Peer Preferences, School Competition and the Effects of Public School Choice

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Abstract

This paper develops a new economic model of public school choice. The key innovation is to model competition between schools in an environment in which parents have peer preferences. The analysis yields three main findings. First, peer preferences dampen schools' incentives to exert effort in response to competitive pressure. Second, when peer preferences are sufficiently strong, choice can reduce social welfare. This is because choice is costly to exercise but aggregate peer quality is fixed. Third, given strong peer preferences, choice can reduce school quality in more-affluent neighborhoods. We conclude that peer preferences weaken the case for choice.

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1 Introduction

Do public school choice programs - also known as open enrollment or intra-district choice programs - improve outcomes for families and their children? Proponents of these programs claim that they help households through at least two channels. First, because they give households a free choice of public school, they allow disadvantaged students to attend high-quality schools. Second, because they provide schools with incentives to compete for students, they improve the quality of education that all schools provide.[1] Opponents of these programs challenge both of these claims. First, they note that the most disadvantaged parents may be unable to exercise choice. For example, it may be too costly to travel to a non-neighborhood school. Moreover, since parents are likely to have preferences for good peer groups, the more-advantaged households that can exercise choice will choose schools with more-advantaged students, thereby exacerbating educational inequality. Second, they argue that these peer preferences will dampen the enrollment response to school effort, such that any incentives to improve quality will be weak.

Empirical evaluations could shed light on the likely effects of these programs, but the evidence base is thin and findings are mixed. It is especially difficult to evaluate these programs in the US context, since they have typically been introduced in large urban districts and are often related to desegregation orders (Pathak, 2011). As such, clean control groups are hard to find.[2] Theoretical analyses could inform this discussion by formalizing these claims and examining them more rigorously, but few papers have modeled schools' incentives to improve quality (i.e., the "effort" decisions of schools) and none of these have allowed parents to have peer preferences.

With this gap in the literature in mind, this paper provides a theoretical analysis of public school choice. Crucially, our model considers schools' effort decisions in an environment in which parents can have peer preferences. We begin with a baseline model that makes strong assumptions but yields a tractable analysis and closed-form solutions. The model features one community divided into two equal-sized neighborhoods, each containing one school. Households are characterized by socio-economic status and are perfectly segregated across the two neighborhoods. School quality depends on school effort and the composition of the students enrolled, with a parameter that governs the strength of peer preferences. Schools do not face capacity constraints. Without public school choice, students attend their neighborhood school. With choice, households

[1] Some proponents of choice claim that they can improve outcomes via a third channel: allowing parents to find a good school match. This assumes that parents have heterogeneous preferences and that schools are differentiated. We assume that parents have the same preferences and hence we ignore this channel. As we discuss in Section 6, a consideration of these match effects would likely strengthen the case for choice, but without overturning our main finding - that peer preferences weaken the case for choice.

[2] One could argue that the US context is not geared to realizing the benefits of choice, since schools in these settings are often operating at capacity. We think it is interesting and important to analyze choice in a context in which capacity constraints do not bind, although we also consider an extension in which they do.

can enroll their children in either school but face a cost of attending the non-neighborhood school. This cost is assumed to vary across households and to be independent of socio-economic status.

These assumptions generate a simple benchmark for the effects of choice in the absence of peer preferences (i.e., when parents care only about school effort). Without choice, schools exert zero effort. With choice, they have incentives to exert effort and thereby increase enrollment. Since both schools exert the same level of effort in equilibrium, there are no quality differences between the schools and hence households do not exercise choice. This implies that choice increases the quality of both schools and the welfare of all households, where welfare is defined as the quality of the school attended less any costs of exercising choice.

Activating peer preferences yields three key findings. First, stronger peer preferences weaken schools' incentives to exert effort. The intuition is that under these baseline assumptions, increased enrollment is accompanied by a reduction in relative peer quality (i.e., the composition of the student body relative to that in the other school). Since households anticipate this, strong peer preferences dampen the enrollment response to school effort and hence weaken schools' incentives to exert effort. Second, when peer preferences are sufficiently strong, choice can reduce aggregate welfare. As well as weakening effort incentives, stronger peer preferences increase the quality difference between the two schools and induce more households to attend the higher socio-economic status school. These peer-driven choices are privately optimal but socially wasteful: the non-neighborhood school is costly to attend and one household's peer gain is another household's peer loss. If the wasteful effects of peer-driven choices overwhelm the positive effects of increased effort, then choice reduces aggregate welfare. Third, with strong peer preferences, choice reduces the quality of the school in the affluent neighborhood. This is because the benefits of greater school effort are offset by the costs of an inferior peer group.

These findings imply that peer preferences weaken the case for choice. To assess the robustness of this conclusion we extend the baseline model in three directions. First, we add a housing market and allow households to choose neighborhood (i.e., Tiebout choice). These neighborhood choices take account of the implications for school attendance and these implications obviously depend on whether or not the district operates a public school choice policy. One consequence of this extension is that schools now have incentives to exert effort even when parents cannot choose schools. Nevertheless, the effects of choice in this extended model turn out to be very similar to those in the baseline model.

Second, we assume that schools face capacity constraints. The key insight is that strong peer preferences make it more likely that capacity constraints bind. Not surprisingly, in any equilibrium in which capacity constraints bind, schools exert zero effort and hence choice reduces aggregate welfare.

Third, we assume that more affluent households face lower costs of exercising choice. In this case, peer preferences strengthen the effort incentives provide by choice, as the mechanism identified in the baseline model goes into reverse (i.e., increased enrollment is associated with improvements in relative peer quality). However, this effect comes at a price. In particular, with sufficiently strong peer preferences, choice induces an exodus of more-affluent parents from the school in the less-affluent neighborhood. This decreases the quality of this school which now serves the most disadvantaged students in the community (i.e., those who cannot afford to exercise choice). This is precisely the concern of those who worry that choice will exacerbate educational inequality. Moreover, this extended model still exhibits the property that choice reduces welfare when peer preferences are sufficiently strong.

Moving beyond these specific extensions, we acknowledge that our analysis abstracts from other important features of real-world settings - multiple neighborhoods, heterogeneous preferences, parental oversight, and public school exit options. However, we argue that a consideration of these features is unlikely to overturn our main conclusion: that peer preferences weaken the case for choice.

We also identify some broader implications of our findings. First, we argue that, because peer preferences shape how choice affects welfare in different neighborhoods (e.g., more-affluent versus less-affluent), our findings can inform studies of the "political economy" of choice. Second, because our analysis identifies several factors that shape the effects of choice - peer preferences, neighborhood inequality (which magnifies the importance of peer preferences) and capacity constraints - we note that our findings could help to interpret various estimates of the impacts of choice on measures of quality (e.g., test scores). Third, since our analysis suggests that peer preferences weaken the case for choice, we suggest that there may be a role for auxiliary policies that can undo some of the effects of peer preferences. One such policy might be a school funding formula that incentivizes schools to enroll lower socio-economic status students.

Our paper builds on three strands of literature. The first is the large literature that estimates households' preferences for school attributes (e.g., see Hastings et al. (2009) for the US and Burgess et al. (2015) for the UK). Many of these papers use data generated by public school choice systems that employ centralized assignment mechanisms.[3] One consistent finding is that households prefer schools that are close. This motivates our assumption that there are costs of attending the non-neighborhood school. Another consistent finding is households prefer schools with more-advantaged students. This motivates our assumption that households can have peer preferences.[4] Further evidence of the relative importance of peer composition is provided by Abdulkadiroglu et al (2017) and Rothstein (2006).

[3]A growing literature considers the exact mechanism by which households are allocated to schools (e.g., whether this induces households to express true preferences or behave strategically). See Pathak (2011) for a review.

[4]These papers reach different conclusions as to whether preferences are heterogeneous. Hastings et al. (2009) estimate that relative to lower socio-economic status parents, higher socio-economic status parents have stronger preferences for school average test scores than for proximity. Burgess et al. (2015) find no statistically significant differences between the preferences for school average test scores among low and high socio-economic status households.

Second, we build on the small number of papers that provide a theoretical analysis of public school choice programs. Some of this research has focused on their implications for neighborhood composition, but has abstracted from their incentive effects.[5] Our approach is closest to Lee (1997). He also employs a model featuring two neighborhoods with local schools. As in our model, choice allows households to enroll their children in non-neighborhood schools but it is costly for them to do so. In his basic model, the quality of schools in each neighborhood is determined solely by the level of spending that is chosen by neighborhood residents and the focus is on how this spending changes when choice is introduced.[6] Lee's paper is complementary to ours in that he abstracts from peer preferences by assuming that school quality does not depend on student characteristics, while we ignore the implications of choice for the level of school spending chosen by voters.

Third, our analysis is inspired by the much larger theoretical literatures devoted to Tiebout choice and private school vouchers (see Epple and Nechyba (2004) and Epple and Romano (2012) for reviews). Although the institutional environments are very different, our focus on the implications of peer preferences for efficiency as well as equity is related to some of these analyses.[7] Epple and Romano (2008) analyze a conditional private school voucher scheme that links vouchers to ability. They show that these can preserve the efficiency-enhancing effects of private school vouchers without the cream-skimming that occurs when vouchers are universal.[8] As noted earlier, our model has implications for some auxiliary policies such as subsidizing the transportation costs of poorer students. This type of policy targeting has also been considered in the voucher context (see Epple et al. (2017) for details of these policies and Neilson (2013) for a theoretical analysis).

The rest of the paper is organized as follows. Section 2 analyzes the baseline model. Sections 3-5 analyze our three extensions of this model: in Section 3 we allow Tiebout choice; in Section 4 we allow schools to be capacity constrained; and in Section 5 we allow for the costs of attending the non-neighborhood school to be negatively correlated with socio-economic status. Section 6 identifies and considers further limitations of the baseline model. Section 7 concludes by summarizing and discussing our findings.

[5] For example, Epple and Romano (2003) analyze the equity implications of public school choice in a rich model in which parents choose neighborhoods, choose schools and vote on taxes to support schools. School quality depends on student characteristics, but schools are passive. Avery and Pathak (2015) present a related analysis that examines how the introduction of choice impacts households' neighborhood choices. Their analysis also assumes student composition determines school quality but ignores school effort.

[6] An extension allows quality to also depend on school effort under the assumption that schools care about the number of students they have enrolled.

[7] Models of private school vouchers share the property that households trade higher quality against higher costs when deciding which school to attend, but must consider how private schools admit students (e.g., whether or not they can turn voucher students away). Models of Tiebout choice share the property that some households must pay a (housing) cost to attend higher-quality schools, but must consider how districts make tax and spending decisions.

[8] McMillan (2005) also considers how public school efforts respond to private school vouchers. In his model, more generous private school vouchers can cause public school effort to fall. The intuition is that a private school voucher can make the outside private school option especially attractive for higher-income students, thereby increasing the effort cost of keeping these students in the public school. This model does not consider peer preferences.

2 The baseline model

The model features a single community with a population of households of size 1. The community is divided into two neighborhoods, A and B, each containing $1/2$ of the population. There are two schools serving the community, one in each neighborhood. The school in neighborhood $J \in \{A, B\}$ is referred to as school J.

Households differ in their socio-economic status. There are a continuum of types indexed by s. Types are uniformly distributed on $[-\mu, \mu]$, where $\mu > 0$, so that the average socio-economic status is 0.[9] The neighborhoods are stratified and A is the more affluent neighborhood. Thus, households of type $[0, \mu]$ live in neighborhood A, while neighborhood B is comprised of types $[-\mu, 0]$. The parameter μ measures the degree of neighborhood inequality.

Each household has a child which it must send to one of the two schools. Households care about school quality (as they perceive it) but incur a utility cost c if using the school not in their neighborhood. This cost captures the additional transaction costs arising from using the non-neighborhood school and varies across households.[10] In our baseline specification, for all household types, costs are uniformly distributed on the interval $[0, \overline{c}]$, so that the fraction of households with cost less than or equal to $c \in [0, \overline{c}]$ is $c/\overline{c}$. Later in the paper, we allow costs to be correlated (negatively) with socio-economic status.

Letting q_J denote the quality of school J, a household living in neighborhood A with cost c obtains a payoff q_A from using school A and a payoff $q_B - c$ from using school B. Similarly, a household living in neighborhood B with cost c obtains a payoff q_B from using school B and a payoff $q_A - c$ from using school A. The quality of school J depends on the effort it exerts and on the average socio-economic status of its children. Thus,

$$q_J = e_J + \alpha s_J, \tag{1}$$

where e_J is school J's effort, s_J is the average socio-economic status of its students, and α is a parameter measuring the importance of peer composition.[11]

Turning to schools, we assume that the community provides schools with a per-student payment that exceeds the costs that an additional student creates. We normalize the per student surplus to one, so that school J's payoff is given by

$$E_J - \gamma \frac{e_J^2}{2}, \tag{2}$$

[9]The average socio-economic status being 0 is just a normalization and involves no loss of generality.

[10]These include any additional time taken to travel to school, additional expenses arising from higher transport costs, psychic costs resulting from loss of community, etc.

[11]Peer composition will be important if parents perceive there to be peer effects in the outcomes that they care about. Some of these outcomes (e.g., aspects of children's behavior) may be unrelated to objectively measurable school quality. This is why it is important to distinguish between school quality as households experience it (which is q_J) and objective measures of school quality (such as test scores) that are used in the empirical literature.

where E_J denotes enrollment in school J and γ is a parameter measuring the marginal cost of effort.

The timing of the interaction between schools and households is as follows. First, the two schools simultaneously commit to their effort levels e_A and e_B. Second, knowing school effort levels, households simultaneously choose in which school to enroll their children. In making this decision, they are assumed to correctly anticipate the enrollment decisions of other households.[12] For now, we ignore capacity constraints, assuming that both schools can accommodate all students who choose to enroll. Capacity constraints will be introduced in a later section.

2.1 School choice

We are interested in how school choice impacts school quality and household welfare. Our benchmark for comparison is a no-choice policy under which households must enroll their children in their neighborhood school. Under this assumption, each school's enrollment consists of the students in its neighborhood and thus is fixed at 1/2. Since enrollment is fixed and effort is costly, schools exert zero effort. Each school's quality is therefore determined by the average socio-economic status of its students (see (1)). Thus, school A's quality is $\alpha\mu/2$ and school B's is $-\alpha\mu/2$. It follows that a household living in neighborhood A obtains a payoff $\alpha\mu/2$ from enrolling their child in school A and a household living in neighborhood B obtains a payoff $-\alpha\mu/2$ from enrolling their child in school B.

To understand what happens under choice, we work backwards, first analyzing the second stage when households simultaneously choose where to enroll their children, knowing school effort levels e_A and e_B. Assume that households anticipate that the quality of school A will be higher than that of school B and let the anticipated quality differential be denoted Δq (i.e., $\Delta q = q_A - q_B$). Then, all households in neighborhood A will use school A and households in neighborhood B will use school A if their costs are less than Δq and school B otherwise. Assuming that Δq is less than or equal to $\bar{c}$, it follows that the average socio-economic status of those enrolling in school A is

$$s_A = \frac{\mu}{2}\left[\frac{1-\frac{\Delta q}{\bar{c}}}{1+\frac{\Delta q}{\bar{c}}}\right], \tag{3}$$

while that of school B's students is

$$s_B = -\frac{\mu}{2}. \tag{4}$$

[12] This formalization of the relationship between school efforts and household enrollment decisions is admittedly highly stylized. In reality, this is a dynamic process in which schools understand that working harder with current students may bring enrollment gains in the future. Capturing this requires a dynamic model in which prospective parents observe today's school efforts and school compositions and these observations guide tomorrow's enrollment decisions. In an earlier version of this paper (Barseghyan et al., 2014), we present a dynamic model that captures this process. The drawback with this model is that it is significantly more complicated and less amenable to extensions. Both dynamic and static models generate similar conclusions regarding the impact of public school choice.

Using (1), this means that, if households correctly anticipate other households' decisions, Δq must satisfy the equation

$$\Delta q = \Delta e + \frac{\alpha\mu}{1 + \frac{\Delta q}{\bar{c}}}, \tag{5}$$

where Δe is the effort differential (i.e., $\Delta e = e_A - e_B$). This is a quadratic equation with solution

$$\Delta q(\Delta e) = \frac{\sqrt{(\bar{c} + \Delta e)^2 + 4\alpha\mu\bar{c}} + \Delta e - \bar{c}}{2}. \tag{6}$$

Equation (6) gives us a closed form solution for the equilibrium quality differential. The solution will lie in the interval $[0, \bar{c}]$ if $\Delta e + \alpha\mu$ is non-negative and if $\Delta e + \alpha\mu/2$ is less than or equal to $\bar{c}$.

Given this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1 + \frac{\Delta q(\Delta e)}{\bar{c}}\right], \tag{7}$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1 - \frac{\Delta q(\Delta e)}{\bar{c}}\right]. \tag{8}$$

Accordingly, if we define an equilibrium to be a pair of effort levels (e_A^*, e_B^*) such that each school J is maximizing its payoff (2) given its rival's effort level, the equilibrium effort levels will be identical and given by

$$e_A^* = e_B^* = \frac{1}{\gamma}\left[\frac{1}{2}\frac{\Delta q'(0)}{\bar{c}}\right]. \tag{9}$$

This condition just reflects the requirement that, for each school, the marginal benefit of an increase in effort must equal the marginal cost. The marginal benefit is the resulting increase in enrollment.

Computing the derivative $\Delta q'(0)$ from (6), we find that the equilibrium effort level is e_S^* (*effort under school choice*) which is defined to equal

$$e_S^* \equiv \frac{1}{\gamma}\left[\frac{1}{4\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{4\bar{c}}\right]. \tag{10}$$

Notice immediately that the parameters measuring strength of peer preferences (α) and the extent of neighborhood inequality (μ) enter into this expression multiplicatively (i.e., as $\alpha\mu$). Moreover, an increase in $\alpha\mu$ reduces equilibrium effort because it reduces the responsiveness of enrollment to effort. Intuitively, when either peer preferences are strong or neighborhood inequality is high, households enrollment decisions are more driven by peer group concerns than school efforts. Given (10), the equilibrium qualities of the two schools under school choice (q_A^*, q_B^*) will be given by

$$q_A^* = e_S^* + \frac{\alpha\mu}{2}\left[\frac{1 - \frac{\Delta q(0)}{\bar{c}}}{1 + \frac{\Delta q(0)}{\bar{c}}}\right], \tag{11}$$

and

$$q_B^* = e_S^* - \frac{\alpha\mu}{2}. \tag{12}$$

This analysis suggests what equilibrium under school choice must look like. However, it stops short of proving that both schools choosing effort level e_S^* is an equilibrium. For this, we have to check that, for each school, e_S^* is a genuine best response to the other school choosing e_S^*. There are two technical issues to worry about. First, for each school, is e_S^* a global maximum in the set of effort levels that give rise to a school quality differential described by (6)? All the analysis so far establishes, is that for both schools e_S^* satisfies the first order necessary condition for maximizing each school's payoff given the other school is choosing e_S^* and the effort differential Δe is such as to generate a school quality differential given by (6). Second, for each school, does e_S^* dominate effort levels that would generate an effort differential giving rise to a negative school quality differential which would not be described by (6)? When the two schools choose effort level e_S^*, the quality differential will be positive because of school A's natural advantage stemming from its location in the more affluent neighborhood. However, if, for example, school B dramatically increased its effort level above e_S^* it could make the quality differential negative. With such an effort choice, the flow of students between schools would be reversed: households from the more affluent neighborhood A would be enrolling their children in school B. Different expressions for the quality differential and the enrollments for the two schools would apply.

In the on-line Appendix, we provide a comprehensive discussion of both issues. We clearly identify the types of deviations that might threaten the existence of equilibrium. We show that for any given values of the parameters α, μ, and γ, the effort levels described in equation (10) are indeed equilibrium effort levels if the upper bound of the cost distribution $\bar{c}$ exceeds a critical level. This critical level is given by

$$\max\left\{\alpha\mu, \sqrt[3]{\frac{\alpha\mu}{\gamma}} - \alpha\mu, \frac{1}{2\gamma\alpha\mu}, \sqrt{\frac{1}{\gamma} + (\alpha\mu)^2}\right\}. \tag{13}$$

We find the assumption that $\bar{c}$ is relatively large natural, since there will likely exist households that would not exercise choice under almost any circumstances. Nonetheless, readers who would prefer not to make such restrictions, should be reassured that in parameter ranges not satisfying this sufficient condition, we did not find any examples in which the effort levels described in equation (10) were not equilibrium effort levels. We will therefore henceforth assume that the school qualities under choice will be (11) and (12).

2.2 The impact of school choice

We are now ready to study the impact of introducing school choice on school quality and household welfare. The policy debate and the empirical literature have focused on the quality impacts of choice, both in the aggregate and across schools and communities. The focus on household welfare is more in the spirit of traditional public economics.[13] Measures of household welfare include the additional costs that households incur when they choose their non-neighborhood school, which we view as a legitimate part of the social calculus.

We begin by defining the precise quantities of interest. Recall that without choice, school A's quality is $\alpha\mu/2$ and school B's is $-\alpha\mu/2$. Using (11) and (12), the changes in the two schools' qualities are

$$dq_A = e_S^* - \alpha\mu \left(\frac{\frac{\Delta q(0)}{\bar{c}}}{1 + \frac{\Delta q(0)}{\bar{c}}} \right), \tag{14}$$

and

$$dq_B = e_S^*. \tag{15}$$

The enrollment-weighted average change in school quality, which we denote by dq, is

$$dq = E_A(0)q_A^* + E_B(0)q_B^* = e_S^*. \tag{16}$$

The expression in (16) is so simple because changes in school qualities resulting from student composition are zero sum and hence wash out of the analysis.

Turning to welfare, three variables are of interest: the average change in welfare of households in the two neighborhoods, which we denote dW_A and dW_B, and the average change in welfare, which we denote by dW. The welfare change from choice for households in neighborhood A is just

$$dW_A = dq_A. \tag{17}$$

This reflects the fact that, since Δq is positive, all households in neighborhood A continue to send their children to school A and hence the only impact on their welfare is how the quality of their school changes.

[13]This said, we do not consider school payoffs in our welfare measure. This is because we see the policy problem to which choice is one possible answer as improving school performance for given levels of educational spending. Eliciting more effort from school personnel is not considered a social loss. In addition, our model of schools' payoffs is too reduced form to permit a satisfactory accounting of the surplus accruing to school personnel and stakeholders.

Households in neighborhood B are more complicated because some switch to school A and some do not. The non-switchers obtain a welfare change of dq_B. Those who do switch obtain a welfare change of $q_A+\alpha\mu/2-c = dq_A + \alpha\mu - c$. Averaging over switchers and non-switchers, we obtain

$$dW_B = \left(1 - \frac{\Delta q(0)}{\overline{c}}\right) dq_B + \int_0^{\Delta q(0)} (dq_A + \alpha\mu - c) \frac{dc}{\overline{c}}. \tag{18}$$

Using (17) and (18), it is straightforward to show that the average welfare gain is

$$dW = dq - \left(E_A(0) - \frac{1}{2}\right) \frac{\Delta q(0)}{2}. \tag{19}$$

This shows that the average welfare gain depends on the difference between two terms. The first is the change in school average quality, which we know from (16) is just the change in school effort. The second represents the additional costs incurred by households in neighborhood B who use school A. For choice to generate positive average welfare gains, the increase in average quality must outweigh the additional costs incurred by switching households.

We are interested in the sign and magnitude of the six variables defined in (14)-(19) and in how they change with the importance of peer preferences and the extent of neighborhood inequality. We will assume that $\overline{c} > \sqrt{6/\gamma}$ and consider the impact of varying $\alpha\mu$ from 0 to $\overline{c}$.[14] Our findings concerning the impacts of choice on school quality are summarized in:[15]

Proposition 1 i) *Choice increases average school quality, but stronger peer preferences and greater neighborhood inequality reduce the extent of the increase.* **ii)** *In the school in the more affluent neighborhood, choice increases quality when peer preferences are weak and neighborhood inequality is low, but reduces it when the product of these variables exceeds a critical level. Moreover, stronger peer preferences and greater neighborhood inequality increase the reduction.* **iii)** *In the school in the less affluent neighborhood, choice increases quality but stronger peer preferences and greater neighborhood inequality reduce the extent of the increase.*

Part i) of the Proposition describes how choice impacts average school quality. The first component is unsurprising: average school quality depends solely on school effort and schools exert zero effort without choice. The second component is more interesting. The intuition is that stronger peer preferences and greater neighborhood inequality reduce the responsiveness of enrollment to school effort and therefore lead to lower efforts.

Parts ii) and iii) of the Proposition describe how choice impacts the distribution of quality across schools.

[14]The assumption that $\overline{c} > \sqrt{6/\gamma}$ plays a role in the findings that choice can reduce quality in school A and reduce average welfare.

[15]The proofs of Propositions 1 and 2 can be found in the Appendix.

The findings reflect the fact that when peer preferences are weak and neighborhood inequality is low, few households in the disadvantaged neighborhood exercise choice in equilibrium. It follows that in both schools, choice increases school effort without changing school peer groups. As such, choice increases quality in both schools. Stronger peer preferences or greater neighborhood inequality mean that more households in the disadvantaged neighborhood exercise choice. This household switching decreases peer quality in school A and leaves it unchanged in school B. Hence for school B, stronger peer preferences and greater neighborhood inequality reduce the quality gain (because they reduce the effort increase) but the quality gain remains positive. For school A, stronger peer preferences and greater neighborhood inequality generate a larger reduction in the quality gain (because they reduce effort and peer quality) and the quality gain is negative when the product of these variables exceeds a critical level.

Turning to welfare, our findings concerning the impacts of choice on household welfare are summarized in:

Proposition 2 i) *Choice increases average welfare when peer preferences are weak and neighborhood inequality is low, but reduces it when the product of these variables exceeds a critical level.* **ii)** *In the more affluent neighborhood, choice increases welfare when peer preferences are weak and neighborhood inequality is low, but reduces it when the product of these variables exceeds a critical level.* **iii)** *In the less affluent neighborhood, choice increases welfare and stronger peer preferences and higher neighborhood inequality first decrease this welfare gain but then, at some point, increase it.*

Again, part i) of the Proposition describes how choice impacts average welfare, and parts ii) and iii) describe the impact on the distribution of welfare across neighborhoods. To gain intuition for part i), recall that the average welfare gain from choice is just the difference between the average quality gain and the costs incurred by the switching households (equation (19)). In the context of Proposition 1, we explained how peer preferences and neighborhood inequality impact the average quality gain. Thus, we just need to understand the switching costs. When peer preferences are weak and neighborhood inequality is low, school A's peer advantage has little attraction. As a result, few households will exercise choice and hence these costs will be low. When peer preferences are stronger and neighborhood inequality greater, school A's peer advantage will drive more households to exercise choice. These switching households obviously benefit from their decision, but their benefit comes at the expense of households in the more affluent neighborhood school and thus there is no aggregate gain. Welfare is reduced by the costs that these households incur and, for high enough peer preferences and neighborhood inequality, these costs overwhelm the benefits of higher average quality.

For part ii), since no households in neighborhood A exercise choice, the change in welfare in neighborhood A equals the change in quality in school A. Thus, the intuition underlying the welfare change in the more affluent neighborhood is just that used to explain the quality change. For part iii) and neighborhood B, the main difference between the welfare and the school quality results, is that stronger peer preferences and higher neighborhood inequality can increase the welfare gain enjoyed by neighborhood B despite decreasing the quality of school B. This is because the benefits enjoyed by those households who switch to school A outweigh the costs (of lower school quality) experienced by those do not.

Figure 1 illustrates the theoretical results established in Propositions 1 and 2. The Figure assumes that $\mu = 1$, $\gamma = 2$, and $\bar{c} = 2.5$, and illustrates how the impact of public school choice varies as α ranges from 0 to 2.[16] In each panel, α is measured on the horizontal axis and the outcome variable of interest on the vertical. The first panel illustrates how enrollment in the two schools changes as α increases and shows that enrollment is higher in the advantaged school with stronger peer effects. The second panel illustrates the change in school effort created by choice and shows that it is positive but decreasing in α. The third panel illustrates the change in school quality created by choice, showing that the average change is positive, but the advantaged school experiences a decrease in quality when α is high. The fourth panel illustrates the change in welfare, showing that citizens in both neighborhoods experience an increase in welfare when α is small, but only citizens in the disadvantaged neighborhood benefit when α is high. Moreover, the change in aggregate welfare turns negative for α high enough.

3 Tiebout choice

A limitation of the baseline model, is that it by assuming that the allocation of households across neighborhoods is exogenous and independent of school quality, it ignores the incentives created by Tiebout choice. If households choose the neighborhood in which they live taking into account the quality of its school and there is some elasticity in housing supply, then enrollment will be sensitive to school efforts even when households must use their neighborhood schools. When school payoffs are increasing in enrollments, this provides schools with an incentive to provide effort even without school choice. Moreover, intuitively, it seems possible that introducing school choice could undermine the incentives created by Tiebout choice and in this way might possibly backfire.

Given this limitation, it is desirable to extend the analysis to incorporate Tiebout choice. We do this in the simplest possible way by adding an initial stage to the model in which households choose in which neighborhood to live. Thus, we continue to assume there are a continuum of household types uniformly distributed on $[-\mu, \mu]$, but now assume that each household must choose in which neighborhood to buy

[16] Obviously, equivalent results are obtained by fixing α equal to 1 and letting μ vary from 0 to 2.

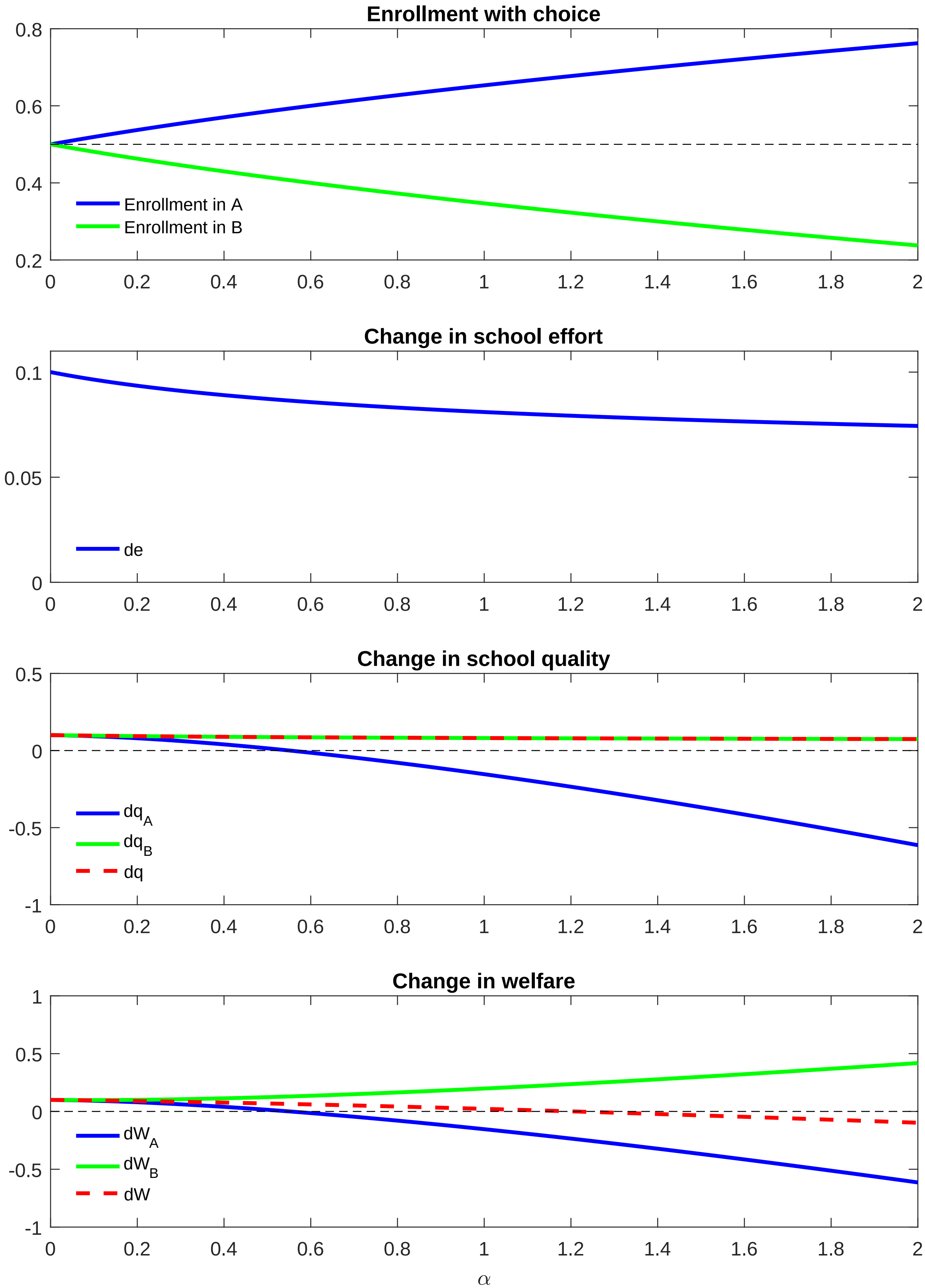

Figure 1: The impact of school choice in the baseline model

a house. Houses are identical within neighborhoods. Neighborhood A is more attractive than B, perhaps because it has better houses or superior amenities.[17] This difference in neighborhoods is captured by assuming that a household of socio-economic status s who buys a house in neighborhood A obtains a housing-related payoff of $b - (\xi - s)P_A$ if he buys a house in neighborhood A and a payoff of $-(\xi - s)P_B$ if he buys in neighborhood B. Here P_J is the price of houses in neighborhood J and b is a positive parameter capturing how much more attractive neighborhood A is than neighborhood B. The parameter ξ is greater than μ, so that $\xi - s$ is positive for all types. This formulation implies that higher socio-economic status types incur a lower disutility from paying for a house which captures the idea that they have more resources and hence a lower value of the dollars left-over after purchasing a house. Higher values of ξ make households more sensitive to house price differences and thus reduce the relative demand for houses in the more expensive neighborhood.

The supply of housing in neighborhood J is assumed to be

$$S_J(P_J) = \beta + \delta_J P_J, \tag{20}$$

where $\beta \in (0, 1/2)$ and $0 < \delta_A < \delta_B$. A higher value of β makes housing supply in both neighborhoods more inelastic at any price, while a higher value of δ_J makes housing supply in neighborhood J more elastic. Assuming supply is more responsive in neighborhood B is necessary to ensure that the two neighborhoods end up equally sized in equilibrium despite the fact that neighborhood A is more desirable.

We continue to assume that, with school choice, if households use the non-neighborhood school, they incur a utility cost c and that this cost is uniformly distributed on the interval $[0, \bar{c}]$. Moreover, we further assume that at the time households choose which neighborhood to live, they are uncertain exactly what the cost of using the non-neighborhood school would be. All they know is the distribution. This assumption means that at the time they choose neighborhoods, households of the same socio-economic status will all behave in the same way, which substantially simplifies the analysis.

Introducing Tiebout choice in this way, means that there is an additional factor to be determined in equilibrium. This is the allocation of households across the two neighborhoods. Under the assumption that the neighborhoods are stratified and that, because it is more attractive, neighborhood A is more affluent, this allocation can be described by the fraction of households living in neighborhood A, which we denote x. The fraction living in neighborhood B is then $1 - x$. Given stratification, neighborhood A will consist of those

[17] This difference in non-school amenities, combined with our assumption that all households share the same preferences, ensures that our model does not feature the multiple equilibria that are commonly found in other models of locational sorting (Bayer and Timmins, 2005).

households with socio-economic status $[\mu(1-2x), \mu]$ and neighborhood B will consist of those households with socio-economic status $[-\mu, \mu(1-2x)]$. Given x, the workings of the rest of the model are exactly as described in the previous section except that the neighborhood populations are described by the intervals $[\mu(1-2x), \mu]$ and $[-\mu, \mu(1-2x)]$ as opposed to $[0, \mu]$ and $[-\mu, 0]$. Accordingly, the key task in solving the extended model is to characterize the equilibrium size of neighborhood A.

3.1 Solving the model with Tiebout choice

3.1.1 Without school choice

We first solve the model without school choice. This is necessary to provide the benchmark with which to compare school choice. It is also relatively simple and provides insight into the solution of the more general case.

A household of type s buying a house in neighborhood A obtains a payoff of

$$q_A + b - (\xi - s)P_A, \tag{21}$$

while if it buys in neighborhood B, it obtains a payoff

$$q_B - (\xi - s)P_B. \tag{22}$$

Comparing these payoffs and letting ΔP denote the housing price differential $P_A - P_B$, we see that the socio-economic status of the indifferent type (i.e., the type who gets the same payoff from buying in either neighborhood) is given by $\xi - (\Delta q + b)/\Delta P$. Assuming that the school quality differential is non-negative, neighborhood A will consist of all those households with socio-economic status higher than the indifferent type. Accordingly, the size of neighborhood A is given by

$$x = \frac{\mu - \left(\xi - \frac{\Delta q + b}{\Delta P}\right)}{2\mu}. \tag{23}$$

We can make further progress by tying down the housing price differential ΔP and the school quality differential Δq. Given (20), if the size of neighborhood A is x, the house prices in the two neighborhoods must be $(x-\beta)/\delta_A$ and $(1-x-\beta)/\delta_B$. The price differential is therefore given by

$$\Delta P = \frac{(\delta_A + \delta_B)\, x - (\beta\,(\delta_B - \delta_A) + \delta_A)}{\delta_A \delta_B}. \tag{24}$$

Turning to the quality differential, given that the indifferent type is $\xi - (\Delta q + b)/\Delta P$, the average socio-economic status of those living in neighborhood A is $\left[\mu + \xi - (\Delta q + b)/\Delta P\right]/2$ and the average socio-economic status of those living in neighborhood B is $\left[-\mu + \xi - (\Delta q + b)/\Delta P\right]/2$. Using (1), it follows that the school quality differential is

$$\Delta q = \Delta e + \alpha\mu. \tag{25}$$

This is particularly simple, since it is independent of the allocation of households across neighborhoods. Substituting (24) and (25) into (23), yields a quadratic equation in the equilibrium size of neighborhood A. The solution of this equation is presented in the on-line Appendix. Here, we just denote it $x(\Delta e)$.

Given this, with effort levels e_A and e_B, the two schools will anticipate enrollments of $x(\Delta e)$ and $1-x(\Delta e)$ respectively. Defining equilibrium as in the previous section, we find that the equilibrium effort levels are identical and satisfy

$$e_A^* = e_B^* = \frac{1}{\gamma}x'(0). \tag{26}$$

Since $x(\Delta e)$ is an increasing function, the equilibrium effort levels are positive, confirming the intuitive idea that Tiebout choice provides schools with incentives to exert effort. Compared with (9), the impact on enrollment is coming from the expansion of the size of the neighborhood rather than the increase in enrollment from the other neighborhood.

In the on-line Appendix, we compute the derivative in question and solve for the equilibrium effort level with Tiebout choice but no school choice. Here, we just denote the solution e_T^* (*effort under Tiebout choice*). The expression in the on-line Appendix reveals that effort is decreasing in the importance of peer effects (α) and the natural advantage of neighborhood A (b). Effort is also decreasing in β, reflecting the idea that more inelastic housing supplies dampens incentives. Finally, effort is decreasing in ξ. Recall that higher values of ξ influence the demand side of the model, by making households more price sensitive. This increases the natural advantage of the cheaper neighborhood, and thus makes neighborhood demand less responsive to school quality differences.

A natural question to ask is how the incentives provided by Tiebout choice compare with those provided by school choice. This is a difficult question to answer, because they are driven by completely different forces. Tiebout choice incentives are limited by the elasticity of housing supply and the substitutability of the neighborhoods (which is measured by b). School choice incentives are limited by the willingness of households to choose a non-neighborhood school (which is measured by $\overline{c}$). It is possible to choose these parameters in such a way as to make the difference between e_S^* and e_T^* either positive or negative. It is notable that both efforts are reduced by stronger peer preferences. In the Tiebout case, this is because stronger peer preferences reduce the substitutability of the two neighborhoods.

To provide a benchmark with which to compare school choice, we will assume that the parameters are such that the equilibrium allocation without school choice is such that the two neighborhoods are equally sized (i.e., $x(0) = 1/2$). In the on-line Appendix, it is shown that this requires that

$$\xi = \frac{2(\alpha\mu + b)\delta_A\delta_B}{(1-2\beta)(\delta_B - \delta_A)}. \tag{27}$$

Under this condition, the qualities of the two schools are given by

$$q_A = e_T^* + \frac{\alpha\mu}{2}, \tag{28}$$

and

$$q_B = e_T^* - \frac{\alpha\mu}{2}. \tag{29}$$

3.1.2 With school choice

If school choice is possible, the payoff from buying a house in neighborhood A remains as described in (21). The payoff from buying a house in neighborhood B is changed because a household will choose to exercise choice if its cost c is less than Δq. Under the assumption that the household does not know its cost at the time of the location decision, its expected payoff from buying a house in neighborhood B is given by

$$\int_0^{\Delta q} (q_A - c)\frac{dc}{\overline{c}} + \int_{\Delta q}^{\overline{c}} q_B \frac{dc}{\overline{c}} - (\xi - s)P_B. \tag{30}$$

Comparing (21) and (30), we see that the socio-economic status of the indifferent type is given by $\xi - ((1 - \frac{\Delta q}{2\overline{c}})\Delta q + b)/\Delta P$. Since neighborhood A will consist of all those households with socio-economic status higher than the indifferent type, its size is given by

$$x = \frac{\mu - \left(\xi - \frac{(1-\frac{\Delta q}{2\overline{c}})\Delta q + b}{\Delta P}\right)}{2\mu}. \tag{31}$$

In the on-line Appendix, we use (24) to substitute in for the price differential and solve for the equilibrium size of neighborhood A, given a school quality differential Δq. Here, we just denote the solution $x(\Delta q)$.

Turning to the determinants of the school quality differential, following the same steps that led to (5), while recognizing that the neighborhood populations are described by the intervals $[\mu(1-2x), \mu]$ and $[-\mu, \mu(1-2x)]$ as opposed to $[0, \mu]$ and $[-\mu, 0]$, we can solve for the quality differential, given x and Δe. We relegate the details to the on-line Appendix and just denote the solution $\Delta q(x,\Delta e)$. Given Δe, the equilibrium size of neighborhood A and school quality differential $x^*(\Delta e)$ and $\Delta q^*(\Delta e)$ are implicitly defined by the system of equations

$$\begin{aligned} x^* &= x(\Delta q^*) \\ \Delta q^* &= \Delta q(x^*,\Delta e) \end{aligned} \tag{32}$$

It is straightforward to show that there exists a solution to this system of equations for all Δe in the relevant range (see the on-line Appendix). A sufficient condition for uniqueness is that at any solution $(x^*,\Delta q^*)$ it is the case that the product $x'(\Delta q^*)\cdot \partial\Delta q(x^*,\Delta e)/\partial x$ is less than 1. While it is difficult to find simple conditions on the primitives to guarantee this condition holds, it is easily satisfied in our simulations. Thus, we will assume it is true in what follows.

Given all this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = x^*(\Delta e) + \frac{\Delta q^*(\Delta e)}{\overline{c}}\left(1 - x^*(\Delta e)\right), \tag{33}$$

and

$$E_B(\Delta e) = \left(1 - \frac{\Delta q^*(\Delta e)}{\overline{c}}\right)\left(1 - x^*(\Delta e)\right). \tag{34}$$

Defining equilibrium in the usual way, we see that the equilibrium effort levels are identical and satisfy

$$e_A^* = e_B^* = \frac{1}{\gamma}\left[\left(1 - \frac{\Delta q^*(0)}{\overline{c}}\right)\frac{dx^*(0)}{d\Delta e} + \left(1 - x^*(0)\right)\frac{\frac{d\Delta q^*(0)}{d\Delta e}}{\overline{c}}\right]. \tag{35}$$

Comparing this with (9) and (26), reveals the dual incentive effects at work with Tiebout and school choice. The first term inside the square brackets reflects the incentive effect due to Tiebout choice and the second that coming from school choice. Relative to (26), the incentive effect arising from changing neighborhood size, is dampened by the possibility of school choice. This is reflected by the multiplicative term $1-\Delta q^*(0)/\overline{c}$. Of course, the impact of a change in the effort differential on neighborhood size may differ from that without school choice, so this does not necessarily imply the incentive effect is weaker. Relative to (9), the incentive effect arising from school choice now depends on the equilibrium size of neighborhood B (the size was equal to 1/2 in the basic model).

Further progress can be made by finding the derivatives of the system (32) (see the on-line Appendix). Calculating these and substituting the resulting expressions into (35), we can solve for the equilibrium effort level. We relegate the details to the on-line Appendix and here just denote the solution by e_{ST}^* (*effort with school and Tiebout choice*). Given that neighborhood A will consist of types $[\mu(1-2x^*(0)),\mu]$ and neighborhood B will consist of types $[-\mu, \mu(1-2x^*(0))]$, the equilibrium school qualities will be given by

$$q_A^* = e_{ST}^* + \alpha\mu \left[\frac{x^*(0)(1-x^*(0))\left(1-\frac{\Delta q^*(0)}{\bar{c}}\right)}{x^*(0)+(1-x^*(0))\frac{\Delta q^*(0)}{\bar{c}}} \right], \tag{36}$$

and

$$q_B^* = e_{ST}^* - \alpha\mu x^*(0). \tag{37}$$

3.2 The impact of school choice with Tiebout choice

We now want to revisit the impact of school choice in the model with Tiebout choice. The benchmark allocation for comparison is that in which, prior to school choice, the two neighborhoods are equally sized. This requires that the parameters satisfy (27) and leads to school quality levels described by (28) and (29). As in Section 2, we are interested in how choice impacts school quality and household welfare. Also of interest in this setting is how choice impacts the size of the neighborhoods and housing prices. Given the complexity of the model with both Tiebout and school choice, we explore the impact of choice numerically.

Figure 2 illustrates the impact of school choice in the model with Tiebout choice. It shows how the impact depends on the importance of peer effects α. We maintain the parameter choices of the baseline model that underlie Figure 1. For the housing market parameters, we assume that $\beta = 0.25$, $\delta_A = 0.1$, $\delta_B = 0.2$, and $b = 1.5$. The housing demand-side parameter ξ adjusts to maintain equality (27) as we change α. This keeps the neighborhoods equal sized in the initial allocation with Tiebout choice alone. Welfare calculations now also take into account the surplus of housing suppliers in each neighborhood. This is done to abstract from welfare gains or losses to households that arise from choice changing housing prices. The on-line Appendix describes the formulas used to compute welfare.

The first four panels in Figure 2 replicate those in Figure 1. Strikingly, they look almost exactly the same.[18] Accordingly, the conclusions of the baseline model appear robust to allowing Tiebout choice. The final two panels describe the change in neighborhood size and the change in house prices created by choice. As expected, choice leads households to substitute to the cheaper neighborhood when peer preferences are strong. They may be able to access school A without paying the higher price of housing in neighborhood A. This raises the price of housing in neighborhood B and reduces that in neighborhood A.

One point which warrants further discussion is the change in school effort created by school choice. With Tiebout choice, school effort will already be positive without school choice. Moreover, intuitively, it seems

[18]This is not a reflection of the particular housing supply parameters we have chosen. Making housing supply more elastic and thus strengthening Tiebout forces, does not appreciably impact the Figure.

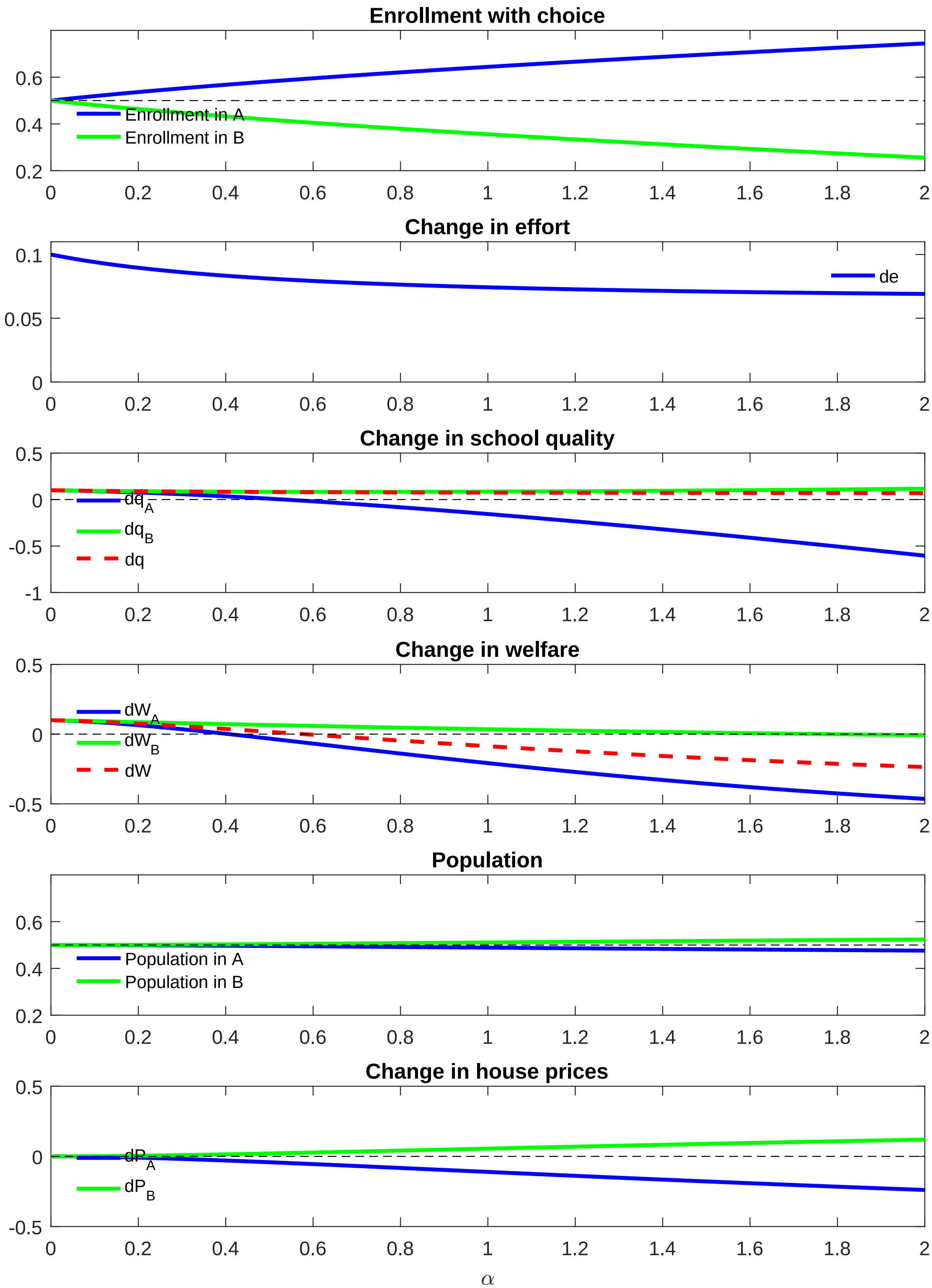

Figure 2: The impact of school choice in the model with Tiebout choice

likely that school choice will dampen the incentives created by Tiebout choice. Thus, it seems that the change in school effort created by school choice should be lower with Tiebout choice than without. While this is hard to see comparing the second panels of Figure 1 and Figure 2, it is in fact the case. The only exception is when there are no peer effects ($\alpha = 0$) when the change in school effort is the same with or without Tiebout choice. Why does school choice dampen the incentives created by Tiebout choice in general, but not in the absence of peer preferences? The answer is that in general, school choice implies that a fraction of students are expected to attend the non-neighborhood school. This weakens the (Tiebout) incentives to attract households to the neighborhood. Absent peer preferences however, school choice does not induce any households to attend the non-neighborhood school (see the top panel of Figure 1). For this reason, absent peer preferences but not generally, school choice does not dampen the incentives created by Tiebout choice.[19]

4 Capacity constraints

A further limitation of the baseline model, is that it assumes that schools enroll all students who wish to attend. This ignores the possibility of capacity constraints. Intuitively, when binding, capacity constraints seem likely to reduce equilibrium efforts under choice since, once at capacity, a school has no incentive to exert further effort. In particular, therefore, if the school in the more affluent neighborhood can attract sufficient enrollees to hit its capacity constraint without exerting effort, it would seem to have no incentive to do so. Similarly, if the enrollees in the school in the less affluent neighborhood have no means of escape, then this school also has no incentive to put in effort to keep them.

To analyze capacity constraints, we return to the baseline model of Section 2 and extend it by assuming that each school faces a capacity constraint on enrollment of $\overline{E}$. We assume this maximal capacity exceeds the enrollment that each school would have with no-choice, so that $\overline{E}$ exceeds $1/2$. We further assume that each school is required to enroll all students from its own neighborhood who wish to enroll. If there is excess demand from students from the other neighborhood, then the available slots are allocated randomly among those wishing to enroll.

To understand what happens, consider the second stage when households simultaneously choose in which school to enroll their children knowing school effort levels e_A and e_B. If households anticipate that the quality of school A will be higher than that of school B, all households in neighborhood A will use school A and households in neighborhood B will try to enroll their children in school A if their costs are less than Δq. The fraction of these households who will be successful, is given by $\min\{1, (\overline{E} - 1/2)/\Delta q/2\overline{c}\}$. It follows that the

[19]Formally, the finding is that when $\alpha = 0$, it is the case that $e^*_{ST} - e^*_T = e^*_S$. Using (9), (26), and (35), it is easy to verify that this will be the case if $dx^*(0)/d\Delta e$ equals $x'(0)$ and $d\Delta q^*(0)/d\Delta e$ equals $\Delta q'(0)$. Both equalities hold when $\alpha = 0$.

average socio-economic status of those enrolling in school A is

$$s_A = \frac{\mu}{2}\left[\frac{1-\frac{\Delta q}{\overline{c}}\min\{1,\frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q}{2\overline{c}}}\}}{1+\frac{\Delta q}{\overline{c}}\min\{1,\frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q}{2\overline{c}}}\}}\right], \tag{38}$$

while that in school B is

$$s_B = -\frac{\mu}{2}. \tag{39}$$

Using (1), this means that, if households correctly anticipate the decisions of other households, Δq must satisfy the equation

$$\Delta q = \Delta e + \alpha\left(\frac{\mu}{1+\frac{\Delta q}{\overline{c}}\min\{1,\frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q}{2\overline{c}}}\}}\right). \tag{40}$$

This has solution

$$\Delta q(\Delta e) = \begin{cases} \frac{\sqrt{(\overline{c}+\Delta e)^2+4\alpha\mu\overline{c}}+\Delta e-\overline{c}}{2} \text{ if } \Delta e \leq \left(2\overline{E}-1\right)\overline{c}-\frac{\alpha\mu}{2\overline{E}} \\ \Delta e+\frac{\alpha\mu}{2\overline{E}} \text{ if } \Delta e > \left(2\overline{E}-1\right)\overline{c}-\frac{\alpha\mu}{2\overline{E}} \end{cases}. \tag{41}$$

Equation (41) provides the equilibrium quality differential associated with any pair of effort levels. This will be the same as in the basic model if Δe is less than $\left(2\overline{E}-1\right)\overline{c}-\alpha\mu/2\overline{E}$ in which case the capacity constraint is not binding. If this condition is not satisfied, then the capacity constraint binds and the equilibrium quality differential takes a simpler form.

Given this, with effort levels e_A and e_B, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1+\frac{\Delta q(\Delta e)}{\overline{c}}\max\{1,\frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q(\Delta e)}{2\overline{c}}}\}\right], \tag{42}$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1-\frac{\Delta q(\Delta e)}{\overline{c}}\max\{1,\frac{\overline{E}-\frac{1}{2}}{\frac{\Delta q(\Delta e)}{2\overline{c}}}\}\right]. \tag{43}$$

Attempting to characterize equilibrium effort levels by taking first order conditions is no longer appropriate, because school payoff functions are not differentiable at the point at which the capacity constraint first binds. Nonetheless, it is straightforward to think through the forces shaping the schools' effort decisions. Recall first that, in the equilibrium of the basic model, the two schools put in the same effort levels and hence Δe is equal to zero. Accordingly, if it is the case that $\left(2\overline{E}-1\right)\overline{c}$ is greater than $\alpha\mu/2\overline{E}$, then the capacity constraint is not binding at the equilibrium of the basic model. However, this does not imply that the equilibrium of the basic model remains an equilibrium since the capacity constraint changes the enrollment consequences of deviations. In particular, if school B deviates to zero effort, its enrollment cannot fall below $1-\bar{E}$. It could be that putting in zero effort and obtaining an enrollment of $1-\bar{E}$ students dominates the payoff from matching the effort level (10) chosen by school A. When this is the case, then the equilibrium of the basic model is no longer an equilibrium.

More generally, when $\left(2\overline{E}-1\right)\bar{c}$ is smaller than $\alpha\mu/2\overline{E}$, the equilibrium of the basic model can never be an equilibrium. If it were, then the capacity constraint would be binding and school A could reduce its effort marginally with no detrimental impact on enrollment. The only impact would be to reduce the excess demand. The same logic implies that in any equilibrium in which the capacity constraint is binding, school A must be putting in zero effort.

Is it possible that there exist equilibria in which the capacity constraint binds, school B puts in positive effort, and school A puts in zero effort? The answer is no. If the capacity constraint binds, school B can reduce its effort and have no impact on its enrollment. All that happens is the excess demand for school A will increase. It follows that the only possible equilibrium in which the capacity constraint binds for sure is that both schools put in zero effort. A necessary condition for this to exist is that $\left(2\overline{E}-1\right)\bar{c}$ is smaller than $\alpha\mu/2\overline{E}$. It must also be that, with school A exerting zero effort, school B does not wish to jack up its effort level sufficient to start attracting students who can enroll in school A. This would require that school B exert an effort level considerably in excess of $\alpha\mu/2\overline{E}-\left(2\overline{E}-1\right)\bar{c}$. Clearly, the larger is $\alpha\mu/2\overline{E}-\left(2\overline{E}-1\right)\bar{c}$ the less likely will this deviation be attractive.

When neither the equilibrium of the baseline model or the zero effort equilibrium exist, then equilibrium will be in mixed strategies. Our procedure for solving for equilibrium in this case is outlined in the online Appendix. For the parameterization that underlies Figure 1 and a value of $\overline{E}$ of 0.7, we find that the equilibrium of the basic model survives for values of α in the interval $[0, 1.346]$ and the zero effort equilibrium exists for values α in the interval $[1.454, 2]$. A mixed strategy equilibrium exists for values of α in the interval $(1.346, 1.454)$. In this equilibrium, only school B randomizes. School A chooses a positive effort level, and school B randomizes between zero effort and a positive effort level. As α increases, the probability that school B chooses a positive effort converges to zero and the effort level chosen by school A converges to zero.

Figure 3 illustrates the impact of school choice in the model with capacity constraints. The four panels replicate those in Figure 1, although, given the uncertainty in school B's effort level when it is using a mixed strategy, we compute expected values of the outcome variables.[20] Again, the two figures look qualitatively quite similar and the lessons of the baseline model are basically robust to introducing capacity constraints. The only difference is that there is in fact no increase in average school quality or the quality of the school in the less affluent neighborhood, when peer preferences are strong enough that the capacity constraint binds.

[20]Note that the expected effort level of school B is equal to the effort level for school A in the mixed strategy equilibrium. It is straightforward to show that this must be true in this type of mixed strategy equilibrium.

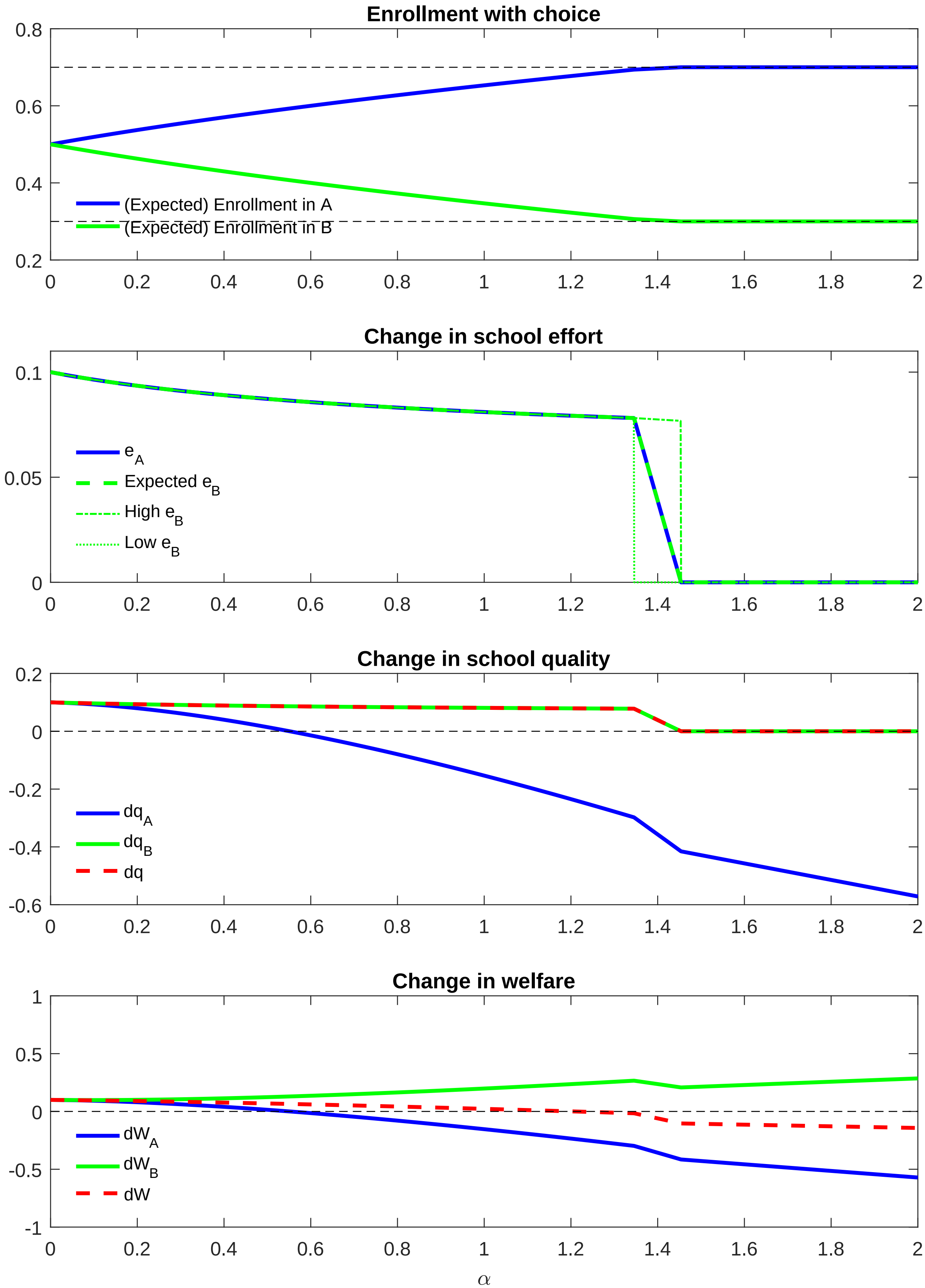

Figure 3: The impact of school choice in the model with capacity constraints

In this sense, it is fair to say that capacity constraints amplify the problems created for school choice by strong peer preferences and unequal neighborhoods.

5 Costs varying by socio-economic status

The baseline model assumes that the utility cost of exercising school choice is ex ante identical across socio-economic status. However, it is often argued that more advantaged households are better able to take advantage of school choice. It is therefore desirable to extend the analysis to capture this. The simplest way of extending the model is to assume that a fraction of households are "immobile" and that immobility is higher among lower socio-economic status households. Intuitively, these immobile households are those who will never choose the non-neighborhood school because they have prohibitively high costs. Introducing cost heterogeneity in this way allows us to hold constant the enrollment response to any given quality differential, but vary the composition of the switching group.

Formally, we assume that in neighborhood B a fraction $\lambda-\theta\,(\mu/2+s)$ of households of type s are immobile. The parameter λ is between 0 and 1 and the parameter θ is non-negative but less than the minimum of $2\lambda/\mu$ and $2(1-\lambda)/\mu$. When θ is equal to 0, the fraction of immobile households is constant across socio-economic status. When θ is positive, the fraction decreases as socio-economic status rises. The higher is θ, the greater is the correlation between low socio-economic status and immobility. Nonetheless, for all θ, the fraction of immobile households in neighborhood B is constant at λ.[21] To preserve space, we relegate the details of how to solve this version of the model to the on-line Appendix. Here, we just explain how this extension impacts the results.

Figure 4 is the counterpart to Figure 1. It assumes a value of λ equal to 0.5 and a value of θ equal to 0.95. These choices imply that a large fraction of households are immobile and that immobility is highly correlated with socio-economic status. Most of the key conclusions of the baseline model are robust to introducing costs that vary with socio-economic status. There are two differences to note. First, the change in average school quality is no longer everywhere decreasing in the strength of peer preferences. This is because, as illustrated by Panel B, the increase in school effort created by choice is actually increasing in α after some point. This reflects the fact that enrollment becomes more responsive to effort at higher levels of α. This in turn is a consequence of the fact that the socio-economic difference between the two schools becomes increasing in α.[22] Second, the quality of the disadvantaged school decreases when peer preferences are

[21] Since we are focusing on the switching of households from neighborhood B to school A under school choice, we simply assume that a constant fraction λ of neighborhood A's households are immobile.

[22] In the baseline model, $\Delta q'(0)$ in equation (9) is decreasing in α. Intuitively, this reflects the fact that as α increases more households are exercising choice and the socio-economic difference between the two schools decreases. In the model of this

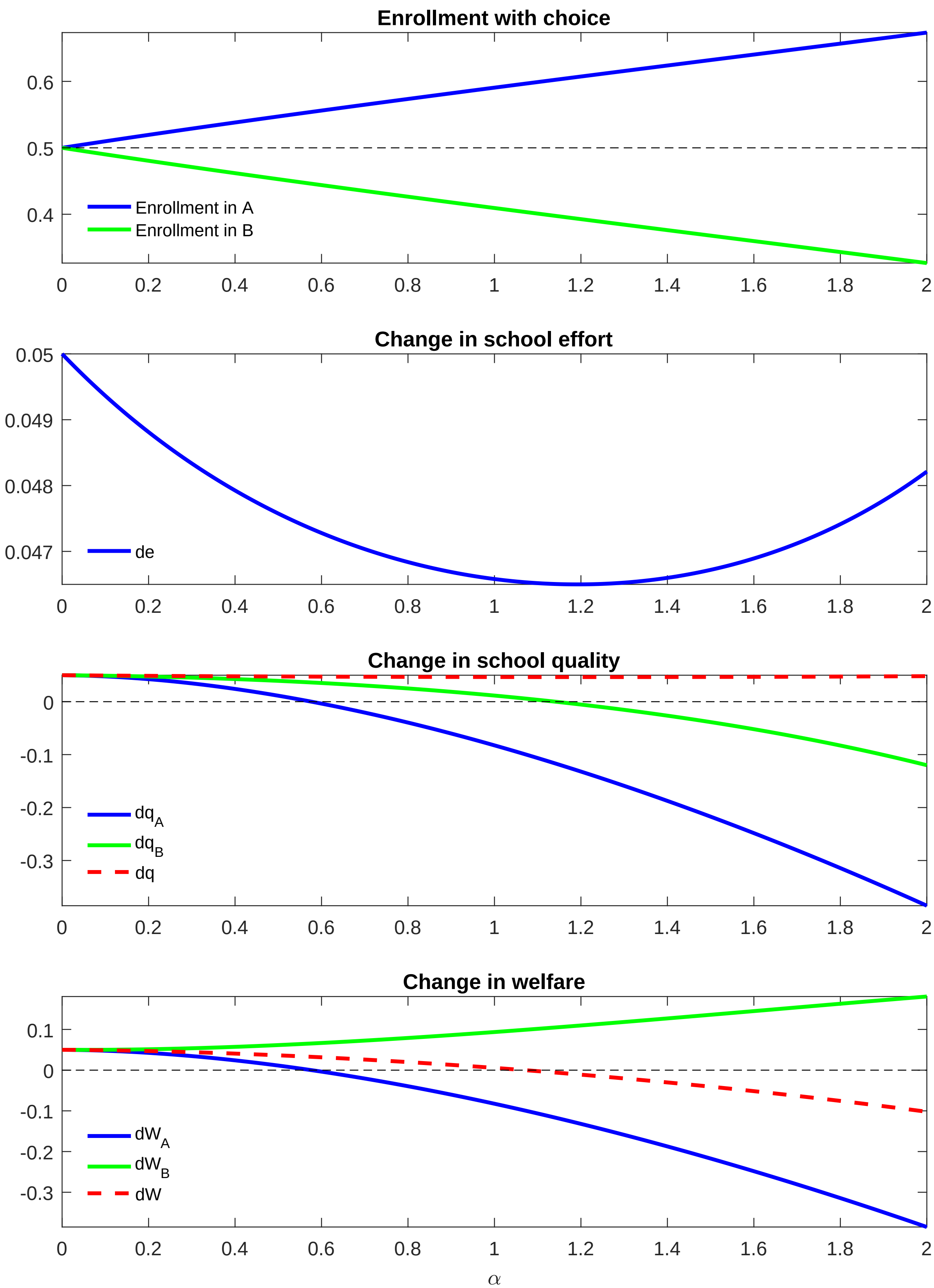

Figure 4: The impact of school choice in the model with costs varying by socio-economic status.

high.[23] This reflects the fact that the disadvantaged school is losing its higher socio-economic status students. This is precisely the concern expressed by critics of choice. Nonetheless, the welfare of households in the disadvantaged neighborhood increases. This increase is driven by the benefits enjoyed by those households exercising choice.

6 Further limitations of the baseline model

The previous three sections have addressed important limitations of our baseline model and have shown how accounting for them changes the impact of choice. This section identifies and discusses some additional limitations.

An obvious limitation of the baseline model is that there are only two neighborhoods. Since this will clearly be counter-factual in most settings, it is important to think through the implications of adding more neighborhoods. Conceptually, there is no problem in introducing more neighborhoods and schools, but the challenge is to do so in a way that keeps the analysis tractable. What complicates things is the possibility that, under choice, households may choose between multiple neighborhood schools. With N neighborhoods, a straightforward extension of our baseline model would introduce, for each household i in neighborhood J, a vector of $N-1$ costs of attending the other neighborhood schools $(c_1^i, ..c_{J-1}^i, c_{J+1}^i, .., c_N^i)$ each a draw from the uniform distribution on $[0, \overline{c}]$. Given a vector of school qualities $(q_1, ..., q_N)$, household i would compare attending the neighborhood school, which would yield payoff q_J, with choosing the best of the other neighborhood schools, which would yield a payoff $\max_{K \neq J}\{q_K - c_K^i\}$. Solving for the equilibrium school effort levels with choice, would require first understanding the mapping between any given vector of school efforts $(e_1, ..., e_N)$ and enrollments $(E_1, ..., E_N)$. This in turn requires solving for the rational expectations equilibrium in which households correctly anticipate the school qualities $(q_1, ..., q_N)$ that are determined by the effort levels $(e_1, ..., e_N)$ and their own choices of where to send their children. In order for the equilibrium effort levels to be characterizable with first order conditions, the mapping between enrollments and effort levels must be smooth. This will not be the case in this set-up. While the model is certainly solvable numerically, an analytical characterization of equilibrium is not going to be possible.

A simple way around this difficulty, which still incorporates the key features of multi-school competition, is to assume that each household i in neighborhood J, simply chooses between school J and one of the other

section, $\Delta q'(0)$ is eventually increasing in α. Intuitively, this reflects the fact that as more households exercise choice, the socio-economic difference between the two schools actually increases. This is because the disadvantaged school is losing its higher socio-economic status students.

[23] Average school quality increases despite the fact that quality in both schools goes down, because more students are at the higher quality school and average quality is enrollment weighted.

$N-1$ schools and that this alternative school varies across households. Specifically, for each neighborhood J and school $K \neq J$, school K is the alternative school for a fraction $1/(N-1)$ of neighborhood J's households. Moreover, for each household, the cost of attending their alternative school is a draw from the uniform distribution on $[0, \bar{c}]$. This means that each school is competing for students with each and every other school, but it simplifies the way in which enrollment behaves as a function of effort levels. In the on-line Appendix we solve a version of this model with three neighborhoods. The neighborhoods are assumed to be stratified by socio-economic status as in the baseline model. We show that the aggregate impacts of choice, along with the impacts on the most and least affluent schools and neighborhoods, are identical to those illustrated in Figure 1. In this sense, the conclusions from our baseline model are robust to introducing more neighborhoods. The additional complication concerns what happens to the middle neighborhood. When peer preferences or neighborhood inequality are strong, it suffers in the same way as does the most affluent neighborhood, although it is not as adversely impacted. It is also the case that school effort levels are no longer symmetric once α is positive. This reflects the fact that the schools are competing over different groups of marginal students.[24] Interestingly, it is the school in the middle neighborhood that puts in the most effort and the school in the most affluent neighborhood that puts in the least.

Other obvious limitations of the baseline model stem from its specific assumptions on household and school preferences. For example, school quality is an additive function of effort and composition and is linear in mean composition; households have homogeneous views on the determinants of school quality; the costs of attending the non-neighborhood school are uniformly distributed; and the school's effort cost function is quadratic. These assumptions are made both to keep the model tractable and because we do not have clear guidance on what else to assume. For example, we could allow for preference heterogeneity, and this could allow choice to improve outcomes by improving school matches. But we choose not to do this, in part because the evidence on preference heterogeneity is not clear cut.[25] Similarly, although more flexible specifications may be superior, it is not obvious how effort and composition interact in households' perceptions of quality and the "linear-in-means" functional form ensures that average school quality is invariant to the distribution of students across schools (i.e., equals the enrollment-weighted average of the effort exerted by the two schools). While we doubt that it is going to change the broad lessons of our analysis, it would obviously

[24] The most affluent school is competing to attract those students from the middle and least affluent neighborhoods for which it is an alternative. The middle school is competing for students from the least affluent neighborhood for which it is an alternative and not to lose those students in its own neighborhood for whom the most affluent school is an alternative. The least affluent school is competing not to lose those students in its own neighborhood for whom the middle and most affluent schools are alternatives.

[25] Under reasonable assumptions (e.g., parents are imperfectly matched under neighborhood choice, parents can observe match values under choice), we would expect match effects to strengthen the case for choice. Nevertheless, provided peer preferences are sufficiently strong, it will be better peers not better matches that drive choice. As such, we would expect our main findings to hold and our main conclusion - that strong preferences weaken the case for choice - to remain.

help if subsequent research yielded more general models of school competition under choice. More flexibility would certainly be necessary if such a model was going to be fitted to data.[26]

More subtle are the limitations arising from features that are left out of the model, but may well play a role in determining the impact of choice. One such feature is parental oversight, a mechanism for incentivizing schools that is likely important in practice (Ferreyra and Liang, 2012). Oversight can be expected to occur in all settings and is a key reason why, in reality, schools exert effort even without the incentives provided by choice. The critical questions from the viewpoint of our analysis are twofold. First, how will oversight be impacted by choice? Second, how will oversight interact with peer preferences under choice?

It seems plausible to suppose that choice would reduce oversight. First, choice provides an exit option for concerned parents. Thus, with choice, parents who are dissatisfied with their child's school's effort may simply choose another school rather than voicing their concerns to school personnel. Second, choice might also increase the physical distance between parents and their schools, thereby raising the costs of attending school meetings to voice concerns. Crucially, we conjecture that oversight reinforces the tendency for choice to have beneficial effects when peer preferences are weak and deleterious effects when they are strong. When peer preferences are weak and/or neighborhood inequality is low, equilibrium school quality will be similar across schools. Voice would then seem preferable to costly exit. When peer preferences and neighborhood inequality is such that school quality is very different in equilibrium, we would expect both schools to receive less oversight - the less affluent school because exit is preferable, the more affluent school because the influx of parents from the less-affluent neighborhood increases the average distance to school.[27]

Another feature left out of the model is the ability of households to exit the public school system, via options such as enrolling in a private school or relocating to another community. As we show, when peer preferences and neighborhood inequality are high, the quality of the school in the affluent neighborhood will decline under choice. This may induce households in the affluent neighborhood to exit the public school system. This may be especially likely in large urban school districts where there are plenty of private schools and households can readily relocate to the suburbs.

Although the possibility of exit is important in practice, we think that peer preferences would still blunt the incentive effects of choice. To see this, suppose we extend the baseline model such that a fraction of the families in the affluent neighborhood exit. Suppose these are the most affluent families and suppose

[26] Developing such an estimable model is an interesting and challenging subject for future research. Such a model would have to simultaneously capture households' neighborhood and school choices, along with schools' effort decisions. Existing work has provided frameworks which capture neighborhood choices when there are peer effects (for example, Bayer and Timmins (2007)) and neighborhood choices when the level of public school spending is endogenous (for example, Epple et al. (2001)). It may be possible to build on these models.

[27] This impact would be reinforced if, as seems true empirically, parents of lower socio-economic status are less likely to provide oversight (Houtenville and Conway, 2008).

that this fraction is decreasing in the quality of the affluent school. This generates a baseline scenario in which the affluent school competes with the exit option. If we compare this to a choice scenario in which the two schools also compete with each other, we expect choice to increase the incentives to exert effort but we expect the increased incentives to be smaller when peer preferences are strong. To see why, consider the two schools separately. The less-affluent school is not affected by the exit option. Hence, following the analysis of Section 2, choice increases the enrollment response to effort but the increase is smaller when peer preferences are strong. The affluent school is now competing on two margins: the usual margin (i.e., competition with the less-affluent school) and the exit margin. When analyzing the enrollment responses to effort, we can consider these two margins separately. Starting with the exit margin enrollment response, we would expect choice to reduce this response with peer preferences but have no impact on this response without peer preferences. That is because choice alters the composition of the affluent school by attracting students from the less-affluent neighborhood. Turning to the enrollment response on the usual margin, it is not clear how this will be affected by peer preferences. This depends whether the compositional impacts of competition on the usual margin (negative) outweigh the composition impacts of competition on the exit margin (positive). On balance, however, it seems likely that peer preference would still blunt the incentive effects of choice.

7 Conclusion

To recap, our baseline model generated three findings regarding the impact of public school choice. First, choice increases average school quality, but the increase is smaller when parents have stronger peer preferences. Average quality increases because the competition for enrollment unleashed by choice spurs schools to increase effort. When peer preferences are stronger, enrollment decisions are more driven by peer group concerns than differences in school efforts and hence the effort effect is smaller. Second, public school choice can increase or decrease aggregate welfare. It increases welfare when households have weak peer preferences and decreases welfare when parents have strong preferences. This reflects that while the threat of choice is socially beneficial (since it elicits socially beneficial effort), the exercise of choice is not (since it costly and since peer quality is a zero-sum construct). Third, when peer preferences are strong, choice harms the more affluent neighborhood by decreasing the quality of its school. These findings largely survive when we extend the model to allow for Tiebout choice, capacity constraints and a negative correlation between socioeconomic status and the costs of exercising choice.

We believe that these findings have important implications for our understanding of the political economy of public school choice programs, for the interpretation of the evidence relating to choice and for the wider policy debate surrounding public school choice. First, our finding that public school choice can reduce welfare in affluent neighborhoods might explain why these policies are relatively uncommon in the US.[28] Indeed, where they have been introduced it is typically in large urban districts and often in response to a desegregation order or the lifting of a desegregation order (Pathak, 2011). These may be districts in which the preferences of more-affluent communities are not decisive. This begs the question of how affluent households might react to the influx of students of lower socio-economic status. One possibility that we discussed in Section 6 is that they might exit the public school system - either by enrolling in private school or relocating to another district. Since we are not aware of any empirical analyses of how public school choice affects exit, it is not clear how important this mechanism is in practice.[29]

Second, our findings could help interpret various estimates of the effects of choice on measures of quality (e.g., academic and other student outcomes). This is because our analysis identifies several factors that shape the effects of choice and that are likely to vary across settings. These include peer preferences, neighborhood inequality and capacity constraints. It would be interesting to assess whether variation in these factors could help to explain why the evidence on choice is mixed.[30]

Third, since we find that peer preferences weaken the case of choice, an obvious question is whether outcomes under choice could be improved by auxiliary policies that affect how it operates. One option is to help disadvantaged students exercise choice (e.g., by subsidizing the costs of low socio-economic status students). The assumption underlying this policy is that, as in the model of Section 5, more advantaged students face a lower cost of using choice. Such a policy will increase the enrollment response to school efforts and thus should increase the equilibrium effort level.[31] However, it is not clear how it will impact welfare when peer preferences are strong because it will induce more costly switching. Also the costs of such

[28] In countries such as the UK and New Zealand, education policy is made at the national level, where the preferences of affluent communities may hold less sway.

[29] At first glance, charter schools may offer a cheaper means of exit. But other aspects of our analysis are relevant here. In particular, since charter schools must ration over-subscribed places by lottery, attempts by more affluent households to resegregate into charter schools would provide incentives for less affluent households to exercise choice and join them. As such, it is difficult to see how a charter school could sustain a more favorable composition of students than a public school in the more affluent neighborhood.

[30] Studying a nationwide choice program in New Zealand, Ladd and Fiske (2003) find no statistically significant correlations between the principal reports of the impact of choice on learning outcomes and principal reports of the level of local competition. Interestingly, Ladd (2002) invokes peer preferences to explain the apparently disappointing effects of public school choice in New Zealand. Lavy (2010) evaluates an Israeli public school choice reform implemented in a single school district in Tel Aviv. He finds that relative to non-reforming control districts, a district that implemented a public school choice program enjoyed significant school productivity gains as measured by dropout rates, test scores and behavioral outcomes. Evidence from the UK is also mixed. Gibbons et al. (2008) exploit across-region variation in choice and find that choice is associated with few productivity gains. Bradley and Taylor (2002) find stronger productivity gains using a difference-in-difference approach.

[31] There are two important caveats here. First, this assumes that the enrollment response does not trigger capacity constraints. If this happens, the policy could lower school effort. Second, if the increased enrollment response of low socio-economic status students comes at the expense of a reduced enrollment response from high socio-economic status students (say, because of a redistribution of expenditures aimed at publicizing the choice option), school effort will be lower. This can be shown formally by reducing the θ parameter in the model of Section 5.

a policy would have to be included in a proper welfare analysis.[32] Another option is to provide schools with incentives to enroll lower socio-economic status students. Many school financing formulas already have this feature, under the assumption that it costs more to educate certain types of students (e.g., those with special educational needs). Here the goal would be to increase the incentive to schools to enroll those students who are on the margin of exercising choice. Intuitively, a well-designed policy of this form seems likely to raise school effort and thus may be helpful.[33]

Finally, we stress that our analysis pertains to only one type of "school choice" policy: public school choice. We studied this because it is the most widely-used choice policy. However, many would argue that it does not go far enough. For example, one could argue that school choice policies would be more effective if schools could choose which curricula to follow, if they had more flexibility over staffing decisions, or if they could take other actions to shape the size and composition of the student body. While this type of analysis is beyond the scope of this paper, we think our findings demonstrate that any such analysis would have to grapple with the complications created by peer preferences.

[32]Some feel for the impact of such a policy can be obtained by comparing Figures 1 and 4. Figure 1 assumes that the costs of exercising choice are independent of socio-economic status, while Figure 4 assumes that some households are immobile and that immobility is inversely correlated with socio-economic status. Thus, moving from Figure 4 to Figure 1 effectively involves lowering the costs of exercising choice, particularly for lower socio-economic status households. It is clear that school effort is higher in Figure 1. It is unclear what will happen to aggregate welfare, even ignoring the costs of such a policy.

[33]While space constraints prevent it, such a policy could be analyzed with our model. Our baseline model assumes that each school receives a per-student surplus of one. We could change that by, say, assuming that for a student of socio-economic status s a school receives a per-student surplus of $1 - s/\mu$. Thus, lower socio-economic status students yield higher surplus. Such a policy would keep the community's total payments to schools the same and hence be budget neutral. It seems clear that this policy will provide greater incentive for school effort. What is not clear is what is the optimal payment schedule and whether it is possible to reverse the conclusions about choice decreasing welfare when peer preferences are strong.

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Appendix

Proof of Proposition 1

For parts (i) and (iii), we have from (15), (16), and (10), that

$$dq_B = dq = e_S^* = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{\bar{c}}\right] > 0.$$

It follows that

$$\frac{dq_B}{d\alpha\mu} = \frac{dq}{d\alpha\mu} = -\frac{\bar{c}}{2\gamma\left(\bar{c}^2 + 4\bar{c}\alpha\mu\right)^{\frac{3}{2}}} < 0.$$

For part (ii), we have from (14), that

$$dq_A = e_S^* - \alpha\mu\left(\frac{\frac{\Delta q(0)}{\bar{c}}}{1 + \frac{\Delta q(0)}{\bar{c}}}\right).$$

Note from (6) that

$$\frac{\frac{\Delta q(0)}{\bar{c}}}{1 + \frac{\Delta q(0)}{\bar{c}}} = \frac{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} - \bar{c}}{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}}.$$

Thus, using (10), and writing the quality change in school A as a function of $\alpha\mu$, we have that

$$dq_A\left(\alpha\mu\right) = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{\bar{c}}\right] - \alpha\mu\left(\frac{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} - \bar{c}}{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}}\right).$$

Note that

$$dq_A\left(0\right) = \frac{1}{2\gamma\bar{c}} > 0$$

and that

$$dq_A\left(\frac{3}{4}\bar{c}\right) = \frac{3}{8\gamma\bar{c}} - \frac{\bar{c}}{4} < 0,$$

where the last inequality follows from the assumption that $\bar{c} > \sqrt{6/\gamma}$. In addition, it is the case that

$$\frac{dq_A\left(\alpha\mu\right)}{d\alpha\mu} = -\frac{\bar{c}}{2\gamma\left(\bar{c}^2 + 4\bar{c}\alpha\mu\right)^{\frac{3}{2}}} - \left(\frac{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} - \bar{c}}{\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}}\right) - \alpha\mu\left(\frac{4\bar{c}^2\left(\bar{c}^2 + 4\alpha\mu\bar{c}\right)^{-\frac{3}{2}}}{\left(\sqrt{\bar{c}^2 + 4\alpha\mu\bar{c}} + \bar{c}\right)^2}\right) < 0,$$

so that $dq_A\left(\alpha\mu\right)$ is decreasing. It follows that there exists a unique value of $\alpha\mu$ less than $\frac{3}{4}\bar{c}$, say $\widetilde{\alpha\mu}$, such that $dq_A\left(\alpha\mu\right)$ is positive for $\alpha\mu \in [0, \widetilde{\alpha\mu})$ and negative for $\alpha\mu \in (\widetilde{\alpha\mu}, \bar{c}]$. ■

Proof of Proposition 2

For part (i), using (19), (16), and (7), we have that

$$dW = e_S^* - \frac{\Delta q(0)^2}{4\overline{c}}.$$

From (6), we have that

$$\frac{\Delta q(0)^2}{4\overline{c}} = \frac{\overline{c} + 2\alpha\mu - \sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}}}{8}.$$

Thus, using (10), we can write the change in average welfare as a function of $\alpha\mu$ as

$$dW(\alpha\mu) = \frac{1}{4\gamma}\left[\frac{1}{\sqrt{\overline{c}^2 + 4\overline{c}\alpha\mu}} + \frac{1}{\overline{c}}\right] - \left(\frac{\overline{c} + 2\alpha\mu - \sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}}}{8}\right).$$

Note that

$$dW(0) = \frac{1}{2\gamma\overline{c}} > 0,$$

and that

$$dW(\frac{3}{4}\overline{c}) = \frac{3}{8\gamma\overline{c}} - \frac{\overline{c}}{16} < 0$$

where the last inequality follows from the assumption that $\overline{c} > \sqrt{6/\gamma}$. In addition, we have that

$$\frac{dW(\alpha\mu)}{d\alpha\mu} = -\frac{\overline{c}}{2\gamma\left(\overline{c}^2 + 4\overline{c}\alpha\mu\right)^{\frac{3}{2}}} - \frac{1}{4} + \frac{\overline{c}}{4\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}}} < 0,$$

so that $dW(\alpha\mu)$ is decreasing. It follows that there exists a unique value of $\alpha\mu$ less than $\frac{3}{4}\overline{c}$, say $\widetilde{\alpha\mu}$, such that $dW(\alpha\mu)$ is positive for $\alpha\mu \in [0, \widetilde{\alpha\mu})$ and negative for $\alpha\mu \in (\widetilde{\alpha\mu}, \overline{c}]$.

For part (ii), we know from (17) that

$$dW_A = dq_A,$$

so the result follows from Proposition 1.

For part (iii), we know from (18) that

$$dW_B = \left(1 - \frac{\Delta q(0)}{\overline{c}}\right) dq_B + \int_0^{\Delta q(0)} (dq_A + \alpha\mu - c)\,\frac{dc}{\overline{c}}.$$

From (15), we know that dq_B is positive. In addition, for all $c \in [0, \Delta q(0)]$ it is the case that $dq_A + \alpha\mu - c$ is at least as big as dq_B. Thus, dW_B is positive. In addition, from (14) and (15), we have that

$$dq_A = e_S^* - \alpha\mu \left(\frac{\frac{\Delta q(0)}{\overline{c}}}{1 + \frac{\Delta q(0)}{\overline{c}}} \right),$$

and that

$$dq_B = e_S^*.$$

Using these, we can write

$$dW_B = e_S^* + \alpha\mu \left(\frac{\frac{\Delta q(0)}{\overline{c}}}{1 + \frac{\Delta q(0)}{\overline{c}}} \right) - \frac{\Delta q(0)^2}{2\overline{c}}.$$

Using (6) and (10), we can write dW_B as a function of $\alpha\mu$ as follows:

$$dW_B(\alpha\mu) = \frac{1}{4\gamma} \left[\frac{1}{\sqrt{\overline{c}^2 + 4\overline{c}\alpha\mu}} + \frac{1}{\overline{c}} \right] + \alpha\mu \left[\frac{\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}} - \overline{c}}{\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}} + \overline{c}} \right] - \left[\frac{\overline{c} + 2\alpha\mu - \sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}}}{4} \right].$$

Note that

$$dW_B(0) = \frac{1}{2\gamma\overline{c}} > 0,$$

and that

$$dW_B\left(\frac{3}{4}\overline{c}\right) = \frac{3}{8\gamma\overline{c}} + \frac{\overline{c}}{8}.$$

Moreover, we have that

$$dW_B(0) - dW_B\left(\frac{3}{4}\overline{c}\right) = \frac{1}{8\gamma\overline{c}} - \frac{\overline{c}}{8} < 0$$

where the latter inequality follows from the assumption that $\overline{c} > \sqrt{6/\gamma}$. Differentiating, we obtain

$$\frac{dW_B(\alpha\mu)}{d\alpha\mu} = -\frac{\overline{c}}{2\gamma\left(\overline{c}^2 + 4\overline{c}\alpha\mu\right)^{\frac{3}{2}}} + \left[\frac{\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}} - \overline{c}}{\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}} + \overline{c}} \right] + \alpha\mu \left(\frac{4\overline{c}^2\left(\overline{c}^2 + 4\alpha\mu\overline{c}\right)^{-\frac{3}{2}}}{\left(\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}} + \overline{c}\right)^2} \right) - \frac{1}{2} + \left(\frac{\overline{c}}{2\sqrt{\overline{c}^2 + 4\alpha\mu\overline{c}}} \right).$$

This derivative is difficult to sign, since it consists of both positive and negative terms. However, note that

$$\frac{dW_B(0)}{d\alpha\mu} = -\frac{1}{2\gamma\overline{c}^2} < 0.$$

It follows that dW_B is decreasing in $\alpha\mu$ for sufficiently small $\alpha\mu$ but then, since $dW_B(0) < dW_B\left(\frac{3}{4}\overline{c}\right)$, must at some point be increasing in $\alpha\mu$. ■