# Online Appendix

# Peer Preferences, School Competition and the Effects of Public School Choice

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# 1 Existence in the baseline model

We need to examine whether both schools choosing effort level $e_S^*$ as defined in (10) is an equilibrium of the game in which both schools simultaneously choose efforts and then households allocate their children across schools in the manner discussed in the text. The analysis in the text establishes that, when school $B$ chooses $e_S^*$, effort level $e_S^*$ satisfies the first order necessary condition for maximizing school $A$'s payoff

$$E_A(e_A - e_S^*) - \gamma \frac{e_A^2}{2},$$

subject to the constraints that $e_A - e_S^* + \alpha\mu$ is positive and $e_A - e_S^* + \alpha\mu/2$ is less than $\overline{c}$. These constraints guarantee that school $A$'s enrollment is given by

$$E_A(e_A - e_S^*) = \frac{1}{2}\left[1 + \frac{\Delta q(e_A - e_S^*)}{\overline{c}}\right],$$

where $\Delta q(e_A - e_S^*)$ is as defined in (6). They also guarantee that $\Delta q(e_A - e_S^*) \in (0, \overline{c})$. Similarly, the analysis in the text establishes that, when school $A$ chooses $e_S^*$, effort level $e_S^*$ satisfies the first order necessary condition for maximizing school $B$'s payoff

$$E_B(e_S^* - e_B) - \gamma \frac{e_B^2}{2},$$

subject to the constraints that $e_S^* - e_B + \alpha\mu$ is positive and $e_S^* - e_B + \alpha\mu/2$ is less than $\overline{c}$.

To establish that both schools choosing effort level $e_S^*$ is an equilibrium we have to check that, for each school, $e_S^*$ is a genuine best response to the other school choosing $e_S^*$. As noted in the text, there are two distinct sets of issues to worry about. First, for each school, is $e_S^*$ a global maximum in the set of effort levels that give rise to a school quality differential described by (6)? Second, for each school, does $e_S^*$ dominate effort levels that would generate a negative school quality differential which would not be described by (6)?

## 1.1 First set of issues

We begin with the first set of issues. The first step is to explore the properties of the equilibrium quality differential (6). We have that for all $\Delta e \in [-\alpha\mu, \overline{c} - \alpha\mu/2]$ the first and second derivatives of the quality differential are

$$\Delta q'(\Delta e) = \frac{1}{2} + \frac{\overline{c} + \Delta e}{2\sqrt{(\overline{c} + \Delta e)^2 + 4\alpha\mu\overline{c}}} > 0 \tag{A1}$$

and

$$\Delta q''(\Delta e) = \frac{2\alpha\mu\bar{c}}{\left((\bar{c}+\Delta e)^2 + 4\alpha\mu\bar{c}\right)^{\frac{3}{2}}} > 0. \tag{A2}$$

Thus, the equilibrium quality differential is an increasing and convex function of the schools' effort differential.

When school $A$ chooses effort level $e_S^*$, $\Delta e$ will be greater than $-\alpha\mu$ if $e_B \in [0, e_S^* + \alpha\mu]$. Thus, school $B$'s payoff on this interval is

$$\frac{1}{2}[1 - \frac{\Delta q(e_S^* - e_B)}{\bar{c}}] - \gamma\frac{e_B^2}{2}.$$

Given (A2), this payoff is concave. Accordingly, since $e_S^*$ satisfies the first order condition (9), $e_S^*$ is optimal in the set of effort levels $[0, e_S^* + \alpha\mu]$.

Matters are more complicated for school $A$. When school $A$ chooses effort level $e_S^*$, $\Delta e$ will belong to the interval $[-\alpha\mu, \bar{c} - \alpha\mu/2]$ if $e_A \in [\max\{0, e_S^* - \alpha\mu\}, e_S^* + \bar{c} - \alpha\mu/2]$. Thus, school $A$'s payoff on this interval is

$$\frac{1}{2}[1 + \frac{\Delta q(e_A - e_S^*)}{\bar{c}}] - \gamma\frac{e_A^2}{2}.$$

The second derivative of this payoff function is

$$\frac{1}{2}[\frac{\Delta q''(e_A - e_S^*)}{\bar{c}}] - \gamma,$$

which, given (A2), is not obviously negative. School $A$'s second order condition is satisfied at $e_A = e_S^*$ if

$$\frac{\alpha\mu}{\left(\bar{c}^2 + 4\alpha\mu\bar{c}\right)^{\frac{3}{2}}} \leq \gamma$$

Obviously, it is straightforward to impose assumptions under which this is true. For concavity over the whole interval, we need that

$$\Delta q''(e_A - e_S^*) \leq 2\bar{c}\gamma$$

over the relevant range. From (A2), this requires that for all $e_A$

$$\alpha\mu \leq \gamma\left((\bar{c} + e_A - e_S^*)^2 + 4\alpha\mu\bar{c}\right)^{\frac{3}{2}}.$$

Notice that provided that $\bar{c} + e_A - e_S^* > 0$, the right hand side of this inequality is increasing in $e_A$. Since $e_A \geq e_S^* - \alpha\mu$, the right hand side will be increasing in $e_A$ if $\alpha\mu < \bar{c}$. Under this assumption, a sufficient condition for concavity is that

$$\frac{\alpha\mu}{(\overline{c}+\alpha\mu)^3} \leq \gamma.$$

This holds if

$$\overline{c} \geq \sqrt[3]{\frac{\alpha\mu}{\gamma}} - \alpha\mu. \tag{A3}$$

## 1.2 Second set of issues

To address the second set of issues, we need to first describe the schools' payoff functions when $\Delta e + \alpha\mu$ is negative and the quality differential is negative. To this end, assume that households anticipate that the quality of school $A$ will be lower than that of school $B$, so that $\Delta q = q_A - q_B < 0$. Then, all households in neighborhood $B$ will use school $B$ and households in neighborhood $A$ will use school $B$ if their costs are less than $-\Delta q$ and school $A$ otherwise. It follows that

$$s_A = \frac{\mu}{2}$$

and that

$$s_B = -\frac{\mu}{2}\left[\frac{\frac{\Delta q}{\overline{c}}+1}{1-\frac{\Delta q}{\overline{c}}}\right].$$

Using (1), this means that, if households have rational expectations, $\Delta q$ must satisfy the equation

$$\Delta q = \Delta e + \alpha\left(\frac{\mu}{1-\frac{\Delta q}{\overline{c}}}\right).$$

This is a quadratic equation with solution

$$\Delta q_{-}(\Delta e) = \frac{\overline{c}+\Delta e - \sqrt{(\overline{c}-\Delta e)^2 - 4\alpha\mu\overline{c}}}{2}. \tag{A4}$$

The solution will lie in the interval $[-\overline{c}, 0]$ if $\Delta e + \alpha\mu$ is non-positive and if $\Delta e + \alpha\mu/2$ is greater than or equal to $-\overline{c}$. Given this, with effort levels $e_A$ and $e_B$ such that $\Delta e$ lies in the range $[-\overline{c} - \alpha\mu/2, -\alpha\mu]$, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1 + \frac{\Delta q_{-}(\Delta e)}{\overline{c}}\right],$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1 - \frac{\Delta q_{-}(\Delta e)}{\overline{c}}\right].$$

We need to understand whether either school has an incentive to deviate from choosing effort level $e_S^*$ to choosing an effort level such that $\Delta e$ ends up in the range $[-\bar{c} - \alpha\mu/2, -\alpha\mu]$. Key to this are the properties of the equilibrium quality differential (A4). Differentiating, we have that:

$$\Delta q_-'(\Delta e) = \frac{1}{2} + \frac{\bar{c} - \Delta e}{2\sqrt{(\bar{c} - \Delta e)^2 - 4\alpha\mu\bar{c}}} > 0 \tag{A5}$$

and that

$$\Delta q_-''(\Delta e) = \frac{2\alpha\mu\bar{c}}{\left((\bar{c} - \Delta e)^2 - 4\alpha\mu\bar{c}\right)^{\frac{3}{2}}} > 0 \tag{A6}$$

Thus, the equilibrium quality differential continues to be increasing and convex in the range in which it is negative. Note further that

$$\lim_{\Delta e \nearrow -\alpha\mu} \Delta q_-(\Delta e) = \lim_{\Delta e \searrow -\alpha\mu} \Delta q(\Delta e) = 0$$

and that

$$\lim_{\Delta e \nearrow -\alpha\mu} \Delta q_-'(\Delta e) = \frac{1}{2} + \frac{\bar{c} + \alpha\mu}{2\left|\bar{c} - \alpha\mu\right|} > \frac{1}{2} + \frac{\bar{c} - \alpha\mu}{2\left(\bar{c} + \alpha\mu\right)} = \lim_{\Delta e \searrow -\alpha\mu} \Delta q'(\Delta e). \tag{A7}$$

The first equality implies that the equilibrium quality differential is continuous at the point at which it switches from negative to positive. The second inequality implies that it has a kink at the switch point: in particular, its slope jumps down.

Now consider the incentives of school $A$ to deviate to choosing an effort level such that $\Delta e$ ends up in the range $[-\bar{c} - \alpha\mu/2, -\alpha\mu]$. This requires that school $A$ reduce its effort level. The lowest effort level it can choose is zero, so that if $e_S^*$ is less than $\alpha\mu$, then such an effort level does not exist. From (10), note that $e_S^*$ is less than $\alpha\mu$ if

$$\frac{1}{\gamma}\left[\frac{1}{4\sqrt{\bar{c}^2 + 4\bar{c}\alpha\mu}} + \frac{1}{4\bar{c}}\right] < \alpha\mu.$$

A sufficient condition for this is that

$$\bar{c} > \frac{1}{2\gamma\alpha\mu}. \tag{A8}$$

If $e_S^*$ exceeds $\alpha\mu$, we need to consider the behavior of school $A$'s payoff function for effort levels $e_A$ on the interval $[0, e_S^* - \alpha\mu]$. The first and second derivatives of school $A$'s payoff function on this interval are

$$\frac{\Delta q_-'(e_A - e_S^*)}{2\bar{c}} - \gamma e_A$$

and

$$\frac{\Delta q''_{-}\left(e_A - e_S^*\right)}{2\overline{c}} - \gamma.$$

Note that school $A$'s payoff must be increasing at $e_A = 0$ since

$$\frac{\Delta q'_{-}\left(-e_S^*\right)}{2\overline{c}} > 0.$$

Under (A3), it is also increasing at $e_A = e_S^* - \alpha\mu$. This follows from the facts that

$$\frac{\Delta q'_{-}(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* - \alpha\mu\right) > \frac{\Delta q'(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* - \alpha\mu\right) > 0.$$

However, this does not rule out the possibility that there is a maximum in the interval $(0, e_S^* - \alpha\mu)$. If this maximum occurs at $\widetilde{e}_A \in (0, e_S^* - \alpha\mu)$, it would have to be the case that

$$\frac{\Delta q'_{-}\left(\widetilde{e}_A - e_S^*\right)}{2\overline{c}} - \gamma\widetilde{e}_A = 0$$

and that

$$\frac{\Delta q''_{-}\left(\widetilde{e}_A - e_S^*\right)}{2\overline{c}} < \gamma.$$

We could rule out the possibility of such a maximum by trying to find conditions under which for all $e_A \in (0, e_S^* - \alpha\mu)$

$$\frac{\Delta q'_{-}\left(e_A - e_S^*\right)}{2\overline{c}} - \gamma e_A > 0.$$

Alternatively, we could allow for the possibility that such a maximum exists but try to find conditions under which the payoff associated with this maximum is less than the equilibrium payoff. Both strategies are difficult and the simplest resolution to the problem is just to assume (A8) holds, so that this range does not arise. However, it should be noted that simulations suggest that school $A$ has no incentive to drop its effort level even when this range arises.

Now consider the incentives of school $B$ to deviate to choosing an effort level such that $\Delta e$ ends up in the range $[-\overline{c} - \alpha\mu/2, -\alpha\mu]$. This requires that it increase its effort level. The relevant range of effort levels is $[e_S^* + \alpha\mu, e_S^* + \overline{c} + \alpha\mu/2]$. For effort levels in this range, the first and second derivatives of school $B$'s payoff function are

$$\frac{\Delta q'_{-}\left(e_S^* - e_B\right)}{2\overline{c}} - \gamma e_B,$$

and

$$-\frac{\Delta q''_{-}(e_S^* - e_B)}{2\overline{c}} - \gamma.$$

From (A6), we see that the second derivative is negative, implying that school $B$'s payoff function is concave on the interval $[e_S^* + \alpha\mu, e_S^* + \overline{c} + \alpha\mu/2]$. This is convenient, but it does not rule out the possibility that deviating to an effort level in the interval $[e_S^* + \alpha\mu, e_S^* + \overline{c} + \alpha\mu/2]$ is desirable, since from (A7) we know that $\Delta q'_{-}(-\alpha\mu)$ exceeds $\Delta q'(-\alpha\mu)$. If

$$\frac{\Delta q'_{-}(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right) > 0, \tag{A9}$$

then school $B$'s payoff function will be increasing on the first part of the interval $[e_S^* + \alpha\mu, e_S^* + \overline{c} + \alpha\mu/2]$. If (A9) holds, then the optimal effort level in the interval $[e_S^* + \alpha\mu, e_S^* + \overline{c} + \alpha\mu/2]$ occurs at $\widetilde{e}_B$ where

$$\frac{\Delta q'_{-}(e_S^* - \widetilde{e}_B)}{2\overline{c}} - \gamma\widetilde{e}_B = 0.$$

We can rule out the possibility of such a maximum by finding conditions under which (A9) does not hold.

Under the assumption that $\alpha\mu < \overline{c}$, we have from (A5) that

$$\Delta q'_{-}(-\alpha\mu) = \frac{1}{2} + \frac{\overline{c} + \alpha\mu}{2(\overline{c} - \alpha\mu)}.$$

Using (A1), we obtain

$$\Delta q'_{-}(-\alpha\mu) - \Delta q'(-\alpha\mu) = \frac{\overline{c} + \alpha\mu}{2(\overline{c} - \alpha\mu)} - \frac{\overline{c} - \alpha\mu}{2\left(\overline{c} + \alpha\mu\right)} = \frac{4\overline{c}\alpha\mu}{2(\overline{c}^2 - (\alpha\mu)^2)}. \tag{A10}$$

Now note that for all $\overline{c}$ the first order condition (9) implies that

$$\frac{\Delta q'(0)}{2\overline{c}} = \gamma e_S^*.$$

Thus, given that $\Delta q'(\Delta e)$ is convex, we have that for all $\overline{c}$

$$\frac{\Delta q'(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right) < \frac{\Delta q'(0)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right) = -\gamma\alpha\mu.$$

Thus, if

$$\left(\frac{\Delta q'_{-}(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right)\right) - \left(\frac{\Delta q'(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right)\right) < \gamma\alpha\mu,$$

then

$$\left(\frac{\Delta q'_{-}(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right)\right) < \left(\frac{\Delta q'(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right)\right) + \gamma\alpha\mu < 0.$$

Now, from (A10), we have that

$$\left(\frac{\Delta q'_{-}(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right)\right) - \left(\frac{\Delta q'(-\alpha\mu)}{2\overline{c}} - \gamma\left(e_S^* + \alpha\mu\right)\right) = \frac{\alpha\mu}{\left(\overline{c}^2 - (\alpha\mu)^2\right)}$$

Moreover,

$$\frac{\alpha\mu}{\left(\overline{c}^2 - (\alpha\mu)^2\right)} < \gamma\alpha\mu \Leftrightarrow \sqrt{\frac{1}{\gamma} + (\alpha\mu)^2} < \overline{c}.$$

Thus, if

$$\sqrt{\frac{1}{\gamma} + (\alpha\mu)^2} < \overline{c}, \tag{A11}$$

then (A9) does not hold. Even when this condition does not hold, a simulation analysis reveals that school $B$ is not better off deviating to a higher effort level.

### 1.3 A sufficient condition for existence

Summarizing all this and collecting together inequalities (A3), (A8), and (A11), we have the following proposition.

**Proposition** *Suppose that*

$$\overline{c} \geq \max\left\{\alpha\mu, \sqrt[3]{\frac{\alpha\mu}{\gamma}} - \alpha\mu, \frac{1}{2\gamma\alpha\mu}, \sqrt{\frac{1}{\gamma} + (\alpha\mu)^2}\right\}.$$

*Then, both schools choosing effort level* $e_S^*$ *as defined in (10) is an equilibrium of the game in which both schools simultaneously choose efforts and then households allocate their children across schools in the manner discussed in the text.*

# 2 Solutions for model with Tiebout choice

## 2.1 Solution of equation (23)

Equation (23) has solution

$$x(\Delta e) = \frac{\left[\begin{array}{c} \sqrt{\left[\left(\delta_A+\delta_B\right)(\xi-\mu)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)(\Delta e+\alpha\mu+b)\delta_A\delta_B} \\ +2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)-\left(\delta_A+\delta_B\right)(\xi-\mu) \end{array}\right]}{4\mu\left(\delta_B+\delta_A\right)}. \quad \text{(A12)}$$

Equation (A12) provides a closed form solution for the equilibrium size of neighborhood $A$ for any given school effort levels. This equation captures the expected relationships between the demand and supply of housing and equilibrium neighborhood size. To see this, suppose that $\delta_A = \delta_A \equiv \delta$ (i.e., the elasticity of housing supply is the same in both neighborhoods). In this case, equation (A12) can be written $x(\triangle e) = \frac{1}{2} + \frac{\sqrt{\xi^2+4\mu\delta v}-\xi}{4\mu}$, where $v = \triangle e + \alpha\mu + b$ captures the relative value of neighborhood A (i.e., the difference in effort, composition and the non-school amenity). If this value is zero then $x = \frac{1}{2}$ (i.e., the population is split equally into neighborhoods A and B). If this value is positive then the size of neighborhood A is increasing in the elasticity of housing supply $\delta$ and decreasing in households' sensitivity to price differences $\xi$.

## 2.2 Solution for $e_T^*$

Computing the derivative $x'(0)$ from (A12) and substituting into (26), the equilibrium effort level with Tiebout choice but no school choice is

$$e_T^* \equiv \frac{1}{\gamma}\left[\frac{\delta_A\delta_B}{\sqrt{\left[\left(\delta_A+\delta_B\right)(\xi-\mu)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)(\alpha\mu+b)\delta_A\delta_B}}\right]. \quad \text{(A13)}$$

## 2.3 Derivation of condition to make $x(0) = 1/2$

Given (A12), we require that

$$\begin{array}{c} \sqrt{\left[\left(\delta_A+\delta_B\right)(\xi-\mu)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)(\alpha\mu+b)\delta_A\delta_B} \\ +2\mu\left(\beta\left(\delta_B-\delta_A\right)\right)-\left(\delta_A+\delta_B\right)(\xi-\mu)=2\mu\delta_B, \end{array}$$

or, equivalently,

$$\sqrt{\left[\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)\left(\alpha\mu+b\right)\delta_A\delta_B}$$
$$=\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)-2\mu\left(\beta\left(\delta_B-\delta_A\right)-\delta_B\right).$$

Squaring both sides and cancelling, we get

$$\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)4\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)+\left[2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)\left(\alpha\mu+b\right)\delta_A\delta_B$$
$$=-\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)4\mu\left(\beta\left(\delta_B-\delta_A\right)-\delta_B\right)+\left[2\mu\left(\beta\left(\delta_B-\delta_A\right)-\delta_B\right)\right]^2.$$

Further manipulation reveals

$$\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)2\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)+2\mu\left[2\beta\left(\delta_B-\delta_A\right)\delta_A+\delta_A^2\right]+4\left(\delta_B+\delta_A\right)\left(\alpha\mu+b\right)\delta_A\delta_B$$
$$-\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)2\left(\beta\left(\delta_B-\delta_A\right)-\delta_B\right)+2\mu\left[\delta_B^2-2\beta\left(\delta_B-\delta_A\right)\delta_B\right].$$

This implies that

$$\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)2\left(2\beta-1\right)\left(\delta_B-\delta_A\right)+2\mu\left[2\beta\left(\delta_B-\delta_A\right)\left(\delta_A+\delta_B\right)-\left(\delta_B^2-\delta_A^2\right)\right]$$
$$+4\left(\delta_B+\delta_A\right)\left(\alpha\mu+b\right)\delta_A\delta_B=0,$$

which reduces to

$$\left(\xi-\mu\right)\left(2\beta-1\right)\left(\delta_B-\delta_A\right)-\mu\left[\left(1-2\beta\right)\left(\delta_B-\delta_A\right)\right]+2(\alpha\mu+b)\delta_A\delta_B=0.$$

This implies that

$$\xi=\frac{2(\alpha\mu+b)\delta_A\delta_B}{\left(1-2\beta\right)\left(\delta_B-\delta_A\right)},$$

which is (27).

### 2.4 Solutions for $x(\Delta q)$ and $\Delta q(x,\Delta e)$

In light of (24), the solution of equation (31), given a school quality differential $\Delta q$, is

$$x(\Delta q)=\frac{\left[\begin{array}{c}\sqrt{\left[\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)\right]^2+8\mu\left(\delta_B+\delta_A\right)\left(\left(1-\frac{\Delta q}{2c}\right)\Delta q+b\right)\delta_A\delta_B}\\+2\mu\left(\beta\left(\delta_B-\delta_A\right)+\delta_A\right)-\left(\delta_A+\delta_B\right)\left(\xi-\mu\right)\end{array}\right]}{4\mu\left(\delta_B+\delta_A\right)}.\quad\text{(A14)}$$

Following the same steps that led to (5), the quality differential, given $x$ and $\Delta e$, is

$$\Delta q(x, \Delta e) = \frac{\sqrt{(x\bar{c} + \Delta e(1-x))^2 + 4\alpha\mu\bar{c}x(1-x)} - (x\bar{c} - \Delta e(1-x))}{2(1-x)}. \tag{A15}$$

## 2.5 Existence and uniqueness with Tiebout and school choice

Define the function

$$\varphi(x; \Delta e) = x - x(\Delta q(x, \Delta e)),$$

where the functions $x(\Delta q)$ and $\Delta q(x, \Delta e)$ are defined in (A14) and (A15). For existence, we need to show that there exists a solution to the equation $\varphi(x; \Delta e) = 0$. Note that when $x$ is close to zero, then the composition of school $A$ is entirely determined by the switchers. Accordingly, there is no difference between the composition of the two schools and thus $\Delta q(0, \Delta e) = \Delta e$. Since $x(\Delta e) \geq \beta$, it follows that $\varphi(0; \Delta e) < 0$. Now consider what is happening at the other end of the distribution. If $x$ is close to one, then almost everyone is in neighborhood $A$. The type therefore converges to the average type 0. The remaining types in neighborhood $B$ are close to the worst types. Thus, they are of type $-\mu$. It follows that it should be the case that $\Delta q(1, \Delta e) = \Delta e + \alpha\mu$. But with this quality differential $x(\Delta q(1, \Delta e)) \leq 1 - \beta$. Thus, $\varphi(1; \Delta e) > 0$. Existence follows from continuity. A sufficient condition for uniqueness is that at any solution point of the equation $\varphi(x; \Delta e) = 0$, we have that

$$\frac{\partial \varphi(x; \Delta e)}{\partial x} = 1 - x'(\Delta q(x, \Delta e))\frac{\partial \Delta q(x, \Delta e)}{\partial x} > 0.$$

## 2.6 Calculating the derivatives with Tiebout and school choice

Totally differentiating the system (32), we obtain:

$$\begin{bmatrix} 1 & -x'(\Delta q^*) \\ -\frac{\partial \Delta q(x^*, \Delta e)}{\partial x} & 1 \end{bmatrix} \begin{bmatrix} dx^* \\ d\Delta q^* \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\partial \Delta q(x^*, \Delta e)}{\partial \Delta e} \end{bmatrix} \begin{bmatrix} d\Delta e \\ d\Delta e \end{bmatrix}.$$

By Cramer's Rule

$$\frac{dx^*(0)}{d\Delta e} = \frac{x'(\Delta q^*)\frac{\partial \Delta q(x^*,0)}{\partial \Delta e}}{1 - x'(\Delta q^*)\frac{\partial \Delta q(x^*,0)}{\partial x}},$$

and

$$\frac{d\Delta q^*(0)}{d\Delta e} = \frac{\frac{\partial \Delta q(x^*,0)}{\partial \Delta e}}{1 - x'(\Delta q^*)\frac{\partial \Delta q(x^*,0)}{\partial x}}.$$

## 2.7 Solution for $e^*_{ST}$

Substituting the derivatives just obtained into (35), the equilibrium effort level with school and Tiebout choice is

$$e^*_{ST} \equiv \frac{1}{\gamma}\left[\left(1-\frac{\Delta q^*(0)}{\overline{c}}\right)\left(\frac{x'(\Delta q^*(0))\frac{\partial \Delta q(x^*(0),0)}{\partial \Delta e}}{1-x'(\Delta q^*(0))\frac{\partial \Delta q(x^*(0),0)}{\partial x}}\right)+(1-x^*(0))\left(\frac{\frac{\partial \Delta q(x^*(0),0)}{\partial \Delta e}}{\overline{c}\left(1-x'(\Delta q^*(0))\frac{\partial \Delta q(x^*(0),0)}{\partial x}\right)}\right)\right] \tag{A16}$$

The derivatives in this expression can all be calculated from (A14) and (A15), but the resulting expressions are cumbersome so we do not report these here.

# 3 Welfare calculations for model with Tiebout choice

## 3.1 Without school choice

Average consumer welfare, $W_A$ and $W_B$, can be written as:

$$\begin{aligned} W_A &= \frac{1}{x}\frac{1}{2\mu}\int_{\mu-2\mu x}^{\mu}\left[q_A + b - (\xi - s)P_A\right]ds, \\ W_B &= \frac{1}{1-x}\frac{1}{2\mu}\int_{-\mu}^{\mu-2\mu x}\left[q_B - (\xi - s)P_B\right]ds. \end{aligned}$$

Substituting for $q_A$, we can write

$$\begin{aligned} W_A &= \frac{1}{x}\frac{1}{2\mu}\int_{\mu-2\mu x}^{\mu}\left[e^*_T + \alpha\frac{\mu}{2} + b - (\xi - s)P_A\right]ds \\ &= \left[e^*_T + \alpha\frac{\mu}{2} + b - \xi P_A\right] + \frac{1}{x}\frac{P_A}{2\mu}\int_{\mu-2\mu x}^{\mu} sds \\ &= \left[e^*_T + \alpha\frac{\mu}{2} + b - \xi P_A\right] + 4\mu P_A\left(1-x\right). \end{aligned}$$

Similarly, substituting for $q_B$, we have

$$\begin{aligned} W_B &= \frac{1}{1-x}\frac{1}{2\mu}\int_{-\mu}^{\mu-2\mu x}\left[q_B - (\xi - s)P_B\right]ds \\ &= \left[e^*_T - \alpha\frac{\mu}{2} - \xi P_B\right] + \frac{1}{1-x}\frac{P_B}{2\mu}\int_{-\mu}^{\mu-2\mu x} sds \\ &= \left[e^*_T - \alpha\frac{\mu}{2} - \xi P_B\right] - 4\mu P_B x. \end{aligned}$$

In neighborhood $A$, housing supplier surplus can be written as

$$HS_A = \frac{1}{2}\left(P_A Q_A + \beta P_A\right) = \frac{1}{2}\delta_A P_A^2 + \beta P_A.$$

For neighborhood $B$ we have

$$HS_B = \frac{1}{2}\delta_B P_B^2 + \beta P_B.$$

### 3.2 With school choice

The expression for average consumer welfare for neighborhood $A$ is the same as above. For neighborhood $B$ it is different because some families will be using school $A$:

$$W_A = \frac{1}{x}\frac{1}{2\mu}\int_{\mu-2\mu x}^{\mu}\left[q_A + b - (\xi - s)P_A\right]ds = \left[q_A + b - \xi P_A\right] + 4\mu P_A\left(1-x\right).$$

$$W_B = \frac{1}{1-x}\frac{1}{2\mu}\int_{-\mu}^{\mu-2\mu x}\left[\int_0^{\Delta q}\left(q_A - c\right)\frac{dc}{\overline{c}} + \int_{\Delta q}^{\overline{c}} q_B\frac{dc}{\overline{c}} - (\xi - s)P_B\right]ds$$
$$= \left[q_B - \xi P_B\right] - 4\mu P_B x + \frac{\left(\Delta q\right)^2}{2\overline{c}}.$$

Finally, the housing supplier surplus for both neighborhoods stays the same as above.

### 3.3 The welfare impact of school choice with Tiebout choice

To compute the welfare change for residents in each neighborhood we need to take into account that some of them will be moving from neighborhood $A$ to neighborhood $B$ when school choice becomes available. We will denote quantities with school choice with superscript $ST$, and quantities without school choice-with superscript $T$. The average welfare changes below are computed for the residents that reside in a given neighborhood before the school choice becomes available. (These welfare changes do not take into account changes in housing supplier surplus, which are straightforward to compute and are added when reporting the results in the paper.)

We have that $x_T > x_{ST}$. It follows that average welfare change for original residents of neighborhood $A$

can be derived as follows:

$$x^T \Delta W_A = \frac{1}{2\mu}\int_{\mu-2\mu x^{ST}}^{\mu}\left[q_A^{ST}+b-(\xi-s)P_A^{ST}\right]ds +$$

$$\frac{1}{2\mu}\int_{\mu-2\mu x^T}^{\mu-2\mu x^{ST}}\left[\int_0^{\Delta q^{ST}}\left(q_A^{ST}-c\right)\frac{dc}{\overline{c}}+\int_{\Delta q^{ST}}^{\overline{c}} q_B^{ST}\frac{dc}{\overline{c}}-(\xi-s)P_B^{ST}\right]ds-\left\{x^T\left[q_A^T+b-\xi P_A^T\right]+4\mu P_A^T x^T\left(1-x^T\right)\right\}$$

$$= x^{ST}\left[q_A^{ST}+b-\xi P_A^{ST}\right]+\left(x^T-x^{ST}\right)\left[q_B^{ST}+\frac{\left(\Delta q^{ST}\right)^2}{2\overline{c}}\right]+4\mu P_A^{ST}\cdot x^{ST}\left(1-x^{ST}\right)+$$

$$4\mu P_B^{ST}\cdot\left(x^T-x^{ST}\right)\left(1-x^T-x^{ST}\right)-\left\{x^T\left[q_A^T+b-\xi P_A^T\right]+4\mu P_A^T x^T\left(1-x^T\right)\right\}.$$

The expression for welfare changes for original residents of neighborhood $B$ is simpler, as all of them stay put:

$$\begin{aligned}\Delta W_B \quad = \quad & \left\{\left[q_B^{ST}-\xi P_B^{ST}\right]-4\mu P_B^{ST}x^T+\frac{\left(\Delta q^{ST}\right)^2}{2\overline{c}}\right\} \\ & -\left\{\left[q_B^T-\xi P_B^T\right]-4\mu P_B^T x^T+\frac{\left(\Delta q^T\right)^2}{2\overline{c}}\right\}.\end{aligned}$$

# 4 Solving for equilibrium in model with capacity constraints

Our solution procedure has three steps. We first construct functions $E_A(\Delta e), E_B(\Delta e)$, and $\Delta q(\Delta e)$ for all $\Delta e \in R$. Second, we construct the region $[\alpha_L, \alpha_U]$ where a pure strategy equilibrium does not exist. Third, for this region, we conjecture what a mixed strategy equilibrium looks like and provide conditions that are necessary and sufficient for our conjecture to be true.

**Step 1**. Functions $E_A(\Delta e), E_B(\Delta e)$, and $\Delta q(\Delta e)$ for $\Delta e \geq -\alpha\mu$ have already been constructed in the main text. We now need to construct them for $\Delta e < -\alpha\mu$. Recall that in Section 1.2 of this Appendix we constructed these quantities, but without taking into account the capacity constraint in school $B$. The question is when the enrollment in school $B$, as derived in Section 1.2, will exceed $\overline{E}$ :

$$E_B(\Delta e) = \frac{1}{2}\left[1-\frac{\Delta q_-(\Delta e)}{\overline{c}}\right] > \overline{E}.$$

Substituting for $\Delta q_-(\Delta e)$ from (A4), the inequality above becomes

$$\frac{\sqrt{\left(\overline{c}-\Delta e\right)^2-4\alpha\mu\overline{c}}-\Delta e-\overline{c}}{4\overline{c}} > \overline{E}-\frac{1}{2},$$

which, after some straightforward manipulation, can be written as

$$\Delta e < -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}}.$$

When the condition above holds, school $B$ enrollment is $\overline{E}$ and school $A$'s is $(1-\overline{E})$, which implies that $s_A = \frac{\mu}{2}$ and $s_B = \frac{\mu}{2}\frac{\overline{E}-1}{\overline{E}}$. It follows that, with capacity constraints, $\Delta q(\Delta e)$ for $\Delta e < -\alpha\mu$ can be written as

$$\Delta q(\Delta e) = \begin{cases} \frac{\overline{c}+\Delta e-\sqrt{(\overline{c}-\Delta e)^2-4\alpha\mu\overline{c}}}{2} \text{ if } \Delta e \geq -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}} \\ \Delta e + \frac{\alpha\mu}{2\overline{E}} \text{ if } \Delta e < -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}} \end{cases}.$$

Furthermore,

$$E_A(\Delta e) = \begin{cases} \frac{1}{2}\left[1+\frac{\Delta q(\Delta e)}{\overline{c}}\right] \text{ if } \Delta e \geq -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}} \\ 1-\overline{E} \text{ if } \Delta e < -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}} \end{cases},$$

and

$$E_B(\Delta e) = \begin{cases} \frac{1}{2}\left[1-\frac{\Delta q(\Delta e)}{\overline{c}}\right] \text{ if } \Delta e \geq -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}} \\ \overline{E} \text{ if } \Delta e < -\left(2\overline{E}-1\right)\overline{c} - \frac{\alpha\mu}{2\overline{E}} \end{cases}.$$

This concludes Step 1.

**Step 2**. Next step is to figure out the region where a pure strategy equilibrium does not exist: $[\alpha_L, \alpha_U]$. The lower bound of this region, $\alpha_L$, is the value of $\alpha$ at which, if school $A$ exerts equilibrium effort as predicted by the baseline model, then school $B$ is indifferent between exerting equilibrium effort and exerting zero effort. The upper bound, $\alpha_U$, is the lowest value of $\alpha$ for which zero effort by both schools is an equilibrium. We construct it by checking whether, assuming school $A$ puts in zero effort, there is a positive effort level that makes school $B$ better off than if it put in zero effort.

**Step 3**. Next we conjecture what a mixed strategy equilibrium would look like. Define $\pi_A(\pi_B)$ as the probability that school $A(B)$ puts in zero effort. An equilibrium consists of four objects: $(\pi_A, \pi_B, e_A, e_B)$. A candidate $(\pi_A, \pi_B, e_A, e_B)$ is an equilibrium iff the following conditions are true:

$$\begin{aligned} e_A &= \arg\max_e \left\{\pi_B E_A(e) + (1-\pi_B)E_A(e-e_B) - \gamma\frac{e^2}{2}\right\} \\ e_B &= \arg\max_e \left\{\pi_A E_B(-e) + (1-\pi_A)E_B(e_A-e) - \gamma\frac{e^2}{2}\right\} \end{aligned}$$

and

$$\begin{aligned}
\pi_B E_A(e_A) + (1-\pi_B) E_A(e_A - e_B) - \gamma \frac{e_A^2}{2} &= \pi_B E_A(0) + (1-\pi_B) E_A(-e_B) \\
\pi_A E_B(-e_B) + (1-\pi_A) E_B(e_A - e_B) - \gamma \frac{e_B^2}{2} &= \pi_A E_B(0) + (1-\pi_A) E_B(e_A)
\end{aligned}$$

The first two simply state that given the other school strategies $e_A$ and $e_B$ are best responses. The last two state that putting in zero effort is also a best response. These conditions constitute a system of four equations in four unknowns. For each value of $\alpha \in [\alpha_L, \alpha_U]$, we solve this system using numerical methods. The resulting solution is the conjectured mixed strategy equilibrium.

# 5 Solving the model with costs varying by socio-economic status

Suppose the anticipated quality differential $\Delta q$ is non-negative. Then, households in neighborhood $B$ will use school $A$ if they are mobile and their costs are less than $\Delta q$. Accordingly, the fraction of households exercising choice is $(1-\lambda)\Delta q/2\bar{c}$ which is independent of $\theta$. What depends on $\theta$ is the average socio-economic status of these households. Consider some type $s$ residing in neighborhood $B$ (i.e., belonging to the interval $[-\mu, 0]$). The probability that this type will exercise choice is $[1-\lambda+\theta(\mu/2+s)]\,\Delta q/\bar{c}$. The probability density of these types in the set of those exercising choice is therefore $[1-\lambda+\theta(\mu/2+s)]\,/(1-\lambda)$. With some work (see the next section), it can be shown that this implies that the average type in the set of those exercising choice is

$$-\frac{\mu}{2} + \frac{\theta\mu^2}{12\,(1-\lambda)}. \tag{A17}$$

Note that this is increasing in $\theta$. This in turn implies that the average socio-economic status of school $A$'s students is

$$s_A = \frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}} + \frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}\right], \tag{A18}$$

and that of school $B$'s students is

$$s_B = -\frac{\mu}{2}\left[1 + \frac{\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]. \tag{A19}$$

Using (1), this means that, if households correctly anticipate other households' decisions, $\Delta q$ must satisfy the equation

$$\Delta q = \Delta e + \frac{\alpha\mu}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}} + \frac{\alpha\mu\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-\left((1-\lambda)\frac{\Delta q}{\bar{c}}\right)^2}. \tag{A20}$$

This gives rise to a cubic equation. It is possible that this equation has two positive solutions in the relevant range (i.e., satisfying $\Delta q < \overline{c}$). In such a situation, there are two possible equilibrium quality differentials. One involves a low quality differential and few students from the less affluent neighborhood exercising choice. The other involves a high quality differential and more students switching schools. This multiplicity is possible because the high socio-economic students are leaving the school in the less affluent neighborhood and hence lowering its quality when they leave. Because they are of lower socio-economic status than the students in the affluent neighborhood, they also lower the quality of school $A$. However, the reduction in the quality of school $A$ can be smaller than the reduction in the quality of school $B$ and this is what underlies the multiplicity. Nonetheless, this is only a possibility and does not arise in the numerical example analyzed in the paper.

Given all this, with effort levels $e_A$ and $e_B$, the two schools will anticipate enrollments of

$$E_A(\Delta e) = \frac{1}{2}\left[1 + (1-\lambda)\frac{\Delta q(\Delta e)}{\overline{c}}\right], \tag{A21}$$

and

$$E_B(\Delta e) = \frac{1}{2}\left[1 - (1-\lambda)\frac{\Delta q(\Delta e)}{\overline{c}}\right]. \tag{A22}$$

The equilibrium effort levels will be identical and given by

$$e_A^* = e_B^* = \frac{1}{\gamma}\left[\frac{(1-\lambda)}{2}\frac{\Delta q'(0)}{\overline{c}}\right]. \tag{A23}$$

Computing the derivative from (A20), we find that the equilibrium effort level is $e_V^*$ (*effort under school choice with costs varying by socio-economic status*) which is defined to equal

$$e_V^* \equiv \frac{1}{\gamma}\left[\frac{1-\lambda}{2\left(\overline{c} + \frac{\alpha\mu(1-\lambda)}{\left(1+(1-\lambda)\frac{\Delta q(0)}{\overline{c}}\right)^2} - \frac{\alpha\mu\frac{\theta\mu}{6}\left(1+\left((1-\lambda)\frac{\Delta q(0)}{\overline{c}}\right)^2\right)}{\left(1-\left((1-\lambda)\frac{\Delta q(0)}{\overline{c}}\right)^2\right)^2}\right)}\right]. \tag{A24}$$

Given (A24), the equilibrium qualities of the two schools under school choice $(q_A^*, q_B^*)$ will be given by

$$q_A^* = e_V^* + \frac{\alpha\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q(0)}{\overline{c}} + \frac{\Delta q(0)}{\overline{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q(0)}{\overline{c}}}\right], \tag{A25}$$

and

$$q_B^* = e_V^* - \frac{\alpha\mu}{2}\left[1 + \frac{\frac{\Delta q(0)}{\overline{c}}\frac{\theta\mu}{6}}{1-(1-\lambda)\frac{\Delta q(0)}{\overline{c}}}\right]. \tag{A26}$$

# 6 Derivations for the model with costs varying by socio-economic status

## 6.1 Derivation of (A17)

Given the discussion in the text, the average type is

$$\begin{aligned}
& \int_{-\mu}^{0} s\left[\frac{1-\lambda+\theta(\frac{\mu}{2}+s)}{1-\lambda}\right]\frac{ds}{\mu} \\
= & \int_{-\mu}^{0}\left[s\left(\frac{1-\lambda+\theta\frac{\mu}{2}}{\mu\,(1-\lambda)}\right)+s^2\left(\frac{\theta}{\mu\,(1-\lambda)}\right)\right]ds \\
= & \left[\frac{s^2}{2}\left(\frac{1-\lambda+\theta\frac{\mu}{2}}{\mu\,(1-\lambda)}\right)+\frac{s^3}{3}\left(\frac{\theta}{\mu\,(1-\lambda)}\right)\right]_{s=-\mu}^{s=0} \\
= & -\left[\frac{\mu^2}{2}\left(\frac{1-\lambda+\theta\frac{\mu}{2}}{\mu\,(1-\lambda)}\right)-\frac{\mu^3}{3}\left(\frac{\theta}{\mu\,(1-\lambda)}\right)\right] \\
= & -\frac{\mu}{2}+\frac{\theta\mu^2}{12\,(1-\lambda)}.
\end{aligned}$$

## 6.2 Derivation of (A18) and (A19)

For (A18), note that we have that

$$\begin{aligned}
s_A &= \frac{\frac{1}{2}\left[\frac{\mu}{2}\right]+\frac{1-\lambda}{2}\frac{\Delta q}{\bar{c}}\left[-\frac{\mu}{2}+\frac{\theta\mu^2}{(12)(1-\lambda)}\right]}{\frac{1}{2}+\frac{1-\lambda}{2}\frac{\Delta q}{\bar{c}}} = \frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]+(1-\lambda)\frac{\Delta q}{\bar{c}}\left[\frac{\theta\mu^2}{(12)\,(1-\lambda)}\right] \\
&= \frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]+\frac{\frac{\Delta q}{\bar{c}}\left[\frac{\theta\mu^2}{12}\right]}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}} \\
&= \frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}}+\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]
\end{aligned}$$

For (A19), note that we have that

$$(1-\lambda)\frac{\Delta q}{\bar{c}}\left(-\frac{\mu}{2}+\frac{\theta\mu^2}{(12)\,(1-\lambda)}\right)+\left(\lambda+(1-\lambda)\left(1-\frac{\Delta q}{\bar{c}}\right)\right)s_B=-\frac{\mu}{2}$$

Thus,

$$
\begin{aligned}
s_B &= \frac{-\frac{\mu}{2}-(1-\lambda)\frac{\Delta q}{\bar{c}}\left(-\frac{\mu}{2}+\frac{\theta\mu^2}{(12)(1-\lambda)}\right)}{\lambda+(1-\lambda)\left(1-\frac{\Delta q}{\bar{c}}\right)} \\
&= -\frac{\mu}{2}-\frac{\frac{\Delta q}{\bar{c}}\left(\frac{\theta\mu^2}{12}\right)}{\lambda+(1-\lambda)\left(1-\frac{\Delta q}{\bar{c}}\right)} \\
&= -\frac{\mu}{2}\left[1+\frac{\frac{\Delta q}{\bar{c}}\left(\frac{\theta\mu}{6}\right)}{\lambda+(1-\lambda)\left(1-\frac{\Delta q}{\bar{c}}\right)}\right] \\
&= -\frac{\mu}{2}\left[\frac{\lambda+(1-\lambda)\left(1-\frac{\Delta q}{\bar{c}}\right)+\frac{\Delta q}{\bar{c}}\left(\frac{\theta\mu}{6}\right)}{\lambda+(1-\lambda)\left(1-\frac{\Delta q}{\bar{c}}\right)}\right] \\
&= -\frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}}+\frac{\Delta q}{\bar{c}}\left(\frac{\theta\mu}{6}\right)}{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}\right] \\
&= -\frac{\mu}{2}\left[1+\frac{\frac{\Delta q}{\bar{c}}\left(\frac{\theta\mu}{6}\right)}{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]
\end{aligned}
$$

### 6.3 Derivation of (A20)

$$
\begin{aligned}
\Delta q &= \Delta e+\alpha\left(\frac{\mu}{2}\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}}+\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}+1+\frac{\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]\right) \\
&= \Delta e+\alpha\left(\frac{\mu}{2}\left[\frac{2+\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}+\frac{\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-(1-\lambda)\frac{\Delta q}{\bar{c}}}\right]\right) \\
&= \Delta e+\alpha\left(\frac{\mu}{2}\left[\frac{2\left(1-(1-\lambda)\frac{\Delta q}{\bar{c}}\right)+\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}\left(1-(1-\lambda)\frac{\Delta q}{\bar{c}}\right)+\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}\left(1+(1-\lambda)\frac{\Delta q}{\bar{c}}\right)}{\left(1+(1-\lambda)\frac{\Delta q}{\bar{c}}\right)\left(1-(1-\lambda)\frac{\Delta q}{\bar{c}}\right)}\right]\right) \\
&= \Delta e+\alpha\mu\left[\frac{1-(1-\lambda)\frac{\Delta q}{\bar{c}}+\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{\left(1+(1-\lambda)\frac{\Delta q}{\bar{c}}\right)\left(1-(1-\lambda)\frac{\Delta q}{\bar{c}}\right)}\right] \\
&= \Delta e+\frac{\alpha\mu}{1+(1-\lambda)\frac{\Delta q}{\bar{c}}}+\frac{\alpha\mu\frac{\Delta q}{\bar{c}}\frac{\theta\mu}{6}}{1-\left((1-\lambda)\frac{\Delta q}{\bar{c}}\right)^2}.
\end{aligned}
$$

## 7 Multiple neighborhoods

This sub-section illustrates how to extend our benchmark model to allow for multiple neighborhoods. The key simplifying assumption is that, under school choice, it is feasible for each household to attend only one school outside their neighborhood. Moreover, this feasible alternative school varies across households within a neighborhood. These assumptions mean that each household in a neighborhood faces a simple binary choice, which keeps things tractable. Nonetheless, since households within each neighborhood are choosing between different alternatives, schools can obtain enrollees from multiple neighborhoods and are therefore effectively competing with all schools in the community. In the exposition to follow, we focus on the case of three neighborhoods to reduce the algebraic burden. However, it should be clear that the approach will generalize to larger numbers.

## 7.1 The model with three neighborhoods

Assume the same set-up as in the benchmark model, but now suppose that the community is divided into three neighborhoods, $A$, $B$, and $C$, each containing 1/3 of the population. There are three schools serving the community, one in each neighborhood, and the school in neighborhood $J \in \{A, B, C\}$ is referred to as school $J$. Neighborhoods are stratified, so that households of types $[\mu/3, \mu]$ live in neighborhood $A$; types $[-\mu/3, \mu/3]$ in neighborhood $B$; and types $[-\mu, -\mu/3]$ in neighborhood $C$.

Each household can freely send its child to the school in its own neighborhood. In addition, under choice, it can use one of the two schools in the other neighborhoods at a cost. Specifically, one half of the households in neighborhood $A$ can send their child to school $B$ while incurring a cost $c_B$ and the other half to school $C$ at a cost $c_C$. These costs are independently and uniformly distributed on the interval $[0, \overline{c}]$. Similarly, one half of the households in neighborhood $B$ ($C$) can send their child to school $A$ at a cost $c_A$ and the other half can use school $C$ ($B$) at a cost $c_C$ ($c_B$).

Households' payoffs are the same as those in the benchmark model. For example, a household living in neighborhood $A$ with a cost $c_B$ ($c_C$) obtains a payoff $q_A$ from using school $A$, and a payoff $q_B - c_B$ ($q_C - c_C$) from using school $B$ ($C$) where $q_J$ again refers to the quality of school $J$. A school's quality depends on school effort and the average socio-economic status of its children as before. Moreover, schools' payoffs and the timing of the interaction between schools and households are also identical to their counterparts in the benchmark model.

## 7.2 Impact of school choice

We now explain how we can use this extended model to analyze the impact of school choice. Under a no-choice policy in which households must enroll their children in their neighborhood school, schools continue to have no incentive to put in effort, so school qualities just depend on the neighborhood's socio-economic status. Accordingly, school $A$'s quality is $2\alpha\mu/3$, school $B$'s is zero, and school $C$'s is $-2\alpha\mu/3$. Hence, a household living in neighborhood $A$ obtains a payoff $2\alpha\mu/3$, a household living in neighborhood $B$ obtains a payoff 0, and a household living in neighborhood $C$ obtains a payoff $-2\alpha\mu/3$.

To understand what happens under choice, we follow the same strategy as for the benchmark model. Thus, we first analyze the second stage of the interaction where, knowing school effort levels, households simultaneously choose where to enroll their children. Then, we study the first stage where schools choose effort levels taking into account their implications for enrollment.

In the second stage, a household who lives in neighborhood $J$ and has access to school $I$, will use school $I$ if and only if $q_I - q_J$ exceeds $c_I$. Assuming that households anticipate that the quality of school $A$ will be higher than that of school $B$ and that the quality of school $B$ will be higher than that of school $C$, households in neighborhood $A$ will use school $A$, households in $B$ will use schools $A$ and $B$, and households in $C$ will use schools $A$, $B$, and $C$. With a little work, it can be shown that the average socio-economic status of the children enrolling in the three schools are

$$\begin{aligned} s_A &= \tfrac{2\mu}{3}\left[\tfrac{2\bar{c}-q_A+q_C}{2\bar{c}+2q_A-q_B-q_C}\right] \\ s_B &= -\tfrac{2\mu}{3}\tfrac{q_B-q_C}{2\bar{c}+2q_B-q_A-q_C} \\ s_C &= -\tfrac{2\mu}{3} \end{aligned} \quad . \tag{A27}$$

Using the fact that $q_J = e_J + \alpha s_J$, it follows that:

$$\begin{aligned} e_A &= q_A - \alpha\tfrac{2\mu}{3}\tfrac{2\bar{c}-q_A+q_C}{2\bar{c}+2q_A-q_B-q_C} \\ e_B &= q_B + \alpha\tfrac{2\mu}{3}\tfrac{q_B-q_C}{2\bar{c}+2q_B-q_A-q_C} \\ e_C &= q_C + \alpha\tfrac{2\mu}{3} \end{aligned} \tag{A28}$$

System of equations (A28) implicitly defines the school qualities $(q_A, q_B, q_C)$ as a function of the schools' effort levels $(e_A, e_B, e_C)$. This system has the convenient property that the partial derivatives of school quality with respect to school effort (i.e., $\partial q_I/\partial e_J$) can be expressed as functions of $(q_A, q_B, q_C)$ only. As an example, consider the impact on qualities of a change in school $A$'s effort $e_A$. The third equation of system (A28) implies that $\partial q_C/\partial e_A = 0$. Totally differentiating the first two equations then implies that $\partial q_A/\partial e_A$ and $\partial q_B/\partial e_A$ must satisfy the following two equations:

$$\begin{aligned} 1 &= \frac{\partial q_A}{\partial e_A} + \alpha\frac{2\mu}{3}\frac{\frac{\partial q_A}{\partial e_A}}{2\bar{c}+2q_A-q_B-q_C} + \alpha\frac{2\mu}{3}\frac{2\bar{c}-q_A+q_C}{(2\bar{c}+2q_A-q_B-q_C)^2}\left[2\frac{\partial q_A}{\partial e_A} - \frac{\partial q_B}{\partial e_A}\right]. \\ 0 &= \frac{\partial q_B}{\partial e_A} + \alpha\frac{2\mu}{3}\frac{\frac{\partial q_B}{\partial e_A}}{2\bar{c}+2q_B-q_A-q_C} - \alpha\frac{2\mu}{3}\frac{q_B-q_C}{(2\bar{c}+2q_B-q_A-q_C)^2}\left[2\frac{\partial q_B}{\partial e_A} - \frac{\partial q_A}{\partial e_A}\right] \end{aligned}$$

These are two linear equations that define $\partial q_A/\partial e_A$ and $\partial q_B/\partial e_A$ as functions of $(q_A, q_B, q_C)$.

Turning to the first stage, the schools' enrollments are given by

$$\begin{aligned} E_A &= \tfrac{1}{3}\left[1 + \tfrac{1}{2}\tfrac{2q_A - q_B - q_C}{\bar{c}}\right] \\ E_B &= \tfrac{1}{3}\left[1 + \tfrac{1}{2}\tfrac{2q_B - q_A - q_C}{\bar{c}}\right] \\ E_C &= \tfrac{1}{3}\left[1 + \tfrac{1}{2}\tfrac{2q_C - q_A - q_B}{\bar{c}}\right] \end{aligned} \quad . \tag{A29}$$

Using the first order conditions for the schools' maximization problems, it follows that the equilibrium effort levels $(e_A^*, e_B^*, e_C^*)$ must satisfy the following conditions:

$$\begin{aligned} e_A^* &= \tfrac{1}{6\gamma\bar{c}}\left[2\tfrac{\partial q_A}{\partial e_A} - \tfrac{\partial q_B}{\partial e_A}\right] \\ e_B^* &= \tfrac{1}{6\gamma\bar{c}}\left[2\tfrac{\partial q_B}{\partial e_B} - \tfrac{\partial q_A}{\partial e_B}\right] \\ e_C^* &= \tfrac{1}{6\gamma\bar{c}}\left[2 - \tfrac{\partial q_B}{\partial e_C} - \tfrac{\partial q_A}{\partial e_C}\right] \end{aligned} \quad . \tag{A30}$$

Combining (A28) and (A30) we obtain

$$\begin{aligned} \tfrac{1}{6\gamma\bar{c}}\left[2\tfrac{\partial q_A}{\partial e_A} - \tfrac{\partial q_B}{\partial e_A}\right] &= q_A - \alpha\tfrac{2\mu}{3}\tfrac{2\bar{c} - q_A + q_C}{2\bar{c} + 2q_A - q_B - q_C} \\ \tfrac{1}{6\gamma\bar{c}}\left[2\tfrac{\partial q_B}{\partial e_B} - \tfrac{\partial q_A}{\partial e_B}\right] &= q_B + \alpha\tfrac{2\mu}{3}\tfrac{q_B - q_C}{2\bar{c} + 2q_B - q_A - q_C} \\ \tfrac{1}{6\gamma\bar{c}}\left[2 - \tfrac{\partial q_B}{\partial e_C} - \tfrac{\partial q_A}{\partial e_C}\right] &= q_C + \alpha\tfrac{2\mu}{3} \end{aligned} \quad . \tag{A31}$$

As already observed, the partial derivatives $\partial q_I/\partial e_J$ for all $I, J$ can be expressed as functions of $(q_A, q_B, q_C)$ only. Thus, system (A31) provides three equations in the three equilibrium school qualities $(q_A, q_B, q_C)$. Obviously, given our assumptions on the rankings of the schools, we need a solution that satisfies $q_A > q_B > q_C$. Given such a solution, the associated equilibrium effort levels can then be obtained from (A28).

While system (A31) is too complicated to permit analytical results, it can readily be solved numerically for the equilibrium school qualities under choice.[1] We can then compute the impact of school choice on school quality and household welfare just as we did for the benchmark model. The changes in school quality and household welfare in the three neighborhoods are defined in the obvious ways. Figure A1 illustrates our findings. It assumes the same underlying parameter values as does Figure 1. Note that the aggregate impacts of choice, along with the impacts on the most and least affluent schools and neighborhoods, are identical to those illustrated in Figure 1. The additional complication concerns what happens to the middle neighborhood - $B$. Basically, when peer preferences or neighborhood inequality are strong, it suffers in the same way as

[1] Of course, this procedure for finding equilibrium does not account for the possibility of deviations in effort levels that change the quality ranking of the three schools. While we do not expect that such deviations will be desirable, finding sufficient conditions to rule them out as we did for the benchmark model would obviously be much more involved and we do not attempt that here.

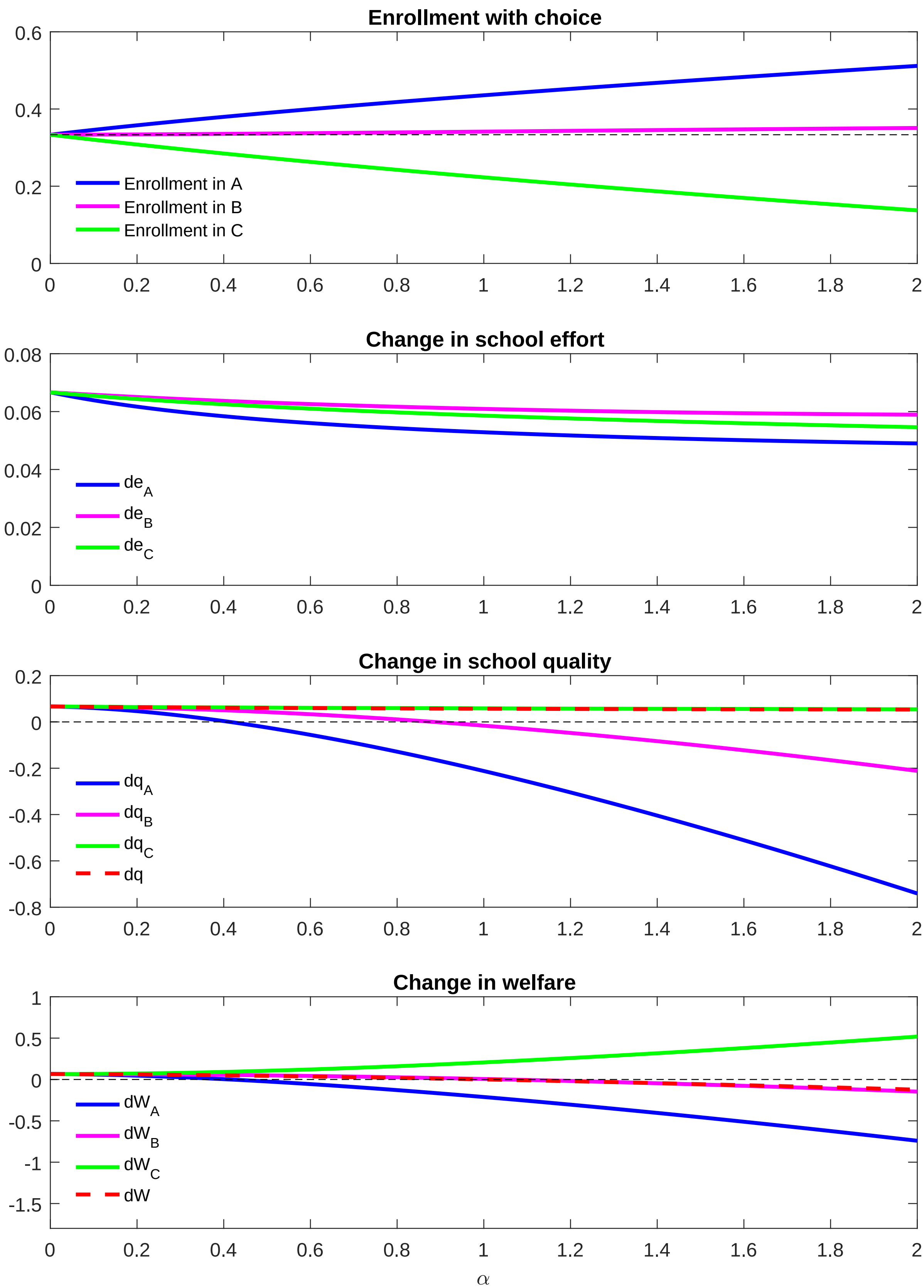


Figure A1: The impact of school choice in the model with three neighborhoods does the most affluent neighborhood, although it is not as adversely impacted. Note also that the school effort levels are no longer symmetric once $\alpha$ is positive. This reflects the fact that the schools are competing over different groups of marginal students.[2] Interestingly, it is the school in the middle neighborhood that puts in the most effort and the school in the most affluent neighborhood that puts in the least.

[2] School $A$ is competing to attract students from schools $B$ and $C$; school $B$ is competing to attract students from school $C$ and retain students who can attend school $A$; and school $C$ is competing to retain students who can attend schools $A$ and $B$.