Appendix to Non-Performing Loans, Prospective Bailouts, and Japan's Slowdown

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1. Aggregation of Households' Consumption and Wealth

It can be shown that the laws of motion of aggregate consumption and non-human wealth V_t are described by the following equations:

$$\begin{aligned} C_t &= [1-\beta(1-p)][R_t V_t + [W_{1,t} + W_{2,t}]] + \tfrac{1}{1+\psi_0}\Psi_n {w_t^n}^{\frac{1+\psi_0}{\psi_0}} + \tfrac{1}{1+\psi_0}\Psi_m {w_t^m}^{\frac{1+\psi_0}{\psi_0}}, \\ V_{t+1} &= R_t V_t + (1-\tau_t)\left[\Psi_n {w_t^n}^{\frac{1+\psi_0}{\psi_0}} + \Psi_m {w_t^m}^{\frac{1+\psi_0}{\psi_0}}\right] - C_t, \\ W_{1,t+1} &= \tfrac{R_{t+1}}{[1-\theta_1][1-p]}\left[W_{1,t} - (\tfrac{\psi_0}{1+\psi_0} - \tau_t)\left[\tfrac{a_1^n p}{\theta_1+p-\theta_1 p}{w_t^n}^{\frac{1+\psi_0}{\psi_0}} + \tfrac{a_1^m p}{\theta_1+p-\theta_1 p}{w_t^m}^{\frac{1+\psi_0}{\psi_0}}\right]\right], \\ W_{2,t+1} &= \tfrac{R_{t+1}}{[1-\theta_2][1-p]}\left[W_{2,t} - (\tfrac{\psi_0}{1+\psi_0} - \tau_t)\left[\tfrac{a_2^n p}{\theta_2+p-\theta_2 p}{w_t^n}^{\frac{1+\psi_0}{\psi_0}} + \tfrac{a_2^m p}{\theta_2+p-\theta_2 p}{w_t^m}^{\frac{1+\psi_0}{\psi_0}}\right]\right], \end{aligned} \tag{1}$$

where $W_{1,t}$ and $W_{2,t}$ denote the following quantities:

$$\begin{aligned} W_{1,t} &= \textstyle\sum_{s=t}^{\infty}(\tfrac{\psi_0}{1+\psi_0} - \tau_s)\left[\tfrac{a_1^n p}{\theta_1+p-\theta_1 p}{w_s^n}^{\frac{1+\psi_0}{\psi_0}} + \tfrac{a_1^m p}{\theta_1+p-\theta_1 p}{w_s^m}^{\frac{1+\psi_0}{\psi_0}}\right][1-\theta_1]^{s-t}R(t,s), \\ W_{2,t} &= \textstyle\sum_{s=t}^{\infty}(\tfrac{\psi_0}{1+\psi_0} - \tau_s)\left[\tfrac{a_2^n p}{\theta_2+p-\theta_2 p}{w_s^n}^{\frac{1+\psi_0}{\psi_0}} + \tfrac{a_2^m p}{\theta_2+p-\theta_2 p}{w_s^m}^{\frac{1+\psi_0}{\psi_0}}\right][1-\theta_2]^{s-t}R(t,s), \end{aligned} \tag{2}$$

and

$$R(t,s) \equiv \begin{cases} \frac{[1-p]^{s-t}}{\prod_{m=t+1}^{s} R_m} & \text{if } s \geqslant t+1, \\ 1 & \text{if } s = t. \end{cases} \tag{3}$$

Finally, Ψ_n and Ψ_m are defined as

$$\begin{aligned} \Psi_n &\equiv p\textstyle\sum_{t=-\infty}^{\bar{s}} \psi_n(\bar{s},t)^{-\frac{1}{\psi_0}}[1-p]^{\bar{s}-t}, \\ \Psi_m &\equiv p\textstyle\sum_{t=-\infty}^{\bar{s}} \psi_m(\bar{s},t)^{-\frac{1}{\psi_0}}[1-p]^{\bar{s}-t}. \end{aligned} \tag{4}$$

Note that in the Blanchard's (1985) original model the sum $W_{1,t} + W_{2,t}$ denotes aggregate human wealth, that is, the present value of all labor income of currently alive households. In this model it represents the same quantity minus the present value of the aggregate disutility from working of the currently alive households.

2. The Alternative Quantitative Model

The alternative model differs from the benchmark model only by the structure of the production side of the economy. which is borrowed from Jaimovich and Floetotto (2008).

The final consumption good in this economy is produced by perfectly competitive firms, owned by the households, according to the following production function:

$$Y_t = \left[\int_0^1 [Q_t(j)]^{\xi} \, dj\right]^{\frac{1}{\xi}}, \tag{5}$$

where ξ is a constant less than one, and $Q_t(j)$ denotes the output of sector j. In each of the j sectors, there are $F_t(j) > 1$ firms producing differentiated goods:

$$Q_t(j) = [F_t(j)]^{1-\frac{1}{\tau}} \left[\sum_{i=1}^{F_t(j)} y_t(j,i)^{\tau}\right]^{\frac{1}{\tau}}, \tag{6}$$

where $y_t(i,j)$ is the output of firm i and τ is a constant less than one. The market structure of each sector exhibits monopolistic competition: each $y_t(j,i)$ is produced by one firm that sets the price of its good in order to maximize profits. It is assumed that $\xi < \tau$.

Let $P_t(j)$ be the price of j^{th} intermediate sector good in terms of the final good. Then, the maximization problem of the final good producer can be written as

$$\max \left[\int_0^1 [Q_t(j)]^{\xi} \, dj\right]^{\frac{1}{\xi}} - \int_0^1 P_t(j) Q_t(j) dj, \tag{7}$$

and the first order optimality condition implies that $Q_t(j) = P_t(j)^{\frac{1}{\xi-1}} Y_t$. Similarly, denoting $p_t(j,i)$ the price of the i^{th} good in the j^{th} sector, the demand function for the i^{th} good can be written as:

$$y_t(j,i) = \left[\frac{p_t(j,i)}{P_t(j)}\right]^{\frac{1}{\tau-1}} \frac{Q_t(j)}{F_t(j)} = \left[\frac{p_t(j,i)}{P_t(j)}\right]^{\frac{1}{\tau-1}} P_t(j)^{\frac{1}{\xi-1}} \frac{Y_t}{F_t(j)}, \tag{8}$$

where the price of the sector j good is defined as $P_t(j) = F_t^{\frac{1}{\tau}-1}(j) \left[\sum_{i=1}^{F_t} p_t(j,i)^{\frac{\tau}{\tau-1}}\right]^{\frac{\tau-1}{\tau}}$.

A firm in the intermediate goods sector lives for two periods and owned by the households. The production function for each good i in sector j is Cobb-Douglas: $k^{\alpha}(i,j) \cdot n^{1-\alpha}(i,j)$.

There is an operating cost ϕ^O denominated in units of the intermediate good. In order to produce the firm must borrow to buy capital. The decision of a firm born in period t is:

$$\begin{aligned} &\pi_{t+1}(j,i) = \max\left\{p_t(j,i)\cdot\left(y_{t+1}-\phi^O\right)-\left(R^f_{t+1}-(1-\delta)\right)k_{t+1}-w_{t+1}n_{t+1},\right\} \\ &\text{s.t. } y_{t+1} = k^{\alpha}_{t+1}n^{1-\alpha}_{t+1}. \end{aligned} \tag{9}$$

Free entry into each sector implies that in equilibrium the firm's profits must be zero:

$$\pi_{t+1}(j,i) = 0. \tag{10}$$

The key feature of this model is that, though there is a continuum of sectors, the number of firms in each sector is finite. This implies that when a firm in sector j sets its price, it takes into account the effect this will have on the price of the good j. The resulting price elasticity of demand faced by each firm in sector j depends on the number of firms in that sector. In a symmetric equilibrium (i.e., in an equilibrium in which all firms in all sectors make identical decisions), the price elasticity of demand is $\frac{1}{\tau-1}+\left[\frac{1}{\xi-1}-\frac{1}{\tau-1}\right]\frac{1}{F_t}$. As F_t goes to infinity the expression above collapses to $\frac{1}{\tau-1}$ and the model simplifies to the standard Dixit-Stiglitz model with a constant elasticity of substitution between goods. The firms optimally equate the price to the marginal revenue:

$$\frac{p_{t+1}(j,i)}{mc_t(j,i)} = \mu(F_t(j)) \equiv \frac{(\xi-1)\,F_t(j)-(\tau-\frac{1}{\lambda})}{\tau\,(\xi-1)\,F_t(j)-(\tau-\frac{1}{\lambda})}. \tag{11}$$

The firms' FOCs with respect to capital and labor can be written, respectively, as

$$\begin{aligned} &\alpha\tfrac{1}{\mu(F_{t+1})}\tfrac{p_{t+1}(j,i)y_{t+1}(j,i)}{k_{t+1}(j,i)} = R^f_{t+1}-(1-\delta), \\ &(1-\alpha)\tfrac{1}{\mu(F_{t+1}(j))}\tfrac{p_{t+1}(j,i)y_{t+1}(j,i)}{n_{t+1}(j,i)} = w_{t+1}. \end{aligned} \tag{12}$$

Free entry implies that the firms' share of the revenues is equal to the operating cost ϕ^O: $(\mu(F_{t+1}(j))-1)\cdot y_t(j,i) = \phi^O$. Suppose the economy is in a symmetric equilibrium. Then, aggregate output in this economy is given by $Y_{t+1} = \frac{1}{\mu(F_{t+1})}K^{\alpha}_{t+1}N^{1-\alpha}_{t+1}$. The rental rate on capital and wage can be written as $\alpha\frac{Y_{t+1}}{K_{t+1}} = R^f_{t+1}-(1-\delta)$ and $(1-\alpha)\frac{Y_{t+1}}{N_{t+1}} = w^n_{t+1}$, respectively. Finally, the number of firms in each sector and aggregate output of the economy are related as follows: $F_{t+1} = \frac{\mu(F_{t+1})-1}{\phi^O} Y_{t+1}$, with $\mu(F_t)$ defined as in (11). The inverse of the mark-up is interpreted as TFP.

The four equations above, combined with the equations from the benchmark model which describe the behavior of the households, the banks, and the government, as well as the resource constraint, fully describe the dynamics of the economy in the alternative model.

As in the benchmark case, the model's parameters can be categorized as "neoclassical", "perpetual youth", "endogenous productivity", and "others." Their complete list is provided Table A1. The "neoclassical" and "perpetual youth" parameters in this model coincide with those in the benchmark model with the following exceptions. The parameter α is set to 0.362. The depreciation rate δ is set to 0.089.

The parameters τ and ω are set, respectively, to 0.949 and 0.001, as in Jaimovich and Floetotto (2008). The value of the fixed cost ϕ^O, is conservatively set to yield the steady state value of the mark-up of 35%. This is the average between the steady state value of the mark-up in Jaimovich and Floetotto (2008) and the average mark-up value in Japan reported by Martins et al. (1996) (30% and 40%, respectively).

Table A1. Calibration of the alternative model.

Parameter	α	δ	ψ_0	Ψ_n	a_1^n	a_2^n	τ_k
	0.36	0.089	0.62	2.39	-927	927	0.48
Parameter	p	θ_1	θ_2	ψ_0	ξ	ω	
	0.024	0.0803	0.08	0.62	0.949	0.001	
Target	R	$\frac{\phi^O}{Y}$	$\frac{g}{Y}$				
	1.05	0.296	0.15				

Notes: The first two rows describe the parameters used in the alternative quantitative experiments described in Section 4. The remaining parameters are chosen to match the steady state quantities reported in the last row.