t is less than $\mathcal{W}(H_t)$. There is no cost to the period t residents of increasing wealth to $\mathcal{W}(H_t)$, since this will not change the future size of the community. If the residents remain in the community they will benefit from the lower taxes its higher wealth allows and if they leave, the higher wealth will be capitalized into the price of housing. This offsets the tax cost of accumulation. Increasing wealth above $\mathcal{W}(H_t)$ will impact period t residents' payoffs. If the residents remain in the community they will share the benefit from its higher wealth with a larger population and if they leave, the higher wealth will not be capitalized into the price of housing which just equals the supply price C. The benefit of increasing the future size of the community via the fiscal externality must be weighed against the tax cost. The extent of the population increase stemming from marginally increasing wealth above $\mathcal{W}(H_t)$ will be governed by the derivative $\mathcal{H}'(\mathcal{W}(H_t))$. Given the concavity of $\mathcal{H}(W)$, this marginal benefit will be decreasing in $\mathcal{W}(H_t)$ and hence H_t. On the other hand, the marginal cost of accumulation is decreasing in H_t. The marginal benefit is falling faster than the marginal cost and the housing level H^s is where the marginal benefit equals the marginal cost. Beyond the housing level H^s, therefore, the marginal benefit is smaller than the marginal cost and increasing wealth beyond $\mathcal{W}(H_t)$ is not attractive.

The proof of Propositions 2 and 3 is in Appendix 1. The period t residents' problem involves a lot of choice variables and many constraints which makes deriving the solution challenging. Despite this, the solution is relatively straightforward. The only possibly counter-intuitive feature is that, when the community accumulates, all the accumulating is done in period t rather than being spread over time. One might have guessed that the period t residents would delay some of the accumulation until they had attracted more residents, thereby spreading the burden of accumulation over a larger population. However this turns out not to be desirable because such a strategy would delay entry into the community. Any accumulation to which potential residents have to contribute is anticipated and reduces the set of potential residents willing to join the community.[22]

5.2 Will future residents follow the plans of earlier residents?

The residents in period $\upsilon > t$ will follow the period t residents' optimal plan $\{g_{\tau+1}, b_{\tau+1}, T_\tau, H_{\tau+1}, P_\tau\}_{\tau=t}^{\infty}$ if $\{g_{\tau+1}, b_{\tau+1}, T_\tau, H_{\tau+1}, P_\tau\}_{\tau=\upsilon}^{\infty}$ is an optimal plan for these residents given the initial state $(g_\upsilon, b_\upsilon, H_\upsilon)$. The optimal plan for the period υ residents will solve the same problem as for the period t residents, except that the community's wealth and housing stock will be (W_υ, H_υ) rather than (W_t, H_t). It is therefore described by Propositions 2 and 3 with the obvious change in notation.

Using Propositions 2 and 3, we can now establish:

Proposition 4 *If Assumptions 1 and 2 are satisfied and $W_t < \mathcal{W}(1)$, future residents will follow the period t residents' optimal plan if and only if either $W_t \geq \mathcal{W}(H^s)$ or $H_t \geq H^s$.*

To see why the "if" part of the Proposition is true, note that if W_t exceeds $\mathcal{W}(H^s)$, then the period t residents' optimal plan implies that H_{t+1} exceeds H^s and that W_{t+1} equals $\mathcal{W}(H_{t+1})$. Thereafter, housing and wealth are supposed to remain constant. This is consistent with what future residents

[22] To be concrete, imagine that the initial residents were to reduce wealth accumulation from $W_n(H_t)$ to $W_n(H_t) - \epsilon$ in period t, where ϵ is small and positive, and make up the difference in period $t+1$. This would decrease the population from $\mathcal{H}(W_n(H_t))$ to $\mathcal{H}(W_n(H_t) - \epsilon/(1-\beta))$ in period $t+1$ which would translate into a lower level of public good surplus in period $t+1$ for period t residents. The benefit would be to reduce taxes by ϵ/H_t in period t. It can be shown that if the period $t+1$ loss of public good surplus were less than the period t benefit from lower taxes, then it would be even better to not make up the difference in period $t+1$ and simply reduce the total amount of accumulation to $W_n(H_t) - \epsilon$. This contradicts the fact that $W_n(H_t)$ is the optimal amount of wealth to accumulate.

want because, by part (ii) of Proposition 3, if the period v residents have a housing stock H_v exceeding H^s and a wealth level $\mathcal{W}(H_v)$, they will indeed be happy to keep wealth and the future housing stock constant. If H_t exceeds H^s but W_t is less than $\mathcal{W}(H^s)$, Proposition 3 tells us that the optimal plan for period t's residents leaves next period's residents with a housing level H_t and wealth no greater than $\mathcal{W}(H_t)$. By Proposition 3, this is a situation they have no incentive to change.

For the "only if" part, if W_t is less than $\mathcal{W}(H^s)$ and H_t is less than H^s then W_t could either exceed or be smaller than the threshold $W^*(H_t)$. In the former case, new construction occurs in period t under the period t residents' optimal plan and (W_{t+1}, H_{t+1}) will equal $(W_t, \mathcal{H}(W_t))$. Thereafter housing and wealth remain constant. However, H_t will be less than H^s and W_{t+1} will equal $\mathcal{W}(H_{t+1})$ and hence be strictly less than $W^*(H_{t+1})$. By Proposition 2, the period $t+1$ residents will want to increase taxes and accumulate wealth to attract new residents. In the latter case, new construction does not occur until period $t+1$ and it is the period $t+2$ residents who want to deviate. Under the period t residents' optimal plan, at the beginning of period $t+2$, (W_{t+2}, H_{t+2}) will equal $(W_n(H_t), \mathcal{H}(W_n(H_t)))$ and thereafter housing and wealth are supposed to remain constant. However, H_{t+2} will be less than H^s and W_{t+2} will equal $\mathcal{W}(H_{t+2})$, so the period $t+2$ residents will want to increase taxes and accumulate wealth.

Proposition 4 suggests that when the state (g, b, H) is such that either W exceeds $\mathcal{W}(H^s)$ or H exceeds H^s, then residents just choosing the policies they would in the first period of their commitment development plan will be consistent with equilibrium. Thereafter, future residents will just keep wealth and housing constant which is what current residents would do if they controlled these choices. When the state (g, b, H) is such that W is less than $\mathcal{W}(H^s)$ and H is less than H^s, then residents will need to choose different policies than they would in the first period of their commitment plan. In particular, in the case in which W lies between the threshold $W^*(H)$ and $\mathcal{W}(H^s)$, they will need to recognize that simply investing in $g^o(\mathcal{H}(W)) - g$ units of the public good and financing investment so as to keep wealth constant, will not result in the community increasing in size to $\mathcal{H}(W)$. This is because with initial wealth W and housing stock $\mathcal{H}(W)$, next period's residents will wish to accumulate more public assets. This will imply a reduction in the price of housing in the next period, the anticipation of which will drive the equilibrium price in the current period below the supply price C. To prevent this from happening, the residents will need to tax-finance some investment so as to carry forward more wealth. However, this will necessitate a reduction in the amount of development as potential residents are deterred by the higher taxes.

5.3 The conjectured equilibrium

5.3.1 Overview

On the basis of all this, we can now make a guess for what equilibrium might look like. Our guess has four components. The first is that investment in the public good will always be efficient for the size of the community. This is convenient because it basically reduces the state variables from three to two - community wealth W and housing H. Given any target for next period's wealth, debt adjusts to meet it, under the assumption that the public good is set to its efficient level.

The second component is that when H is greater than or equal to H^s, residents will choose the policies from the initial period of their optimal development plan. This is because Proposition 4 tells us that future residents will follow their plan. Specifically, what this means is that if W is larger than $\mathcal{W}(H)$, then the population increases to $\mathcal{H}(W)$ and the residents finance the investment necessary to service the new population to keep wealth constant. If W is less than or equal to $\mathcal{W}(H)$, then the population remains the same. From Proposition 3, there is not a uniquely optimal choice of wealth in the optimal plan. For the sake of concreteness, we assume that the residents increase wealth to $\mathcal{W}(H)$.[23] This choice makes future housing prices equal to C.

The third component of the guess is that when H is less than H^s and W is greater than or equal to $\mathcal{W}(H^s)$, residents will also choose the policies from the initial period of their optimal development plan. Again, this is because Proposition 4 tells us that future residents will follow their plan. Thus, the population increases to $\mathcal{H}(W)$ and the residents finance the investment necessary to service the new population to keep wealth constant.

The fourth component concerns what happens when H is less than H^s and W is less than $\mathcal{W}(H^s)$. This is the most complicated case and our guess here has multiple sub-components. Based on Proposition 2, we guess there will exist a *threshold wealth function*, which we denote $W^*(H)$, such that new construction will occur if the community's wealth is greater than or equal to this threshold.[24] If the community's wealth is less than the threshold, there will be no new construction and the community will accumulate wealth. Based on Proposition 2, we conjecture this function will be increasing on $[H_0, H^s)$, will exceed $\mathcal{W}(H)$, and will satisfy the limit condition

[23] Note that this will not be categorized as development by public wealth accumulation, because there is no development.

[24] It should be clear that this threshold wealth function $W^*(H)$ will be related to, but not exactly the same as, the threshold wealth function $W^*(H_t)$ associated with Proposition 2. While a different notation could have been employed to more clearly distinguish the two functions, for the remainder of the paper $W^*(H)$ will refer to the threshold wealth function associated with the equilibrium, so no confusion should arise. Similar remarks apply to the function $W_n(H)$ discussed below.

$\lim_{H \nearrow H^s} W^*(H) = \mathcal{W}(H^s)$.

When wealth is above the threshold and new construction occurs, we guess that the community's policy choices will not leave next period's residents with an incentive to accumulate wealth. Were they to do so, the fall in house prices caused by the higher taxes would be anticipated and the market deterred from providing new construction in the current period. Leaving next period's residents with no incentive to build wealth requires that the new state (W', H') is such that W' is at least as big as $W^*(H')$. Since there would seem to be no gain to current residents from leaving more wealth than necessary, we conjecture that W' will equal $W^*(H')$. Under this assumption, equilibrium in the housing market will require that

$$(1 - H')\overline{\theta} + S(H') + \frac{W - \beta W^*(H')}{H'} - C(1 - \beta) = \underline{u}. \tag{18}$$

This condition turns out to be sufficient to pin down what the new housing level H' must be.

When wealth is below the threshold, we guess there will exist some increasing function $W_n(H)$ describing how much the community will accumulate. This function will have the property that $W_n(H)$ exceeds $W^*(H)$, so that the wealth accumulation spurs development in the next period. The wealth level $W_n(H)$ will represent the optimal amount of accumulation for residents given the equilibrium play of future residents.

We have now described all four components of our guess. The key element is the threshold wealth function $W^*(H)$. Once we have this, the policy rules and value function follow from it. To make this precise, let Ψ denote the set of all real valued functions W^* defined on the interval $[H_0, H^s]$ with the properties that W^* is increasing, differentiable, and exceeds $\mathcal{W}$ on the interval $[H_0, H^s]$, and satisfies $W^*(H^s) = \mathcal{W}(H^s)$. Below we show that for any $W^* \in \Psi$, we can define a corresponding candidate equilibrium, which we denote $\mathcal{E}(W^*)$. Our conjectured equilibrium is the candidate equilibrium $\mathcal{E}(W^*)$ associated with the function W^* that satisfies a particular property. This is that for all housing levels H less than H^s, when the community has the wealth level $W^*(H)$, the residents are indifferent between choosing the equilibrium policies and choosing to keep housing constant and increase wealth to $W_n(H)$. This indifference property implies that $W^*(H)$ is a threshold wealth level in the sense discussed above. We show that, under a regularity condition detailed below, such a candidate equilibrium is indeed an equilibrium. An equilibrium threshold wealth function is then defined to be a function W^* satisfying the indifference property and the regularity condition.

5.3.2 Details[25]

We now show how to define the candidate equilibrium $\mathcal{E}(W^*)$ corresponding to any $W^* \in \Psi$. This just involves drawing out the implications of the four components of our guess. We start with the public good. Given the first component of our guess, for all states (g, b, H), the public good rule is

$$g'(g, b, H) = (1 - \delta)g^o(H'(g, b, H)). \tag{19}$$

We can also take care of the tax rule since this just follows from the budget constraint (1). Thus, for all states (g, b, H), the tax is

$$T(g, b, H) = \frac{(1 + \rho)b + c\left(\frac{g'(g,b,H)}{1-\delta} - g\right) - b'(g, b, H)}{H'(g, b, H)}. \tag{20}$$

Next, we deal with states in which $H \in [H^s, 1]$. Given the second component of our guess, the housing rule is

$$H'(g, b, H) = \begin{cases} H & \text{if } W \leq \mathcal{W}(H) \\ \mathcal{H}(W) & \text{if } W \in (\mathcal{W}(H), \mathcal{W}(1)] \end{cases}, \tag{21}$$

and the debt rule is

$$b'(g, b, H) = \begin{cases} \frac{c(1-\delta)g^o(H) - \mathcal{W}(H)}{1+\rho} & \text{if } W \leq \mathcal{W}(H) \\ \frac{c(1-\delta)g^o(\mathcal{H}(W)) - W}{1+\rho} & \text{if } W \in (\mathcal{W}(H), \mathcal{W}(1)] \end{cases}. \tag{22}$$

These rules imply that when W exceeds $\mathcal{W}(H)$ and new construction takes place, the community has wealth W at the beginning of the next period. When W is less than or equal to $\mathcal{W}(H)$, next period wealth is $\mathcal{W}(H)$. We also need to specify the price and value function for this part of the state space. In defining the price, we employ the notation

$$\mathcal{P}(H, W', W) = (1 - H)\overline{\theta} + S(H) + \frac{W - \beta W'}{H} + \beta C - \underline{u}, \tag{23}$$

to denote the price at which housing demand would equal H if wealth levels this and next period were W and W', the housing price next period were C, and the community provides the efficient level of the public good. Then we have

$$P(g, b, H) = \begin{cases} \mathcal{P}(H, \mathcal{W}(H), W) & \text{if } W \leq \mathcal{W}(H) \\ C & \text{if } W \in (\mathcal{W}(H), \mathcal{W}(1)] \end{cases}. \tag{24}$$

[25] Readers just interested in seeing what happens in the equilibrium can skip ahead to Section 6.

Note that, by definition, $\mathcal{P}(H, \mathcal{W}(H), \mathcal{W}(H))$ is equal to C, so the price will not exceed C. The value function in this part of the state space is

$$V(g,b,H) = \begin{cases} V^*(\mathcal{W}(H)) + \frac{W - \mathcal{W}(H)}{H} & \text{if } W \leq \mathcal{W}(H) \\ V^*(W) & \text{if } W \in (\mathcal{W}(H), \mathcal{W}(1)] \end{cases} \tag{25}$$

where on the interval $[\mathcal{W}(H^s), \mathcal{W}(1)]$ the function $V^*(W)$ is defined as

$$V^*(W) = C + \frac{\underline{u}}{1-\beta} + \frac{\mu\overline{\theta}}{1-\mu\beta}(\mathcal{H}(W) - 1). \tag{26}$$

It is worth noting that $V^*(W)$ is concave on the interval $[\mathcal{W}(H^s), \mathcal{W}(1)]$ given that $\mathcal{H}(W)$ is concave.

States in which $H \in [H_0, H^s)$ but the community's assets are such that $W \in [\mathcal{W}(H^s), \mathcal{W}(1)]$ are governed by the third component of our guess. Things are very simple for this case and we have that

$$(H'(g,b,H), b'(g,b,H), P(g,b,H)) = (\mathcal{H}(W), \frac{c(1-\delta)g^o(\mathcal{H}(W)) - W}{1+\rho}, C) \tag{27}$$

and that

$$V(g,b,H) = V^*(W), \tag{28}$$

where the function $V^*(W)$ is defined as in (26).

Finally, we deal with states in which $H \in [H_0, H^s)$ but the community's assets are such that W is less than $\mathcal{W}(H^s)$. This is the complicated case, covered by the fourth component of our guess. It is here that the equilibrium policy rules depend on the particular threshold wealth function W^* we start with. The housing rule is

$$H'(g,b,H) = \begin{cases} H & \text{if } W < W^*(H) \\ H_c(W) & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}, \tag{29}$$

where the function $H_c(W)$ is defined implicitly as the solution to the system:

$$(1 - H_c)\overline{\theta} + S(H_c) + \frac{W - \beta W^*(H_c)}{H_c} - C(1-\beta) = \underline{u} \;\;\&\;\; W^*(H_c) \geq W. \tag{30}$$

It will be shown in the proof of the Theorem below that $H_c(W)$ is an increasing function on the interval $[W^*(H_0), \mathcal{W}(H^s))$ bounded above by H^s. The debt rule is

$$b'(g,b,H) = \begin{cases} \frac{c(1-\delta)g^o(H) - W_n(H)}{1+\rho} & \text{if } W < W^*(H) \\ \frac{c(1-\delta)g^o(H_c(W)) - W^*(H_c(W))}{1+\rho} & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}, \tag{31}$$

where the function $W_n(H)$ will be defined formally below once the value function has been specified. These rules imply that when W exceeds $W^*(H)$ and new construction takes place, the community's wealth next period is the threshold level associated with its new housing stock $W^*(H_c(W))$. When W is smaller than $W^*(H)$, wealth next period is $W_n(H)$, so this represents the level the community will build up to when starting below the threshold. The price rule is

$$P(g,b,H) = \begin{cases} \mathcal{P}(H, W_n(H), W) & \text{if } W < W^*(H) \\ C & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}. \tag{32}$$

The definition of $W_n(H)$ below will imply the price cannot exceed C. The value function is given by

$$V(g,b,H) = \begin{cases} V^*(W^*(H)) + \frac{W - W^*(H)}{H} & \text{if } W < W^*(H) \\ V^*(W) & \text{if } W \in [W^*(H), \mathcal{W}(H^s)) \end{cases}, \tag{33}$$

where on the interval $[W^*(H_0), \mathcal{W}(H^s))$ the function $V^*(W)$ is defined recursively as

$$V^*(W) = (1-\mu)\left[C + \frac{u}{1-\beta}\right] + \mu\left[S\left(H_c(W)\right) + \frac{W - \beta W^*(H_c(W))}{H_c(W)} + \beta V^*(W^*(H_c(W)))\right]. \tag{34}$$

Finally, the function $W_n(H)$ is defined as

$$W_n(H) = \arg\max_{W'} \left\{ \begin{array}{c} (1-\mu)\left[\mathcal{P}(H, W', W^*(H)) + \frac{u}{1-\beta}\right] + \mu\left[S\left(H\right) + \frac{W^*(H) - \beta W'}{H} + \beta V^*(W')\right] \\ s.t.\ \mathcal{P}(H, W', W^*(H)) \leq C \end{array} \right\}. \tag{35}$$

This maximization problem represents the problem faced by residents choosing next period's wealth under the assumption that there is no new construction this period. Notice that the price constraint implies that $W_n(H)$ must be at least as large as $W^*(H)$ which means that next period's housing price is C. The function $V^*(W)$ is defined on the interval $[W^*(H_0), W(1)]$ by (26) and (34).

We have now defined the policy functions and value function for all possible states, so this completes the definition of our candidate equilibrium $\mathcal{E}(W^*)$. As noted above, our conjectured equilibrium is the candidate equilibrium $\mathcal{E}(W^*)$ associated with the function W^* with the property that for all housing levels $H \in [H_0, H^s)$, when the community has wealth $W^*(H)$, the residents are indifferent between the equilibrium policies and the wealth accumulation policies $(W_n(H), H)$.[26]

Our next result verifies that, if the function $V^*(W)$ associated with this function W^* is strictly

[26] By the wealth accumulation policies $(W_n(H), H)$, we mean the policies that would keep housing constant, provide the public good efficiently, and result in a wealth level $W_n(H)$ next period.

concave, this guess is indeed an equilibrium.

Theorem *Suppose that Assumptions 1-3 are satisfied. Let* $W^* \in \Psi$ *and let* $\mathcal{E}(W^*)$ *be the associated candidate equilibrium. Suppose that (i) the function* $V^*(W)$ *defined on the interval* $[W^*(H_0), \mathcal{W}(1)]$ *by (26) and (34) is strictly concave and (ii) for all* $H \in [H_0, H^s)$

$$V^*(W^*(H)) = (1-\mu)\left[\mathcal{P}(H, W_n(H), W^*(H)) + \frac{u}{1-\beta}\right] + \mu\left[S(H) + \frac{W^*(H) - \beta W_n(H)}{H} + \beta V^*(W_n(H))\right]. \tag{36}$$

Then, $\mathcal{E}(W^*)$ *is an equilibrium.*

In light of this result, we will refer to a function W^* that satisfies the conditions of the Theorem as an *equilibrium threshold wealth function.* To interpret the indifference condition (36), note that $V^*(W^*(H))$ (which is defined in (34)) represents the continuation payoff from the equilibrium policies when the initial state (g, b, H) is such that the community's wealth is $W^*(H)$. The expression on the right hand side represents the continuation payoff from building wealth up to $W_n(H)$, holding constant housing at H. The proof of the Theorem can be found in Appendix 2. The main task lies in establishing that the policy rules defined above are optimal for the residents in the sense of solving problem (8) when the function $V^*(W)$ is strictly concave and (36) holds. This is not a straightforward task because the choice problems faced by residents are not always well-behaved and the optimal solutions are sometimes corner solutions.

5.4 Existence of equilibrium

The analysis so far leaves open the question of whether there exists an equilibrium threshold wealth function. While we do not have an analytical proof that such a function must exist, we have been able to find equilibrium threshold wealth functions numerically for specific parameterizations of the model. Indeed, we have considered a vast number of such parameterizations and in almost all those satisfying our assumptions we have found an equilibrium threshold wealth function. Specifically, we have studied 20878 different parameterizations under which there exist an interval of initial housing levels H_0 that satisfy Assumptions 1-2. For over 95% of these, there exists an equilibrium threshold wealth function for every initial housing level in the interval. A detailed account of our numerical analysis of the model can be found in Appendix 4.

6 Community development by public wealth accumulation

This section describes how the community develops in the equilibrium found in the previous section and identifies the circumstances under which equilibrium involves development by wealth accumulation. This is defined as arising whenever the community develops beyond the size that could be supported by its initial wealth. More formally, equilibrium involves development by wealth accumulation if the community's housing stock grows larger than $\max\{H_0, \mathcal{H}(W_0)\}$. The way the community develops depends on its initial wealth W_0. There are three ranges - high, medium, and low - that are associated with distinct patterns of development. Development by wealth accumulation occurs in the medium and low ranges.

6.1 High initial wealth

When W_0 exceeds $\mathcal{W}(H^s)$, future residents will follow the period 0 residents' optimal plan and the equilibrium outcome is exactly as under this plan. Thus, in period 0, residents invest in the public good and the market provides new construction. The increase in the stock of the public good is financed entirely with debt, which keeps the community's wealth constant. Thereafter, there is no further development.[27]

Proposition 5 *Suppose that Assumptions 1-3 are satisfied. Let W^* be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 \geq \mathcal{W}(H^s)$, then, in this equilibrium, in period 0, the community invests in $g^o(\mathcal{H}(W_0)) - g_0$ units of the public good and the market provides $\mathcal{H}(W_0) - H_0$ new houses. The community finances investment so as to keep its wealth constant, meaning that all but $c\delta g^o(\mathcal{H}(W_0))$ is financed with debt. Thereafter, the public good is maintained at $g^o(\mathcal{H}(W_0))$ and no more housing is provided. Taxes are set so that wealth remains at W_0. Throughout, the price of housing is C.*

In this case, the equilibrium does *not* involve development by wealth accumulation because the community just grows to a size $\mathcal{H}(W_0)$. To be sure, the community invests in the public good to service the new population, so that its public assets grow. However, the community finances this increase in public good stock solely with debt, so that its liabilities grow by the exact same amount. Accordingly, the community's public wealth remains constant.

[27] The proofs of Propositions 5, 6, and 7 can be found in Appendix 3.

6.2 Medium initial wealth

When W_0 is less than $\mathcal{W}(H^s)$, future residents will not follow the period 0 residents' optimal development plan, so the equilibrium outcome must differ from this plan. If W_0 is between $W^*(H_0)$ and $\mathcal{W}(H^s)$, the market provides new construction in the initial period, but less than the $\mathcal{H}(W_0) - H_0$ units provided under the path associated with high initial wealth. When the residents invest in the public good, they finance some of the investment with taxation and, as a result, the market provides less new construction. However, the next period, the community begins with a higher level of wealth. The same thing happens again in period 1: the residents finance some investment with taxes, which dampens new construction but increases the community's wealth a little further. This process keeps going indefinitely, with the community's housing and wealth levels gradually increasing. The size of the community converges to H^s asymptotically.

Proposition 6 *Suppose that Assumptions 1-3 are satisfied. Let W^* be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 \in [W^*(H_0), \mathcal{W}(H^s))$, then, in this equilibrium, the market provides new construction in every period and the housing stock converges asymptotically to H^s. In all periods, the community provides the efficient level of the public good and finances some of the increase in stock with taxes. The community's wealth increases, converging asymptotically to $\mathcal{W}(H^s)$. Throughout, the price of housing is C.*

In this case, the equilibrium involves development by wealth accumulation because the community grows to a size $H^s = \mathcal{H}(\mathcal{W}(H^s))$ which exceeds $\mathcal{H}(W_0)$. To illustrate how the community develops, we compute our equilibrium for a particular example. The public good benefit function has the form $B(z) = B_0 z^\sigma/\sigma$ for some $\sigma \in (0,1)$ and the parameters have the following values:

Parameter	$\bar{\theta}$	β	μ	δ	σ	B_0	α	C	c	$\underline{u}$
Value	1	$\frac{1}{1.06}$	.96	.1	.5	.34	.6	20	1	0

The results are described in Figure 1.

The top panel describes the equilibrium sequence of wealth and housing levels $\{W_t, H_t\}_{t=0}^{\infty}$. Wealth is measured on the vertical axis, and housing on the horizontal. The left most dot is (W_0, H_0), the next one is (W_1, H_1), etc, etc. The line in this panel describes the equilibrium threshold wealth function $W^*(H)$, so the position of (W_0, H_0) implies that W_0 exceeds $W^*(H_0)$. In all periods, residents choose a level of wealth to carry forward equal to the threshold level associated with the new housing stock, so that (W_1, H_1) and the dots that follow all lie on this

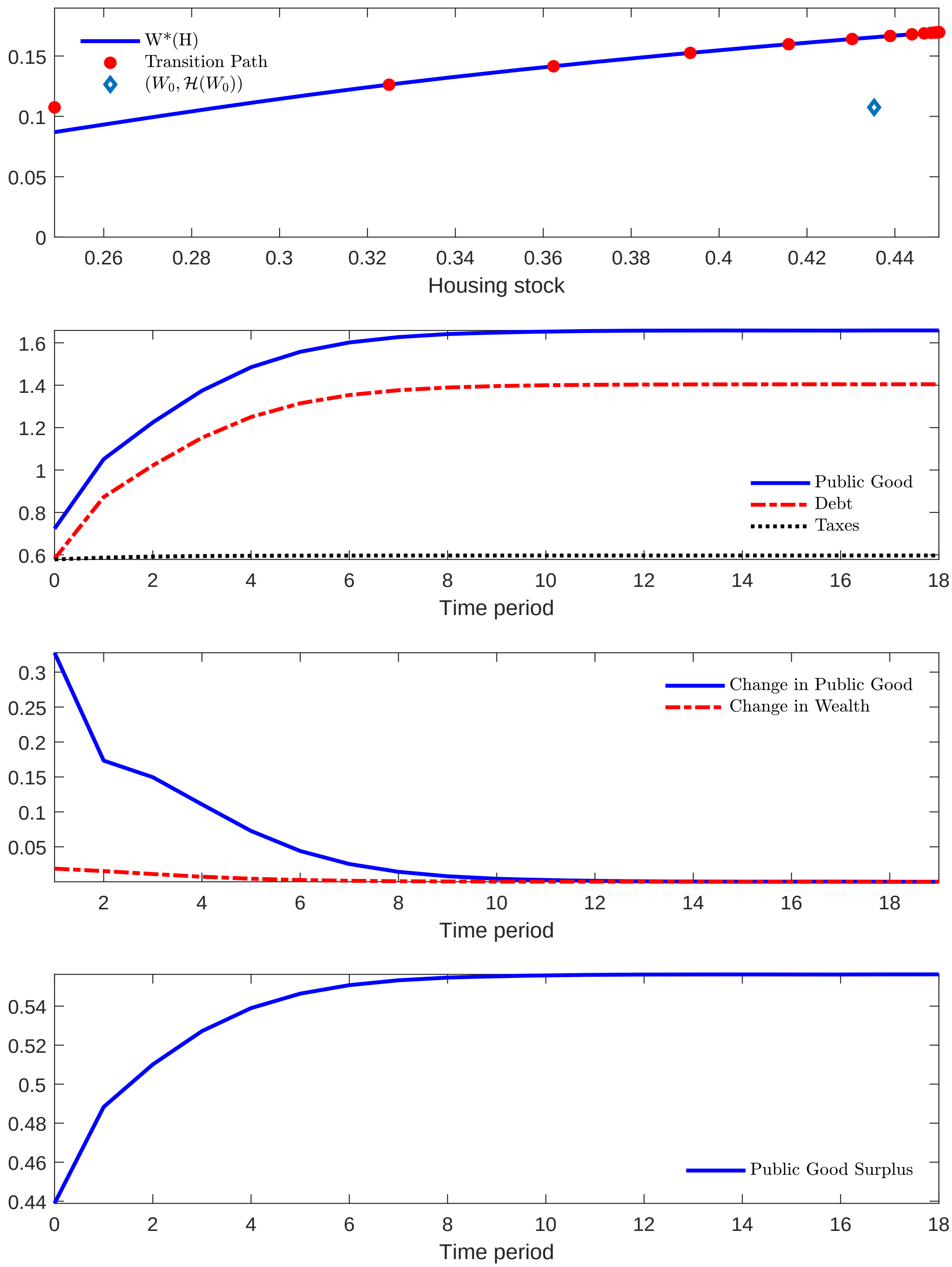

Figure 1: Development with medium initial wealth

line (i.e., $(W_t, H_t) = (W^*(H_t), H_t)$). In this example, $\mathcal{H}(W_0)$ is equal to 0.435, so that it takes the community six periods to exceed the housing level that it could attract with its initial wealth. The second panel describes the evolution of the public good, taxes, and debt. The public good increases steadily as the population increases. Taxes increase, albeit very little, while debt grows. The growth in debt parallels the growth in the public good. The third panel illustrates how the community is building wealth. The upper line describes the increase in the community's public good stock, $g_t - g_{t-1}$, and the bottom line describes the increase in public wealth, $W_t - W_{t-1}$. Given that the difference between the increase in public good stock and the increase in wealth is the increase in debt, the difference between the two lines illustrates how much of the increase in the public good is paid for with debt. While most of the increase is financed by debt, a proportion is financed by taxes which is what allows wealth to build. Given that taxes are essentially constant over the course of development, this tax finance is driven by the increase in tax revenues stemming from a larger tax base. Finally, the bottom panel illustrates how the public good surplus available from living in the community evolves over time. The key point to note is that as the community's wealth increases, the surplus increases. This reflects the fiscal externality and explains why increasing numbers of potential residents choose to live in the community.

6.3 Low initial wealth

If W_0 is smaller than $W^*(H_0)$, no new construction occurs in the initial period. Rather, the residents simply raise taxes to build up wealth. In the next period, new construction gets underway. Depending on the amount of accumulation, there can either be a single period of new construction and the housing stock jumps to H^s, or there can be new construction in every period and gradual convergence to H^s.

Proposition 7 *Suppose that Assumptions 1-3 are satisfied. Let W^* be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 < W^*(H_0)$, then, in this equilibrium, the market provides no new construction in period* 0 *and the community raises taxes to build wealth to a level $W_n(H_0) \leq \mathcal{W}(H^s)$. If $W_n(H_0) < \mathcal{W}(H^s)$, the market provides new construction in every subsequent period, the housing stock converges asymptotically to H^s, and the community's wealth converges asymptotically to $\mathcal{W}(H^s)$. If $W_n(H_0) = \mathcal{W}(H^s)$, the market provides $H^s - H_0$ new houses in period* 1*, no new houses in subsequent periods, and wealth remains at $\mathcal{W}(H^s)$. In ether case, in all periods the community provides the efficient level of the public good. The price of housing is less than C in period* 0 *and C thereafter.*

In this case, the equilibrium again involves development by wealth accumulation because the community grows to size $\mathcal{H}(\mathcal{W}(H^s))$. Figure 2 illustrates how the community develops in this case. The right panels illustrate a case in which $W_n(H_0)$ is less than $\mathcal{W}(H^s)$ and the left panels illustrates a case in which $W_n(H_0)$ is equal to $\mathcal{W}(H^s)$. We get both cases with the same parameterization by simply varying the initial housing stock. A higher housing stock means a greater population over which to spread the costs of accumulation and that yields a higher $W_n(H_0)$. Notice that, after the initial period of accumulation, the path of development is gradual in the right panels and rapid in the left panels. Aside from this, the basic dynamics of policies in the two cases are similar.

6.4 The incentives underlying wealth accumulation

Propositions 5, 6, and 7 imply that equilibrium involves development by wealth accumulation if and only if W_0 is less than $\mathcal{W}(H^s)$.[28] It is worth understanding the incentives driving the accumulation that takes place. Interestingly, when new construction is occuring and residents are financing some of the public good investment with taxes, they are not choosing to hold back current development to subsidize future development. On the contrary, they choose the largest amount of development they can and it is the off-equilibrium path behavior of future residents that forces accumulation.

To understand this, suppose we are in the medium wealth case and consider what happens in the initial period. The equilibrium policy involves the market providing a level of new construction equal to $H_c(W_0) - H_0$ and residents increasing wealth to the threshold level associated with the new housing stock $W^*(H_c(W_0))$ where $H_c(W_0)$ is defined in (30). Note first that, by choosing higher taxes, the residents could increase the wealth level carried forward to some level beyond $W^*(H_c(W_0))$ at the cost of reduced development in the initial period. Development is reduced because the higher taxes deter some potential residents from purchasing a house in the community. The proof of the Theorem shows that residents have no incentive to hold back development in this way. Intuitively, the residents are already providing next period's residents a wealth level sufficient to deter them from stopping development and accumulating more wealth. Since next period's residents have a lower cost of accumulation, current residents have no incentive to accumulate more than this threshold level.

The more subtle issue is whether residents could attract more development by choosing lower taxes? The answer is no. Suppose they were to choose a small tax reduction with the aim of

[28] Note also that Assumption 2 is guaranteeing that the community's initial housing stock H_0 is less than H^s. If H_0 were greater than or equal to H^s, equilibrium would not involve development by wealth accumulation.

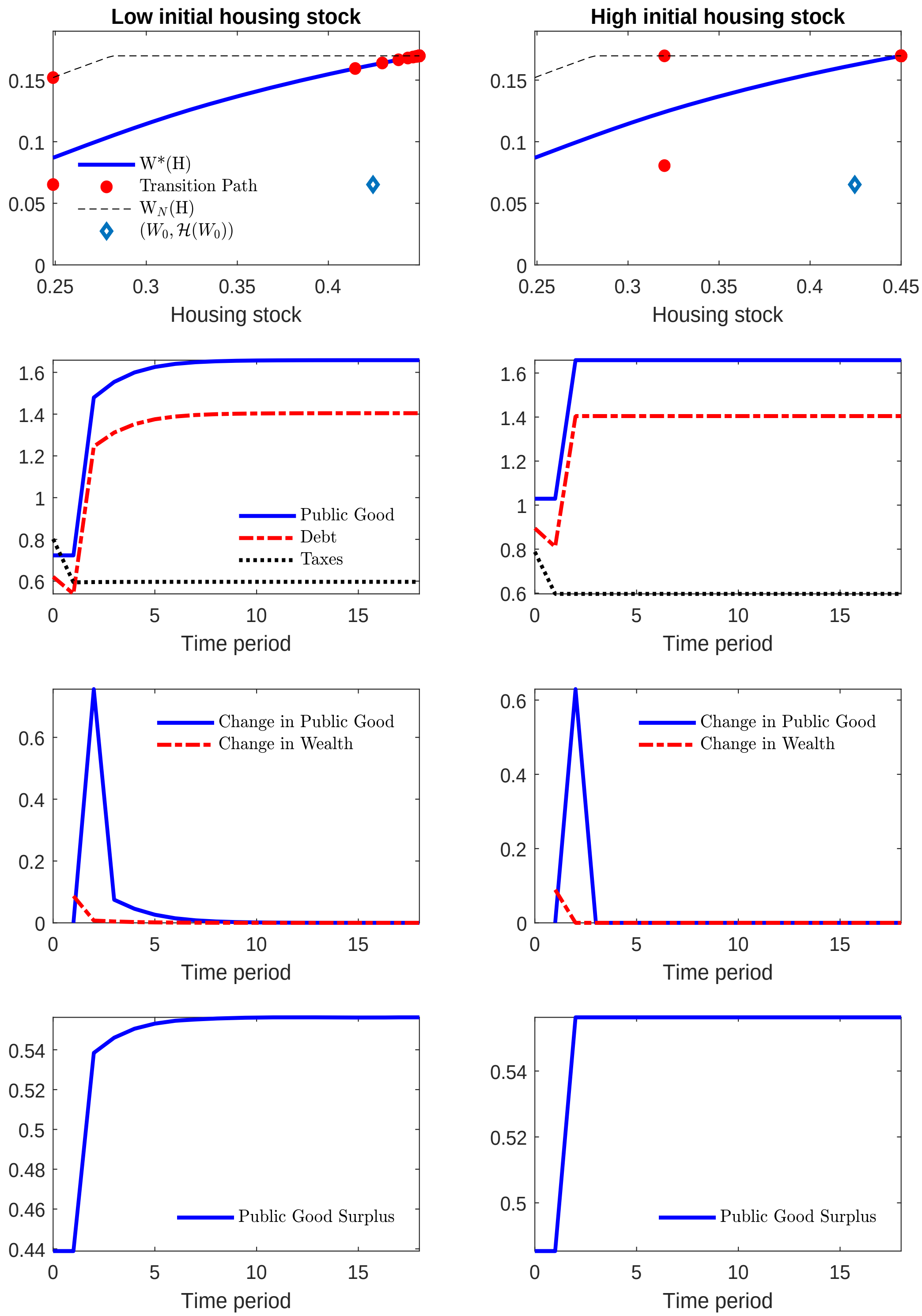

Figure 2: Development with low initial wealth

carrying forward a wealth level W' marginally lower than the threshold $W^*(H_c(W_0))$ and attracting marginally more potential residents. Then, in response to this deviation from equilibrium play, next period's residents would choose to raise taxes to accumulate wealth and no development would take place. The price of housing in the next period would fall discontinuously below the construction cost C in response to this wealth accumulation initiative. This fall would necessitate a reduction in the current price of housing in order to continue to attract a population of size $H_c(W_0)$. Intuitively, potential residents would need to be compensated for the reduction in the future value of their homes. But this would drive the current price of housing below the construction cost C and would mean that the market would not supply any new construction. Thus, a marginal reduction in taxes would mean that residents could no longer attract a population of $H_c(W_0)$ - let alone a larger population. A bigger reduction in taxes would not help, because the lower level of wealth carried forward would be perfectly capitalized in next period's price of housing.

This discussion reveals an interesting non-monotonic relationship between debt-financed tax reductions and potential residents' desire to live in the community. Debt-financed tax reductions are attractive to potential residents provided that development continues in the next period. For as long as this is the case, the future value of potential residents' homes remains at the construction cost C. A tax reduction then simply creates an increase in public good surplus which increases the desire of potential residents to live in the community (see (5)). Once the tax reduction is high enough to drive next period's wealth below the threshold, development stops in the next period and a wealth accumulation initiative is triggered. This causes a discontinuous reduction in the future value of potential residents' homes and potential residents' desire to live in the community. Further tax reductions beyond this point simply cause an even bigger reduction in future home values via the capitalization of higher debt.[29]

It is in the initial period in the low wealth case, that the residents may choose to hold back development. In equilibrium, there is no new construction and the residents build the community's wealth to $W_n(H_0)$ by raising taxes, paving the way for development in period 1. As in the commitment case, this occurs even when development is feasible. In particular, if W_0 is smaller than but sufficiently close to $W^*(H_0)$, there exists a housing level H' larger than H_0 such that if the residents provided a public good level $g^o(H')$ and financed it in such a way as to increase wealth to $W^*(H')$, the market would respond by providing $H' - H_0$ units of new construction.[30]

[29] This capitalization logic also explains why residents have no incentive to implement debt-financed transfers. In addition, it explains why a transversality condition on the community's debt is not needed.

These alternative policies offer a higher payoff in the initial period than the equilibrium policies as they involve both a higher housing price and more development. They also allow development to continue in the next period. However, they offer less payoff in the future, because the wealth level $W_n(H_0)$ exceeds $W^*(H')$ and permits more development in the next period.

6.5 The significance of congestibility

Development by wealth accumulation occurs when the community's initial wealth is less than $\mathcal{W}(H^s)$. This raises the question of what determines $\mathcal{W}(H^s)$? The key determinant is the congestibility of the public good (α). As is clear from (14), the size of α determines the extent of the positive fiscal externality arising from sharing the costs of public good provision. The case of $\alpha = 1$ is of particular interest because there is no positive fiscal externality arising from the public good. Thus, the incentive for accumulation arises only when the fiscal externality created by the collective ownership of community assets is positive, which requires that the community has negative initial wealth. Intuitively, when the community has negative wealth, it has a debt burden and residents would like to attract more residents to share the burden of servicing it. Formally, this can be seen by computing $\mathcal{W}(H^s)$ which equals

$$\mathcal{W}(H^s) = -\frac{H^s\left((1 - H^s)\,\overline{\theta} + S - C(1-\beta) - \underline{u}\right)}{1-\beta} < 0, \tag{37}$$

where S is the optimized public good surplus (which is independent of H when $\alpha = 1$). At the other extreme, we have the case of $\alpha = 0$ in which the public good is a pure public good. This creates the largest positive fiscal externality from cost sharing and the strongest incentives to attract new residents. In this case, using (13), (14), and (16), it is easy to show that

$$\mathcal{W}(H^o) = \frac{cg^o(H^o)(1-\beta(1-\delta))}{1-\beta}. \tag{38}$$

Recall that $\mathcal{W}(H^s)$ approaches $\mathcal{W}(H^o)$ as μ converges to 1, so that for sufficiently large μ, $\mathcal{W}(H^s)$ exceeds $cg^o(H^o)$. Thus, even if the community begins with the optimal level of public good for its initial population and no debt, the equilibrium will involve wealth accumulation.

Figure 3 generalizes these observations by graphing $\mathcal{W}(H^s)$ as a function of α holding constant H^s. The latter is achieved by varying the utility obtained from living outside the community.

[30] We know that $\mathcal{P}(H_0, W^*(H_0), W^*(H_0))$ is larger than $\mathcal{P}(H_0, \mathcal{W}(H_0), \mathcal{W}(H_0))$ which is equal to C. Thus, for W_0 smaller than but sufficiently close to $W^*(H_0)$, it must be the case that $\mathcal{P}(H_0, W^*(H_0), W_0)$ is larger than C. Moreover, W_0 is less than $\mathcal{W}(H^s)$ which equals $W^*(H^s)$. It follows that $\mathcal{P}(H^s, W^*(H^s), W_0)$ is less than C. Thus, by continuity, there exist $H' \in (H_0, H^s)$ such that $\mathcal{P}(H', W^*(H'), W_0) = C$. Such a H' has the properties discussed in the text.

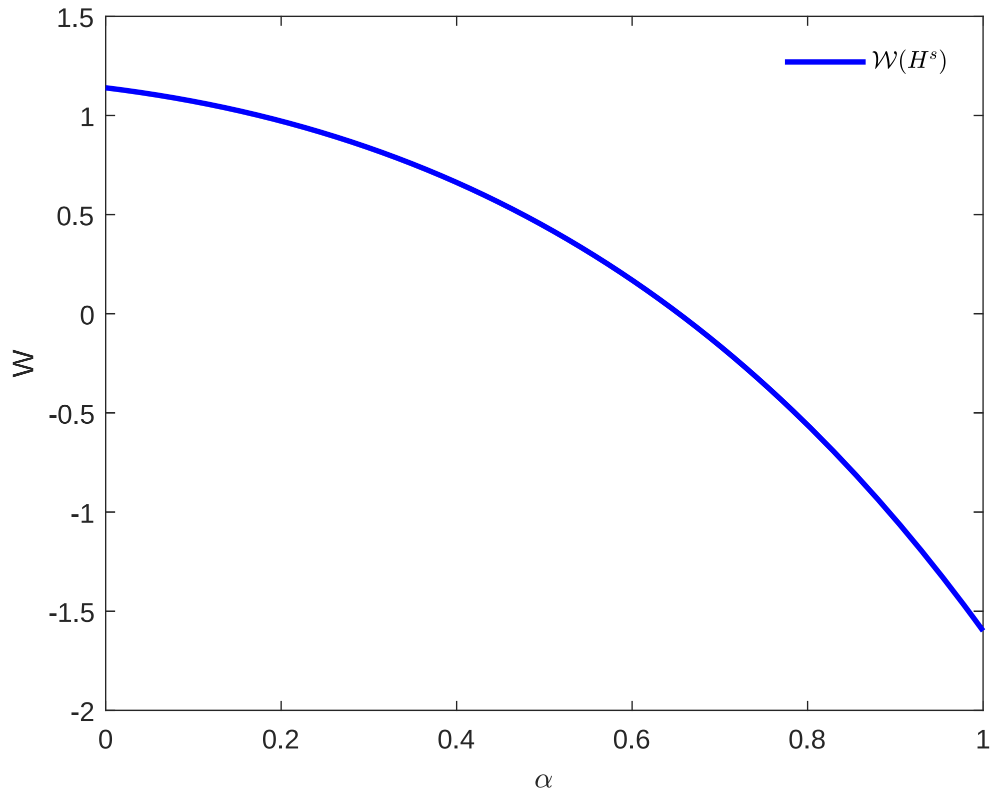

Figure 3: The critical wealth level

Parameters other than α and $\underline{u}$ are set at their benchmark values. Bearing in mind that community development by wealth accumulation occurs if and only if W_0 is less than $\mathcal{W}(H^s)$, the Figure nicely illustrates the role played by the cost-sharing fiscal externality in inducing such behavior. The more important the cost-sharing externality, the larger the set of initial wealth levels for which development by wealth accumulation arises.

7 The optimality of development by wealth accumulation

This section considers how well community development by wealth accumulation works from a normative perspective. Comparing Proposition 1 with Propositions 6 and 7, yields the following conclusion.[31]

Proposition 8 *Suppose that Assumptions 1-3 are satisfied. Let $W^* \in \Psi$ be an equilibrium threshold wealth function and let $\mathcal{E}(W^*)$ be the associated equilibrium. If $W_0 < \mathcal{W}(H^s)$ the long-run size of the community in equilibrium will be smaller than optimal. In addition, the equilibrium exhibits delay because development occurs after period* 0*.*

This proposition reveals that there are two problems with development by wealth accumulation: it does not get the community to its optimal size and, such development as does occur, proceeds too slowly. To understand the first problem, imagine that the community were at its steady state with wealth $\mathcal{W}(H^s)$ and housing level H^s. Why would the residents not accumulate a little more wealth which would increase the future size of the community a little more? Residents must weigh the benefit of increasing the future size of the community via the fiscal externality against the tax cost. The equilibrium increase in the future population would be $\mathcal{H}'(\mathcal{W}(H^s))$. The marginal cost of accumulation is $1/H^s$. As pointed out after Proposition 3, at housing level H^s, the marginal benefit of increasing the population is just equal to the marginal cost. Thus, given the concavity of $\mathcal{H}'(W)$, there is no incentive to accumulate.

Why is development delayed? To shed light on this, it is instructive to consider the policy choices in the initial period and analyze why residents prefer the equilibrium choices to those involving less delay. In the medium initial wealth case, equilibrium policies are such that the community's wealth and housing increase but to levels smaller than $(\mathcal{W}(H^s), H^s)$. Since the community's wealth and housing will eventually increase to $(\mathcal{W}(H^s), H^s)$, why do current residents not prefer to just jump directly to $(\mathcal{W}(H^s), H^s)$? The reason is that raising wealth to this level

[31] To understand this result and the discussion to follow, the reader will need to recall that H^o is the optimal housing level defined in (13) and to bear in mind that $H^s \in (H_0, H^o)$.

would require higher taxes and, given these taxes, the market would not provide $H^s - H_0$ new homes. As noted in Section 6.4, residents are choosing the maximum level of development they can.

In the low initial wealth case, the equilibrium involves no new construction in period 0 and the community's wealth growing to $W_n(H_0)$. Development is delayed at least one period. Again, potential residents do not want to join the community given its high taxes. If $W_n(H_0)$ is equal to $\mathcal{W}(H^s)$, then all development occurs in period 1 and there is no further delay. If $W_n(H_0)$ is less than $\mathcal{W}(H^s)$, then there is further delay as the housing stock gradually grows to H^s. The initial residents could instead choose to increase the community's wealth all the way to $\mathcal{W}(H^s)$, reducing delay to just one period. However, the tax burden of the increase in wealth must be bourne solely by the period 0 residents, whereas the benefits of the higher wealth are shared by the new residents. Thus, this will not be an attractive strategy, unless H_0 is quite close to H^s.[32]

Further insight into the forces driving these distortions can be obtained by considering the limit as μ tends to 1 in which case residents know that they will remain in the community forever. From (17), we see that H^s tends to H^o. It follows that it is population turnover that is responsible for the fact that the community will be too small in the long run. In addition, it can be shown that the range of values of H_0 for which $W_n(H_0)$ equals $\mathcal{W}(H^s)$ vanishes. This means that if W_0 is below $\mathcal{W}(H^o)$, the community will gradually grow towards the optimal size H^o. It follows that population turnover is not responsible for the fact that development is too slow. Gradual development reflects the fact that the costs of accumulation are lower when the population is larger. In this way, development is necessary to spur future development.

8 Discussion

This section discusses the empirical plausibility of the idea of community development by public wealth accumulation and identifies some open questions. It also draws out some broader lessons of the model.

8.1 Plausibility

The logic underlying development by wealth accumulation rests on two basic hypotheses. The first is that potential residents will be influenced in their location decisions by a community's wealth.

[32] A sufficient condition for $W_n(H_0)$ to be less than $\mathcal{W}(H^s)$ is that H_0 is less than $(1-\beta)\,H^s/\mu\,(1-\mu\beta)$. Note that for sufficiently large μ, this condition is satisfied for any H_0 less than H^s.

This is necessary to create the fiscal externality. The second is that residents will be willing to support higher taxes to improve their community's wealth in order to attract more potential residents. This is necessary for the fiscal externality to shape policy choices.

The economics behind the first hypothesis is that higher public wealth allows a community to provide more public good surplus and this is what draws in potential residents. The capitalization literature provides empirical support for the idea that residents are attracted to communities by the public good surplus they offer. This literature leverages the theoretical idea that, if potential residents value public services and dislike taxes, then in communities which provide higher public services (controlling for taxes) or lower taxes (controlling for services), there must be some compensating differential to maintain locational equilibrium if space in these communities is scarce.[33] Typically, the compensating differential focused on is higher housing prices, but wage differentials are also a theoretical possibility in settings where firms choose where to produce.[34]

A particularly relevant example of this style of work is Gyourko and Tracy (1991) who study capitalization of public sector surplus into wages and housing prices. Their underlying theoretical framework is one in which homogeneous worker/residents and firms compete for scarce sites across jurisdictions. Jurisdictions are characterized by natural amenities, public service levels, and tax rates. Wages and gross of tax land prices (which are measured by housing expenditures) adjust to make workers and firms indifferent to their location. This set-up motivates regressing wages and housing expenditures against measures of public services (police, fire, health, and education), taxes, and community-level natural amenities (weather, pollution, proximity to oceans and lakes, etc). Gyourko and Tracy's results support the idea that potential residents value public services and dislike taxes and suggest that fiscal variables are nearly as important to worker/residents as natural amenities.

As stressed by Gyourko and Tracy, public service levels and taxes differ from natural amenities in that they are under the control of local authorities. This raises the question of what is allowing jurisdictions to offer higher public sector surplus to their residents. Our model suggests that it is higher community wealth.[35] Intuitively, a community with better school facilities and community

[33] Recall that in the model of this paper, there is sufficient space to accomodate those who are open to living in the community. Moreover, the supply of housing is perfectly elastic once price reaches construction cost. Thus, after this point, higher public good surplus is dissipated by development rather than housing price increases.

[34] The bulk of the capitalization literature focuses on the impact of property taxes and school quality on housing prices. See Ross and Yinger (1999) and Nguyen-Hoang and Yinger (2011) for surveys.

[35] A possibility suggested by Gyourko and Tracy is that communities offering higher public sector surplus have weaker public sector unions or less corrupt politicians. While such differences across jurisdictions are very plausible, it seems like community wealth must matter as well.

health centers, more modern fire stations, fire trucks, police cars, etc, and lower public debt, is in a stronger position to offer higher public sector surplus. This suggests that there should be a positive relationship between a community's wealth and the public sector surplus that it offers. However, we are not aware of any empirical studies that analyze this relationship.

Evidence supporting the first hypothesis could also come from analyzing the direct relationship between public wealth and housing prices. Unfortunately, the capitalization literature has largely overlooked this relationship.[36] The one exception is the work of Stadelmann and Eichenberger (2014). They take it for granted that, as a theoretical matter, public net assets (i.e., wealth) should be capitalized into housing prices, but note the difficulties of finding a setting in which to test this idea empirically. Most problematic is the absence of good data on the value of public assets. However, they discover that the Swiss Canton of Zurich provides a setting in which the required data exists and analyze the empirical relationship. Their findings provide strong support for the capitalization of public net assets.

Turning to the second hypothesis, in the model, a community's wealth can be improved in two ways: tax-financed public investment or reducing debt. In reality, another way is to finance public investment by reducing non-capital spending on other services. Regarding public investment, the practical literature on community development documents numerous examples of communities improving their public assets with an eye to attracting more residents and/or businesses (see, for example, Phillips and Pittman 2015). Communities undertake projects to revitalize their downtowns and waterfronts, improve their infrastructure, and build museums and parks. They do this both to benefit their residents and also to attract more development.[37] The rationale for attracting more development may vary across communities, but all that matters for this paper's argument is that there be some rationale.[38]

[36] There is some work looking at the impact of unfunded municipal pension liabilities on property prices. See Epple and Schipper (1981), Leeds (1985), and MacKay (2014).

[37] Larger communities also routinely put together packages to attract or retain specific businesses (see, for example, Garcia-Mila and MacGuire 2002 and Greenstone and Moretti 2003). These deals include tax breaks, low-cost or free land, or targeted infrastructure investments. To the extent that the targeted business creates agglomeration economies that benefit the community at large (see Geenstone, Hornbeck, and Moretti 2010 for evidence), these packages have a similar flavor to the public wealth accumulation studied here. The idea is that current residents undertake an investment that spurs future development which benefits the community via an externality. Indeed, if the agglomeration economy is sufficiently large, then such an investment should be able to spur development even if financed by debt. In practice, financing is typically shared with state governments. Moreover, tax breaks to attract a new business do not require changing current taxes or services to finance, which reduces the up-front cost to current residents. An interesting subject for further research is to consider the offering of such policies to attract businesses in a dynamic political economy model like the one studied here.

[38] This is a point worth emphasizing. While our model incorporates a positive fiscal externality created by the sharing of the public good, any positive externality from additional residents would motivate wealth accumulation. As reviewed in Duranton and Puga (2004), the urban economics literature identifies a number of different reasons

Of course, it is not clear that such investment projects are financed by higher taxes on current residents or by reducing spending on other services. Indeed, many will be primarily financed by grants from state or federal government.[39] These grants will permit the community's wealth to increase and should attract potential residents via the fiscal externality, but this is a different mechanism than the locally-generated development studied here. Obviously, residents will prefer their projects to be financed by higher levels of government than by taxation, so it is natural that their elected representatives seek out such grants. Moreover, as we argue below, the motivation for such inter-governmental grants could be precisely that communities will under-accumulate the wealth they need to develop if left unaided.

Regarding the strategy of improving community wealth by reducing debt, we are not aware of any studies that shed light on the prevalence of this.[40] It might be useful to study communities in poor fiscal health due to negative economic shocks and/or large unfunded public employee pension liabilities and document how they respond to their situations. As suggested by the model, these type of communities should be the most likely to levy higher taxes on residents or reduce spending on other services to improve their fiscal health. Again, however, one would expect state governments to play a role in supporting struggling communities, which may obscure the picture empirically.

To sum up, we see two open empirical questions raised by the idea of development by wealth accumulation. First, is it the case that wealthier communities offer higher public sector surplus? Second, how common is it for communities to raise taxes or reduce non-capital spending on other services in order to finance new public investments or pay down debt? Further evidence on these questions would be helpful. It would also be interesting to try and directly test the idea of development by wealth accumulation. The central prediction is that development should be positively related to a community's wealth. A basic difficulty in testing this lies in measuring the value of a community's public assets. A further problem would be in adequately controlling for all the other determinants of development.[41]

for agglomeration economies.

[39] Another possibility that arises in practice is that development-related community investments are financed by private contributions from groups of affluent local citizens. See, for example, Goldstein (2017) and Sullivan (2018).

[40] Temple (1994) provides an empirical analysis of the debt/tax choice in the financing of local government spending, but she is not focused on the implications of this choice for development.

[41] In a well-known study of the determinants of U.S. city growth, Glaeser, Scheinkman, and Shleifer (1995) find that 1960 debt levels (holding revenue constant) are positively associated with subsequent population growth over the period 1960-1990. However, their study does not include information on city assets.

8.2 Broader lessons

The most basic lesson of the model is that the collective ownership of a community's assets and liabilities creates a fiscal externality. This emerges organically from the assumption that the community provides a durable public good and can finance investment with debt. One point not emphasized in the analysis so far is that this fiscal externality can lead a well-endowed community to develop too much. Proposition 5 implies that if initial wealth W_0 exceeds $\mathcal{W}(H^o)$, the community will be too large. The excess entry into the community that arises in this case is analogous to the excess entry familiar in the standard common pool problem and is one way of thinking about the problem of urban sprawl (Nechyba and Walsh 2004). Here the negative fiscal externality created by the collective ownership of community wealth offsets the positive fiscal externality that arises from the cost-sharing of the public good. Residents might like to keep the new entrants out, but there is no way they can do so with the available policy instruments.

The fiscal externality created by collective ownership is relevant for the long-standing policy debate about the desirability of decentralizing fiscal decisions to local communities. On the negative side, it is another reason for believing that a decentralized system in which local governments choose fiscal policies and the free market determines the allocation of households across communities is not likely to produce optimally-sized communities. On the positive side, the fiscal externality does provide a simple mechanism by which decentralized communities can influence their size - at least if they wish to grow. While there are good reasons to suppose that this mechanism will not allow communities to achieve fully efficient sizes, it should allow some development.

The fact that residents will under-accumulate wealth may also have implications for inter-governmental grants. As noted in the previous sub-section, many projects chosen by local communities are financed partially by higher levels of government. The traditional way to motivate such grants is that they account for inter-juridictional spillovers in the benefits of local public goods. This model suggests that they may be a way of allowing communities to build the wealth necessary for them to grow to optimal size. To further analyze this and the general question of the desirability of decentralization, it will be important to build dynamic general equilibrium models featuring multiple communities providing durable public goods with taxes and debt.

8.3 Alternative policy instruments

The model assumes an unregulated competitive housing market. This means that the externalities created by the collective ownership of public wealth and the cost-sharing of the public good are unpriced. This in turn results in the use of wealth accumulation to attract new residents.

The standard way to deal with an externality is to impose a Pigouvian tax or subsidy to internalize it. For the externalities arising in the model, a tax or subsidy on new construction would be appropriate.[42] A tax should be levied when the community has positive wealth and the cost-sharing externality is not sufficient to offset the negative externality this creates. A subsidy is called for when the community has negative wealth or when it has positive wealth, but the cost-sharing externality offsets it. This naturally raises the question of what would happen in the model if residents could levy a tax or subsidy on new construction.

While a full analysis of this is beyond the scope of this paper, there are several points that can be made. First, with residents controlling the price of entry, it seems probable that there would exist equilibria in which the community's wealth does not matter for its development path. Rather, public wealth would just be capitalized into housing prices. Thus, the externality created by the collective ownership of public wealth would be priced and a community with less wealth would just set a lower tax or higher subsidy to reflect its lower appeal.

Second, while wealth accumulation would not occur in these equilibria, the type of gradual development that arises in our equilibrium seems likely if the cost-sharing externality leads residents to employ subsidies to increase new construction. While the benefits of a larger population are shared by future residents, the cost of a subsidy is bourne by the current residents. As the population expands, their cost can be shared among a broader base. The same logic driving gradual wealth accumulation therefore applies.

Third, it is not obvious that residents would be better off in an equilibrium in which subsidies are available than in the equilibrium studied in this paper. A subsidy works differently than wealth accumulation. The obvious difference is that wealth accumulation requires residents to invest in the present to create incentives for the future, while a subsidy works immediately and is paid for contemporaneously. In addition, a subsidy on new construction will reduce the price of residents' properties since new construction and existing housing are substitutes. However, in our equilibrium, wealth accumulation need not reduce housing prices. When the community has medium initial wealth, the price of housing remains constant at construction cost as the community's wealth accumulates (Proposition 6), and, when the community has low wealth, the

[42] In practice, subsidies for new construction do not seem common which could reflect practical (e.g., informational) and/or legal barriers to their use. An exception would be support offered for the provision of affordable (i.e., low income) housing through the Community Development Block Grant Program operated by the U.S. Department of Housing and Urban Development. Explicit taxes also do not seem widespread, but zoning, permits, and impact fees are certainly routine ways communities regulate development.

price of housing falls below construction cost only in the initial period (Proposition 7). If residents anticipate leaving the community with significant probability, this difference could be important. Indeed, the fact that subsidies will reduce property prices may simply curtail their use when they are available. All this suggests that residents could be better off with wealth accumulation.

Barseghyan and Coate (2019) makes a start at formally analyzing the political economy of corrective taxes and subsidies. The paper employs the same basic model of community development presented here. However, to simplify the analysis, it abstracts from public goods, debt, and general taxation, and incorporates externalities in a reduced form manner. Confirming the intuition discussed above, with positive externalities, the paper finds conditions under which there exists an equilibrium in which subsidies are gradually increased, resulting in falling housing prices and increasing community size. Furthermore, there exist equilibria in which development is stalled and subsidies are not deployed. These equilibria are motivated by the fear that development will harm property prices by leading to further subsidization. While this needs to be confirmed in an analysis that incorporates the full set of policies, this finding certainly suggests that there might be circumstances under which development by wealth accumulation will dominate what would happen if residents have access to subsidies.

9 Conclusion

This paper presents and analyzes a novel political economy model of community development. The main lesson from the analysis concerns the potential role of public wealth accumulation in development. The key observation is that the collective ownership of a community's assets and liabilities create a fiscal externality. This fiscal externality is positive when a community has negative public wealth and negative when a community has positive wealth. When making fiscal decisions, residents will anticipate their impact on this fiscal externality. A community with negative public wealth, may want to increase it to benefit from the positive externality. A community with positive wealth may want to increase it to attract residents because of other positive fiscal externalities or agglomeration economies. If such development occurs, future residents may have an incentive to engage in more of it. This is because the cost of further wealth increases can be bourne by a larger group of residents. In this way, the community can develop fueled by public wealth accumulation.

The model reveals the forces that shape such development and sheds light on its normative properties. Development by wealth accumulation arises if the community starts out small enough and with low enough initial wealth. Several patterns of development are possible, depending on the community's initial conditions. With a medium level of initial wealth, development will be gradual and continual, with the community converging asymptotically to a steady state level. With a low level of initial wealth, the community will have a period of accumulation which precedes development. The subsequent development can be gradual or rapid depending on the extent of initial accumulation. Development by wealth accumulation will not take the community to an efficient size and will proceed too slowly. Nonetheless, the mechanism allows even very poorly endowed communities to develop to some extent.

In addition to revealing the idea of development by wealth accumulation, the model presented here provides a platform for further theoretical investigation of community development. There are many questions to be addressed and we have pointed out some along the way. Understanding what would happen if residents could use additional policy instruments such as zoning or a tax/subsidy on new construction is obviously of significant interest. It would also be interesting to extend the model to understand declining communities and the role played by local governments in managing or combating decline. Finally, constructing a dynamic general equilibrium model with multiple communities providing durable local public goods with taxes and debt would allow a number of important issues to be addressed.

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