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Bureaucrats, Voters, and Public Investment*

Abstract

This paper explores the provision of a durable public good in Romer and Rosenthal's agenda setter model. It identifies a type of equilibrium, called a Romer-Rosenthal equilibrium, in which in every period the bureaucrat proposes the maximum level of public investment the voter will support. The paper establishes that such an equilibrium exists for a variety of public good benefit functions. Equilibrium public good levels converge or almost converge to a steady state. These steady states can involve a unique public good level being provided each period or may exhibit a two period cycle. Steady state public good levels exceed the voter's optimal level. More surprisingly, steady state equilibrium reversion levels can exceed the voter's optimal steady state level, meaning that reversion levels cannot be used to bound the optimal level. Reflecting the inability of the agents to commit to their future proposing and voting behavior, equilibrium paths are Pareto inefficient.

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1 Introduction

A central focus of research on the theory of public goods has been understanding the performance of different political processes in providing such goods.[1] The major goal of this research has been to understand how political decision-making distorts provision away from the normative ideal. The motivation is both to better understand real world policy outcomes and to provide insights concerning the relative merits of different political institutions. The bulk of this literature has focused on the provision of static public goods, such as firework displays and police protection, which must be provided anew each period. In practice, however, many important public goods are durable, lasting for many years and depreciating relatively slowly. Public infrastructure, defense capability, and environmental quality are good examples. Understanding the political provision of such goods is more challenging, because of their durable nature. Today's political choices have implications for future choices, creating a dynamic linkage across policy-making periods.

The practical importance of durable public goods and the theoretical challenges they pose has led to increased interest in their political provision. For example, a number of recent papers have studied the provision of such goods by legislatures when tax revenues can also be used to finance district-specific transfers (Battaglini and Coate 2007, Battaglini, Nunnari, and Palfrey 2012, LeBlanc, Snyder, and Tripathi 2000). This paper contributes to the developing literature on the political provision of durable public goods by exploring their determination in Romer and Rosenthal's agenda setter model (Romer and Rosenthal 1978 and 1979b). This well-known model offers a lens through which to understand the spending of local governments.[2] It studies the interaction between a bureaucrat who manages the provision of a public good or service for a community and the citizens in that community. The level of the public good is chosen by the bureaucrat but is subject to citizen approval via a ratification vote. If the bureaucrat's proposed spending level is not approved by a majority of residents, then spending reverts to an exogenously specified reversion level. A tension exists between the bureaucrat and the residents, because

[1] This literature is a sub branch of the vast body of work exploring theoretically, experimentally, and empirically alternative arrangements for public good provision. Important contributions include Bowen (1943) and Bergstrom (1979) on public good provision under majority rule, Lizzeri and Persico (2001 and 2005) on provision under two party and multi-party political competition, Baron (1996) and Volden and Wiseman (2007) on legislative provision, and Niskanen (1971) and Romer and Rosenthal (1978 and 1979b) on bureaucratic provision.

[2] The agenda control model is an alternative to the conventional median voter model which assumes that policy outcomes reflect the preferences of the median voter in the community. For analysis of the relative performance of these two models, see Romer and Rosenthal (1979a and 1982) and Romer, Rosenthal, and Munley (1992).

the bureaucrat cares only about the size of his budget, while voters also care about taxes.[3] The popularity of this model reflects the fact that it captures well the process by which some communities in the U.S. determine their school expenditures. Each year, a school board, who might reasonably be presumed to have as an objective maximizing school spending, proposes a level of spending that must be approved by the voters. Failure to approve results in either the implementation of a reversion level or the board having to make an alternative proposal.

The substantive motivation for incorporating durable public goods into the agenda setter model is that in many communities in the U.S. when local governments undertake capital investments they finance them by issuing bonds and these bond issues must be approved by the voters. This suggests that the agenda setter model may be particularly suitable for understanding local government investment. From a theoretical viewpoint, incorporating durable public goods is interesting because the reversion level is simply the depreciated current level of the good rather than an exogenously set level as in the static model. The lessons of the static model suggest that the proposed level of the public good will depend on the reversion level. But the reversion level itself depends upon prior proposed levels. Predicting equilibrium levels of provision therefore requires solving a complicated feedback loop. It is by no means obvious what will happen.

This paper explores the simplest possible agenda setter model of public investment. There is a bureaucrat, a representative voter, and a durable public good. The bureaucrat just cares about the level of the public good, while the voter also cares about the taxes necessary to finance it. The state variable is the current level of the public good. Each period, the bureaucrat can propose a level of public investment, but to be implemented, it must be approved by the voter. If it is not approved, next period's public good level equals the depreciated current level. Otherwise, next period's level is augmented by the new investment.

The analysis begins by identifying a type of equilibrium - the *Romer-Rosenthal equilibrium* - which represents the natural dynamic analogue to that arising in the static agenda setter model. The defining feature of this equilibrium is that in every period the bureaucrat proposes the maximum possible level of investment the voter will support. This is distinct from holding back investment in order to push through greater levels of investment in the future.

The paper shows that if the voter's benefits from the public good are quadratic, a Romer-Rosenthal equilibrium exists. Moreover, it is possible to solve for the equilibrium path of public

[3] The assumption of budget maximizing bureaucrats builds on Niskanen (1971).

good levels in closed form. This equilibrium path converges to a unique steady state public good level which is provided each period. Comparing the equilibrium path with that which would be chosen by a benevolent planner yields three main results. First, the equilibrium steady state level exceeds the voter's optimal level. The extent of over-provision is higher the faster the public good depreciates and the more patient are the agents. Second, and more surprisingly, the steady state equilibrium reversion level can also exceed the voter's optimal steady state level. When this happens, the voter is approving investment in each period even though, without this investment, next period's public good level would exceed his (first best) optimal level. Third, the equilibrium path of public investment is Pareto inefficient in the sense that there exists an alternative investment path under which both the bureaucrat and the voter are better off. While over-provision also arises in the static agenda control model, the second and third results do not and thus represent distinctive implications of the dynamic analysis.

The results from the quadratic case naturally raise the question of what might happen with different public good benefit functions. In particular, does the Romer-Rosenthal equilibrium exist more generally and, if so, does it exhibit similar dynamics and inefficiencies? The paper sheds light on this question by analyzing what happens with a CRRA benefit function. The CRRA specification allows the concavity of the benefit function to be varied and therefore yields a more complete picture of possible equilibrium outcomes. While for one particular parameterization it remains possible to solve for the equilibrium in closed form, in general the equilibrium with CRRA benefits must be solved for numerically. The lessons from the analysis of the CRRA case are fourfold. First, while the Romer-Rosenthal equilibrium does not exist for all parameter values, it continues to do so quite generally. Second, the equilibrium path converges to a unique steady state public good level when the concavity of the benefit function is below some critical level. Above this level, the equilibrium path almost converges to a two period cycle in which periods of investment are followed by periods of non-investment. Third, long run public good levels continue to be too high and equilibrium reversion levels exceed the voter's optimal level for certain parameterizations. Fourth, when there are cycles, the equilibrium displays an additional source of inefficiency created by long run volatility in public good levels.

The paper relates to a growing literature in political economy studying policy outcomes in situations in which legislators bargain over policy in the shadow of an endogenous status quo. Such models assume that today's policy choice becomes tomorrow's default outcome if agreement is not reached. Even with static policies, this creates a dynamic linkage between periods because tomorrow's choice is impacted by the default outcome. Following Baron (1996), the bulk of this literature assumes that a legislator is randomly picked to make a proposal and this proposal is voted on against the status quo. Baron (1996) considers a model of public good provision along these lines and shows that outcomes converge to the level preferred by the median voter. Understanding distributive policy-making in this context has proven very challenging and the problem has attracted the attention of a number of authors.[4] Closer to this paper is Diermeier and Fong (2011) which assumes that the agenda-setter (i.e., the legislator chosen to propose) remains constant across periods.[5] With few restrictions on either the policy space or legislator preferences, Diermeier and Fong demonstrate the existence of an equilibrium and present an algorithm by which to compute it. The essential difference between Diermeier and Fong's analysis and this paper, is that the former is concerned with static policies and the latter a durable public good. Durability creates a direct linkage across periods and this feature makes for a challenging problem even with just two players (i.e., the bureaucrat and voter) and a one dimensional policy space (i.e., the level of the public good).[6]

The organization of the remainder of the paper is as follows. Section 2 presents the basic model, defines a Romer-Rosenthal equilibrium, and establishes the existence of such an equilibrium when public good benefits are quadratic. Section 3 compares equilibrium and optimal paths of investment in the quadratic case. Section 4 analyzes the model with CRRA public good benefits and Section 5 concludes.

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[4] The list includes Baron and Herron (2003), Battaglini and Palfrey (2012), Bowen and Zahran (2012), and Kalandrakis (2004). Battaglini and Palfrey's work also provides an experimental analysis. Bowen, Chen, and Eraslan (2014) study a policy space with both a public good and distributional policies.

[5] In a much earlier paper, Ingberman (1985) presents a dynamic agenda setter model with a static public good in which today's spending level becomes the reversion level for tomorrow. However, he parts company with the modern literature by assuming first that the voter votes myopically and second that the bureaucrat commits up front to his sequence of public good proposals.

[6] Riboni (2010) considers a dynamic model with fixed agenda setter in his study of monetary policy-making. Each period, the agenda setter makes a proposal concerning a target inflation rate to a committee. If the committee rejects the proposal, the prior period's target inflation rate remains in place. What makes this model particularly interesting and challenging to solve is that the policy-makers' preferences over the target depend on citizens' inflation expectations which are endogenous.

2 A dynamic agenda setter model

2.1 The model

The model considers the interaction between a bureaucrat and a representative voter. The time horizon is infinite. There is a durable public good which depreciates at rate $\delta \in (0,1)$. The bureaucrat's job is to manage the provision of this good. In any period, the bureaucrat can propose investing in the good. Investment costs c per unit and is financed by a tax on the voter.[7] To be implemented, the bureaucrat's proposal must be approved by the voter. The bureaucrat's per period payoff just depends on the level of the public good, while the voter also cares about taxes.

Let g denote the current level of the public good and g' next period's level. Rather than thinking of the bureaucrat choosing investment, it is more convenient to think of him choosing g'. If he does not propose any new investment or his proposal is rejected by the voter, then g' just equals $(1-\delta)g$. If he does propose new investment and it is approved, then g' will exceed $(1-\delta)g$ and investment will be given by $g' - (1-\delta)g$. The tax on the voter is then $c(g' - (1-\delta)g)$. The bureaucrat's per period payoff is g and the voter's per period payoff is $B(g) - c(g' - (1-\delta)g)$, where $B(g)$ represents the voter's public good benefits.[8] Initially, we will assume that these benefits have a quadratic form so that $B(g)$ equals $b_0 g - b_1 g^2$ for some positive parameters b_0 and b_1. To ensure a positive demand for public goods, we further assume that βb_0 exceeds $c\,(1-\beta(1-\delta))$. Both agents discount future payoffs at rate β.

Following the dynamic political economy literature, we look for a *Markov perfect equilibrium* in which the agents' strategies just depend on the current level of the public good g. Let $g'(g)$ denote the bureaucrat's strategy and let $U(g)$ and $V(g)$ denote the value functions of the bureaucrat and voter respectively. Then, $g'(g)$ solves the problem

$$\max_{g'} \left\{ \begin{array}{c} g + \beta U(g') \\ s.t.\ \beta \left[V(g') - V((1-\delta)g) \right] \geq c(g' - (1-\delta)g) \ \ \&\ g' \geq (1-\delta)g. \end{array} \right\} \tag{1}$$

[7] We do not model the constraint that the voter is able to pay his taxes, implicitly assuming he has sufficient lifetime income that this constraint never binds.

[8] It might arguably be more natural to assume that the bureaucrat's per period payoff were $B(g)$ rather than g. However, as will become clear below, in a Romer-Rosenthal equilibrium, this change has no impact on the equilibrium path of public good levels. All it does is change the bureaucrat's value function and complicate the task of proving the existence of an equilibrium.

The objective function is the bureaucrat's payoff. The first constraint ensures that the voter will approve the bureaucrat's proposal and the second rules out disinvestment. Note that both constraints are trivially satisfied if the bureaucrat chooses not to invest in which case $g' = (1-\delta)g$. Given the strategy $g'(g)$, the bureaucrat's and voter's value functions are defined recursively by the equations

$$U(g) = g + \beta U(g'(g)), \tag{2}$$

and

$$V(g) = B(g) - c(g'(g) - (1-\delta)g) + \beta V(g'(g)). \tag{3}$$

An *equilibrium* consists of a strategy $g'(g)$ and value functions $U(g)$ and $V(g)$ satisfying equations (1), (2), and (3).

2.2 Discussion

While the model is simple, we see it as a natural extension of the static agenda setter model. Indeed, we find it surprising that it has not been proposed and analyzed before. The application that motivated us to write down the model is debt-financed public investment. As noted in the introduction, in many states, local government bond issues must be approved by the voters.[9] Thus, when local governments (such as muncipalities, school districts, and special districts) finance public investment with bond issues, their investment proposals require voter support. While the model assumes that investment proposals are financed by taxation, *Ricardian Equivalence* holds and so bond and tax finance are the same. To be more precise, under the assumption that the interest rate is equal to $1/\beta - 1$, a formally equivalent model would result if we had assumed that the investment was financed with (for example) one period bonds. In such a model, in each period, the voter would repay the previous period's bond issue and the bureaucrat would propose a new issue. This new issue would be used to finance new investment and, if it were not approved,

[9] Local governments in the U.S. use two different types of bonds to finance public investments: general obligation and revenue bonds. General obligation bonds pledge the full-faith and credit of the issuing government as security. This means that the government is required to obtain the funds to repay the bonds by increasing taxation on the residents or reducing public spending. Residents are thus ultimately responsible for repaying these bonds. Accordingly, in most contexts, general obligation bond issues require resident approval. Revenue bonds are designed to be repaid from a specific revenue stream generated by the investment, typically through user charges (for example, borrowing for a bridge investment might be repaid by bridge tolls). If the revenues from this stream are insufficient to repay the bonds, then the bondholders suffer the loss. Local government officials can often issue such bonds without resident approval. Our model therefore applies to investment financed with general obligation bonds.

no new investment would be undertaken.[10] While the cost of bond repayment would not be incurred until the next period, the present value of this repayment would exactly equal the tax in this model.

It is also worth noting that the model can be applied with only minor modifications to Romer and Rosenthal's original school budget example under the assumptions that i) the current period's reversion budget is just the previous period's enacted budget, and ii) there is a constant rate of inflation.[11] In this application, in each period the school board would propose a level of spending to the voters. If the voters approve it, it would be implemented. Otherwise, the level of school spending from the prior period would be implemented. Given inflation, this spending level would purchase less real educational outputs (teachers, books, computers, etc) than in the previous period. Inflation therefore acts just like depreciation in our dynamic model. The state variable in this application would be the real value of the reversion budget. In equilibrium, the school board's proposal would be approved each period and this would determine the reversion budget for the next period. The greater the level of inflation, the greater the bargaining power of the school board, because it becomes more costly for the voters to live with the reversion budget. The only change necessary to apply the model to this setting is to assume that the benefits of government spending are enjoyed in the same period as the costs of taxation. In the model studied here, there is a one period lag between investment being paid for and the new units of the public good generating benefits for the residents.

3 Romer-Rosenthal equilibrium

In the static agenda setter model, Romer and Rosenthal showed that equilibrium involves the bureaucrat proposing the largest level of public spending which leaves the median voter at least as well off as with the reversion level. This is just the reversion level when it exceeds the median voter's optimal level. Otherwise, the equilibrium level exceeds the reversion level. In our dynamic

[10] Our model abstracts from the possibility that the local government can finance new investment from some other source, such as with tax revenues or inter-governmental grants. We assume, in effect, that no inter-governmental grants are available and that any tax revenues not associated with bond service are earmarked to the provision of static public goods and services.

[11] More generally, the model is relevant for the school budget problem whenever the reversion budget level is linked to the previous period's spending and there is some dynamic factor, not accounted for in the reversion budget formula, which makes residents' desired spending levels increase over time. Knowing exactly in what fraction of real world contexts these assumptions are reasonable is difficult because school budget procedures are complex, vary significantly across the states (see Hamilton and Cohen 1974), and contemporary comparative information on them is difficult to find.

model, we call the analogue to this the Romer-Rosenthal equilibrium. The defining feature of this equilibrium is that in any period the bureaucrat always proposes the maximum level of investment the voter will approve. While this is a well established property of equilibrium in the static model, in a dynamic model with depreciation, it seems possible that it might sometimes pay the bureaucrat to hold back investment today to leverage a bigger package tomorrow.[12]

To define the Romer-Rosenthal equilibrium concept formally, assume that the voter's value function $V(g)$ is increasing and strictly concave, and let g^* denote the voter's optimal level of the public good in equilibrium. From (3), this is given by:[13]

$$g^* = \arg\max\{\beta V(g') - cg'\}. \tag{4}$$

When g exceeds $g^*/(1-\delta)$ the voter will prefer the reversion level $(1-\delta)g$ to any higher level. Accordingly, the bureaucrat must simply choose the reversion level. When g is less than $g^*/(1-\delta)$, there exist public good levels higher than the reversion level that will be supported by the voter. In a Romer-Rosenthal equilibrium, the bureaucrat will choose the largest of these. This means that he will choose a public good level g' (greater than $(1-\delta)g$) such that

$$\beta\left[V(g') - V((1-\delta)g)\right] = c(g' - (1-\delta)g). \tag{5}$$

Intuitively, at this public good level, the future benefits to the voter are just offset by the tax cost. Accordingly, we say that an equilibrium $(g'(g), U(g), V(g))$ is a *Romer-Rosenthal equilibrium* if $g'(g)$ equals $(1-\delta)g$ if g exceeds $g^*/(1-\delta)$ and satisfies (5) otherwise.

3.1 Characterization and existence

A Romer-Rosenthal equilibrium has a very convenient property.[14] Notice that when g is less than $g^*/(1-\delta)$, equation (5) implies that the bureaucrat's strategy $g'(g)$ is such that

$$-c(g'(g) - (1-\delta)g) + \beta V(g'(g)) = \beta V((1-\delta)g). \tag{6}$$

[12] Indeed, we have shown that holding back investment can occur in a four period, finite horizon version of our model. For certain parameter values and initial stocks of the public good, the bureaucrat holds back investment in period 1. While this strategy reduces the period 2 public good level, it yields higher public good levels in periods 3 and 4. Our analysis of the four period version of our model can be found in the On-line Appendix.

[13] The voter's utility is linear in private consumption (i.e., money left over after taxes) and hence there are no income effects. This means that the voter's "optimal level" of the public good can be define unambiguously (i.e., independently of the state g).

[14] This property is also true in an equilibrium in which the bureaucrat holds back investment, provided that whenever he does invest, he chooses the maximum possible level.

Substituting (6) into equation (3), we see that the voter's value function satisfies

$$V(g) = B(g) + \beta V((1-\delta)g). \tag{7}$$

Moreover, equation (7) also holds when g exceeds $g^*/(1-\delta)$ since $g'(g)$ equals $(1-\delta)g$. Applying equation (7) repeatedly, we conclude that in a Romer-Rosenthal equilibrium, the voter's value function is

$$V(g) = \sum_{t=0}^{\infty} \beta^t B((1-\delta)^t g). \tag{8}$$

Intuitively, the voter gets the same lifetime utility in equilibrium as he would do if there were never any more investment. This reflects the fact that the bureaucrat extracts all the surplus from any new investment. Note that the voter's value function is increasing and strictly concave as required.

When the voter has quadratic public good benefits, equation (8) implies that the voter's value function is

$$V(g) = \left(\frac{b_0}{1-\beta(1-\delta)}\right) g - \left(\frac{b_1}{1-\beta(1-\delta)^2}\right) g^2. \tag{9}$$

Equation (9) has some useful implications. Substituting it into (4) and solving, we find that the voter's optimal public good level g^* is given by:

$$g^* = \frac{\left(\beta b_0 - c\left(1-\beta(1-\delta)\right)\right)\left(1-\beta(1-\delta)^2\right)}{2\beta b_1\left(1-\beta(1-\delta)\right)}. \tag{10}$$

In addition, substituting (9) into (5), we can show that when g is less than $g^*/(1-\delta)$, the new public good level $g'(g)$ is equal to $2g^* - (1-\delta)g$.[15] We may therefore conclude that in a Romer-Rosenthal equilibrium, the bureaucrat's strategy is given by

$$g'(g) = \begin{cases} 2g^* - (1-\delta)g & \text{if } g \leq g^*/(1-\delta) \\ (1-\delta)g & \text{if } g > g^*/(1-\delta) \end{cases}. \tag{11}$$

This strategy is illustrated in Figure 1. The current level of the public good g is measured on the horizontal axis and the future level g' on the vertical. The dashed line is the 45^o line and the flatter solid line measures $(1-\delta)g$. The strategy $g'(g)$ begins at the point $(0, 2g^*)$, slopes down to the point $(g^*/(1-\delta), g^*)$, and thereafter follows the upward sloping line $(1-\delta)g$. On the interval $[0, g^*/(1-\delta)]$ the amount of public good proposed by the bureaucrat is decreasing linearly in the existing level and the rate of decrease is just $1-\delta$.

[15] The derivation can be found in the Appendix.

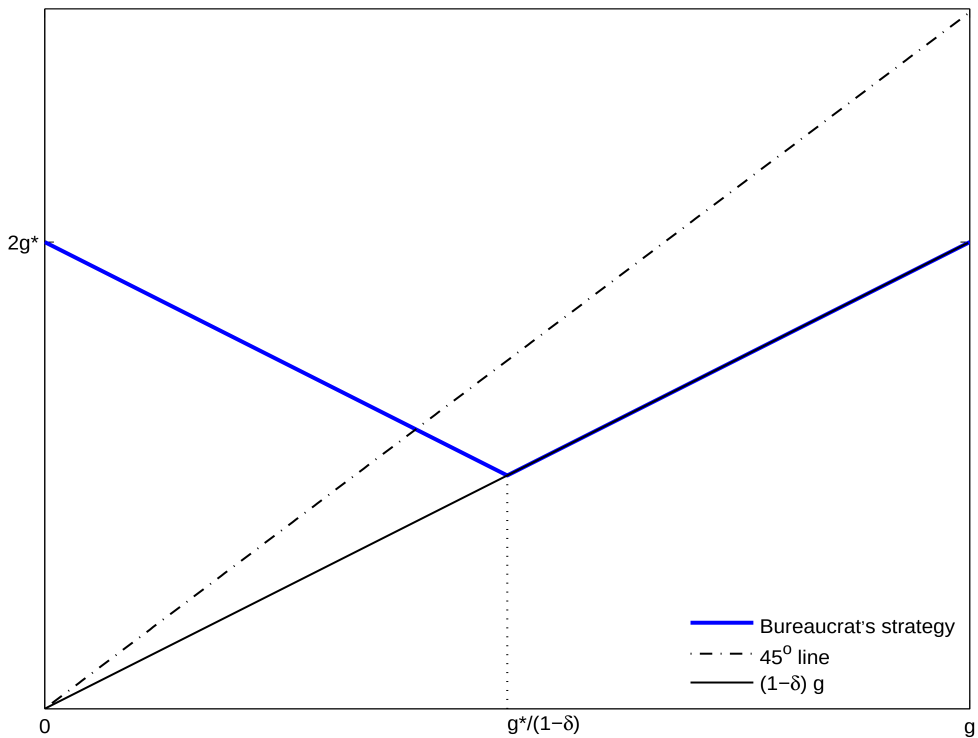

Figure 1: Bureaucrat's strategy

With this knowledge of the bureaucrat's strategy, the dynamic evolution of the equilibrium public good level can be traced out. Figure 2 illustrates the situation. Suppose that the level of the public good at the beginning of period 0 is g_0. Then, in period 0, a large amount of investment takes place and the public good level at the beginning of period 1 is g_1. Since g_1 exceeds $g^*/(1-\delta)$, there is no investment in period 1 and the level of the public good at the beginning of period 2 is $g_2 = (1-\delta)g_1$. Investment resumes in period 2 and the level of the public good at the beginning of period 3 can be obtained from the Figure in the obvious way (i.e., $g_3 = g'(g_2)$). Proceeding in this way, the entire future path of public good levels can be obtained and the bureaucrat's value function will be given by

$$U(g_0) = \sum_{t=0}^{\infty} \beta^t g_t. \tag{12}$$

One further step is required to establish the existence of a Romer-Rosenthal equilibrium. We need to verify that when g is less than $g^*/(1-\delta)$, the bureaucrat is indeed better off choosing a

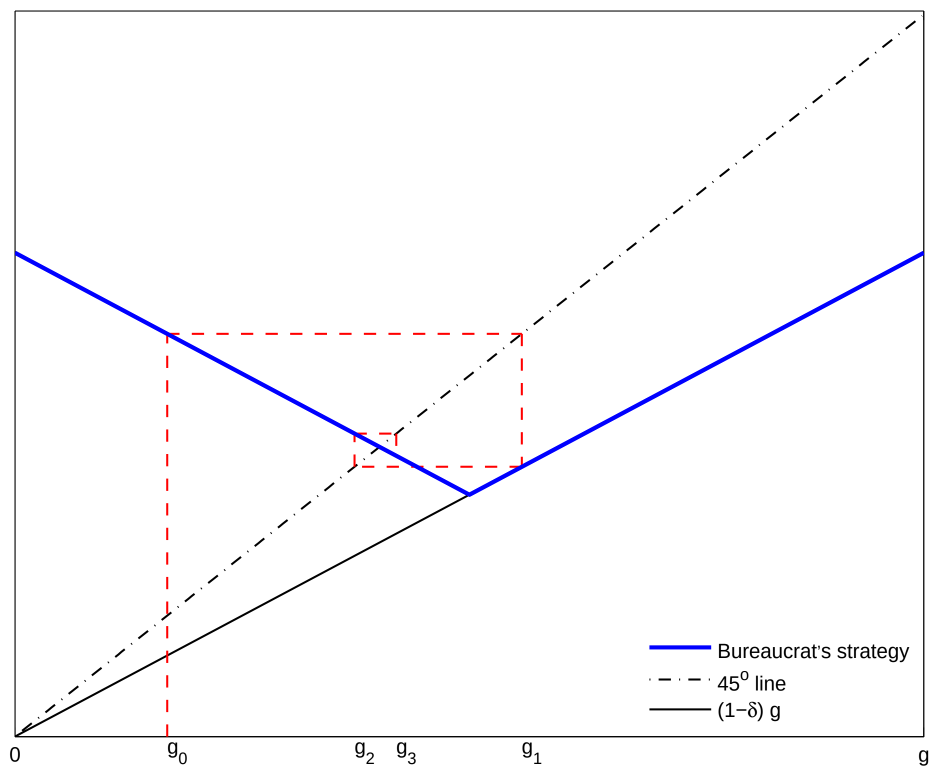

Figure 2: Dynamics

public good level g' that satisfies equation (5) rather than a smaller level. From (1), this requires showing that the bureaucrat's value function takes on a value at $g'(g)$ at least as large as the value for any g' in the interval $[(1-\delta)g, g'(g)]$. Intuitively, the problem is to show that the bureaucrat cannot gain by holding back investment today to pass a bigger amount tomorrow. To verify this is the case, we need to compute the bureaucrat's value function from (12) and check it is maximized at $g'(g)$.[16] While in general this is a difficult problem, the simplicity of the bureaucrat's strategy in the quadratic case makes this possible and allows us to prove:

Proposition 1 *With quadratic public good benefits, there exists a Romer-Rosenthal equilibrium*

[16] Notice that if we had assumed the bureaucrat's per period payoff were $B(g)$ rather than g (as discussed in footnote #8), we would have to compute the bureaucrat's value function from the equation $U(g_0) = \sum_{t=0}^{\infty} \beta^t B(g_t)$ and check it is maximized at $g'(g)$. This further complicates an already difficult task. Nonetheless, it seems intuitively that adding curvature to the bureaucrat's per period payoff would make holding back investment even less desirable and thus would not threaten our existence result. Our numerical analysis supports this conjecture: in any case we have computed in which the Romer-Rosenthal equilibrium exists when the bureaucrat has per period payoff g it also does so when he has per period payoff $B(g)$.

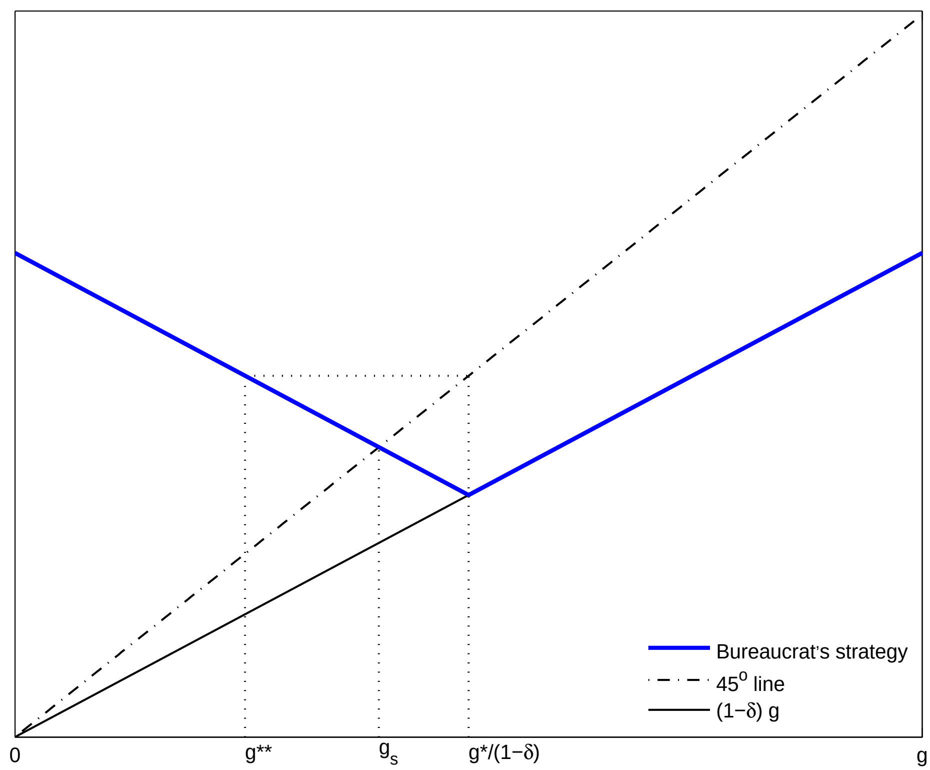

Figure 3: Steady state

with bureaucrat strategy given by (11).

Proof See the On-line Appendix. ■

3.2 Equilibrium dynamics

The long run behavior of public good levels implied by the equilibrium is easy to figure out. Let g^{**} denote the level of public good at which $g'(g)$ equals $g^*/(1-\delta)$. This level is illustrated in Figure 3 and can be shown to equal $g^*(1-2\delta)/(1-\delta)^2$. Its significance is that if g is less than g^{**}, new investment is such that $g'(g)$ exceeds $g^*/(1-\delta)$ which means there is no investment in the subsequent period. By contrast, if g lies in the interval $(g^{**}, g^*/(1-\delta)]$, new investment is such that $g'(g)$ is less than $g^*/(1-\delta)$ implying there is investment in the subsequent period. Moreover, $g'(g)$ exceeds g^{**}, so that there will be investment in the following and all future periods. It follows from this that once the public good level enters the interval $[g^{**}, g^*/(1-\delta)]$ it will remain there and investment will take place each period. The public good level will converge with damped oscillations to the steady state level g_s illustrated in Figure 3. At this steady state, the voters approve a constant level of investment δg_s each period.

If the public good level starts out below g^{**}, the initial period's investment will put the level above $g^*/(1-\delta)$ and a number of periods with no investment will follow until it falls below $g^*/(1-\delta)$. After that point, the level is in the interval $[g^{**}, g^*/(1-\delta)]$ and will converge to g_s. If the public good level starts out above $g^*/(1-\delta)$, a number of periods with no investment will follow until it falls below $g^*/(1-\delta)$. After that point, it will again converge to g_s. The steady state level satisfies the equation $g_s = g'(g_s)$ and, using (11), it is easy to show that it is given by

$$g_s = \frac{2}{2-\delta} g^* = \frac{(\beta b_0 - c(1-\beta(1-\delta)))\left(1-\beta(1-\delta)^2\right)}{(2-\delta)\beta b_1 (1-\beta(1-\delta))}. \tag{13}$$

We have therefore proved:

Proposition 2 *With quadratic public good benefits, in the Romer-Rosenthal equilibrium with bureaucrat strategy given by (11), the public good level converges to the steady state level g_s defined in (13).*

4 Normative analysis of equilibrium

4.1 Optimal investment

We can now compare the equilibrium path of public good levels with the path that would maximize the voter's utility. The planning problem can be posed recursively as:

$$W(g) = \max_{g'} \left\{ \begin{array}{c} B(g) - c(g' - (1-\delta)g) + \beta W(g') \\ s.t. \;\; g' \geq (1-\delta)g \end{array} \right\}, \tag{14}$$

where $W(g)$ denotes maximized voter welfare. Assuming the constraint in (14) is not binding, the first order condition for the optimal g' is that

$$\beta W'(g') = c. \tag{15}$$

Again, assuming the constraint is not binding, the *Envelope Theorem* implies that

$$W'(g) = B'(g) + c(1-\delta). \tag{16}$$

Substituting (16) into (15) and solving, we see that with quadratic public good benefits if the constraint is not binding, $g' = g_s^o$ where

$$g_s^o = \frac{\beta b_0 - c(1-\beta(1-\delta))}{2\beta b_1}. \tag{17}$$

Notice that this optimal level is independent of the initial level g.[17] The initial level matters only if it is larger than the optimal level. In that case, the constraint binds. Thus, we have that the optimal policy function is

$$g'(g) = \begin{cases} g_s^o & \text{if } g \leq \frac{g_s^o}{1-\delta} \\ (1-\delta)g & \text{if } g > \frac{g_s^o}{1-\delta} \end{cases} \tag{18}$$

The dynamic path of public good levels generated by this policy function is very simple. If the initial level of the public good is less than $\frac{g_s^o}{1-\delta}$, then in subsequent periods the public good level is just g_s^o. The public good level jumps to its steady state level in just one period. If the initial level exceeds $\frac{g_s^o}{1-\delta}$, there is no investment until the level of public good has fallen below $\frac{g_s^o}{1-\delta}$ at which time investment kicks in to maintain the public good at its optimal level g_s^o.

4.2 Equilibrium vs optimal investment

We begin by comparing the equilibrium steady state g_s with the optimal steady state g_s^o. Using (17) and (13), we may write

$$\frac{g_s - g_s^o}{g_s^o} = \frac{\delta/2(1+\beta(1-\delta))}{(1-\delta/2)\,(1-\beta(1-\delta))}. \tag{19}$$

The left hand side of this equation measures the extent of over or under-provision as a fraction of the optimal level. Since the right hand side of (19) is positive, the public good is over-provided in steady state. The degree of over-provision is between 0 and 100% of the optimal level, taking on its minimum value when $\delta = 0$ and its maximum when $\delta = 1$. Moreover, it is independent of the public good benefit and cost parameters, just depending on the agents' discount rate β and the depreciation rate δ. It can be shown to be increasing in β and δ, implying that over-provision is greater the more the agents care about the future and the faster the public good depreciates. Thus, we have:

Proposition 3 *With quadratic public good benefits, in the Romer-Rosenthal equilibrium with bureaucrat strategy given by (11), the steady state public good level exceeds the optimal steady state level. The degree of over-provision measured as a fraction of the optimal level ranges between* 0 *and* 100% *and depends positively on the agents' discount rate and the depreciation rate.*

[17] Again, this reflects the fact that the voter's utility is linear in private consumption and hence there are no income effects.

Proof See the Appendix. ■

The result that the magnitude of over-provision is increasing in β reflects the fact that the more the voter cares about the future the more painful the prospect of having to live with the depreciated current level of the public good. The positive dependence on δ reflects the fact that the faster the public good depreciates the more severe is the threat of being stuck with the depreciated current level.[18] It is instructive to compare these results concerning over-provision with those of the static model. When the reversion level is less than the voter's optimal level, the static model predicts that the public good will be over-provided. Moreover, with a quadratic public good benefit function, the magnitude of over-provision as a fraction of the voter's optimal level is less than 100%.[19] Thus, the lessons concerning over-provision are quite similar.

These results concerning over-provision not withstanding, there are two important differences between the dynamic and static models. First, a central lesson of the static model is that if the voter approves the bureaucrat's proposal, the reversion level must be below his optimal public good level. In cases where the bureaucrat passes a proposal, the reversion level therefore provides a useful lower bound estimate of the voter's optimal public good level. This is also true in the dynamic model in the sense that the equilibrium reversion level $(1-\delta)g_s$ is less than the voter's optimal level g^* (this is clear from (13)). However, g^* is the optimal level for the voter *in equilibrium* and it exceeds the first best optimal level g_s^o (compare (10) with (17)). This means that it is possible that the equilibrium reversion level exceeds the first best level. Using (17) and (13), we can show that $(1-\delta)g_s$ exceeds g_s^o when

$$(2(1-\delta)+1)\,\beta(1-\delta) > 1. \tag{20}$$

Studying this inequality yields the following result:

[18] It may seem puzzling that when the public good depreciates very slowly (small δ), there will be little difference between the equilibrium and optimal steady states. After all, the bureaucrat has agenda setting power and very different objectives from the voter. To understand the result, note that with small δ if the public good were only a little below the voter's optimal level the bureaucrat would be able to implement only a very small level of investment. This is because the reversion level would differ little from the voter's optimal level. Since in equilibrium the bureaucrat always takes any level of investment he can get, the equilibrium level of the public good would therefore remain close to optimal. Now note that, even if the community starts out with a very small level of the public good and the bureaucrat initially implements a high level of investment, eventually depreciation must bring the public good level down close to the optimal level.

[19] With quadratic public good benefits, the voter's optimal public good level in the static model is $(b_0-c)/2b_1$. The maximum level the bureaucrat can extract is when the reversion level is 0. This level satisfies $b_0 g - b_1 g^2 = cg$, which implies that it equals $(b_0-c)/b_1$ exactly twice the voter's optimal level. The maximum over-provision magnitude is therefore 100%. This should not be surprising because when depreciation is 100% (i.e., $\delta = 1$) the dynamic model is equivalent to the static model with a reversion level of 0.

Proposition 4 *With quadratic public good benefits, in the Romer-Rosenthal equilibrium with bureaucrat strategy given by (11), for all discount rates β in excess of $1/3$, there exists a critical value of the depreciation rate $\delta(\beta) \in (0,1)$ such that the equilibrium reversion level $(1-\delta)g_s$ exceeds the optimal steady state level g_s^o for all depreciation rates in the interval $(0, \delta(\beta))$.*

Proof See the Appendix. ■

Proposition 4 is striking in the sense that in equilibrium the voter is approving investment in every period even though the public good level that would arise in the next period if the investment were not undertaken exceeds his first best level. The intuition is that the voter values public goods more in the equilibrium because higher levels reduce exploitation by the bureaucrat.[20] The lesson from this Proposition is that the equilibrium reversion level cannot be used as a lower bound for the first best level of public goods. This result is the most important insight from the model. It seems natural to assume that if voters in a community approve an investment, then it must be the case that the public good level that would prevail without the investment would be "too low". This is correct if too low means below the level that the voters would like *in equilibrium.* But it is not correct if too low means below the *first best* level. This makes it hard to use bond elections to make inferences about political distortions in the provision of public goods.[21]

The second key difference between the static and dynamic models concerns efficiency. In the static model, the equilibrium policy choice is on the Pareto frontier: by construction, the equilibrium policy maximizes the bureaucrat's welfare subject to a given level of utility for the voter. In the dynamic model, however, the equilibrium path of investment is Pareto inefficient. To see this, consider a modified planning problem which puts weight λ on the citizen's payoff and $1-\lambda$ on the bureaucrat's. The only effect of this modification is to change the optimal steady state to

$$g_s^o(\lambda) = \frac{\beta(\lambda b_0 + (1-\lambda)) - \lambda c(1-\beta(1-\delta))}{\lambda 2\beta b_1}. \tag{21}$$

[20] This phenomenon can also occur in the finite horizon version of our model. We have shown in a four period version that equilibrium can involve the voter approving investment in period 1 even though a benevolent planner would choose not to invest given the initial stock of the public good. The voter does this in order to reduce public good over-provision in periods 3 and 4. Our analysis of the four period version of our model can be found in the On-line Appendix.

[21] In an influential recent paper, Cellini, Ferreira, and Rothstein (2010) propose and implement empirically a procedure for evaluating whether a local government is over or under providing durable public goods based on the reaction of local housing prices to a bond issue's passage or failure. However, as a theoretical matter, the reaction of housing prices to a bond issue's passage or failure reflects voters' preferences for more or less public good *in equilibrium* and tells us nothing about socially optimal levels. Coate (2013) provides a detailed analysis of this issue in a model which integrates aspects of the framework analyzed in this paper with a local housing market.

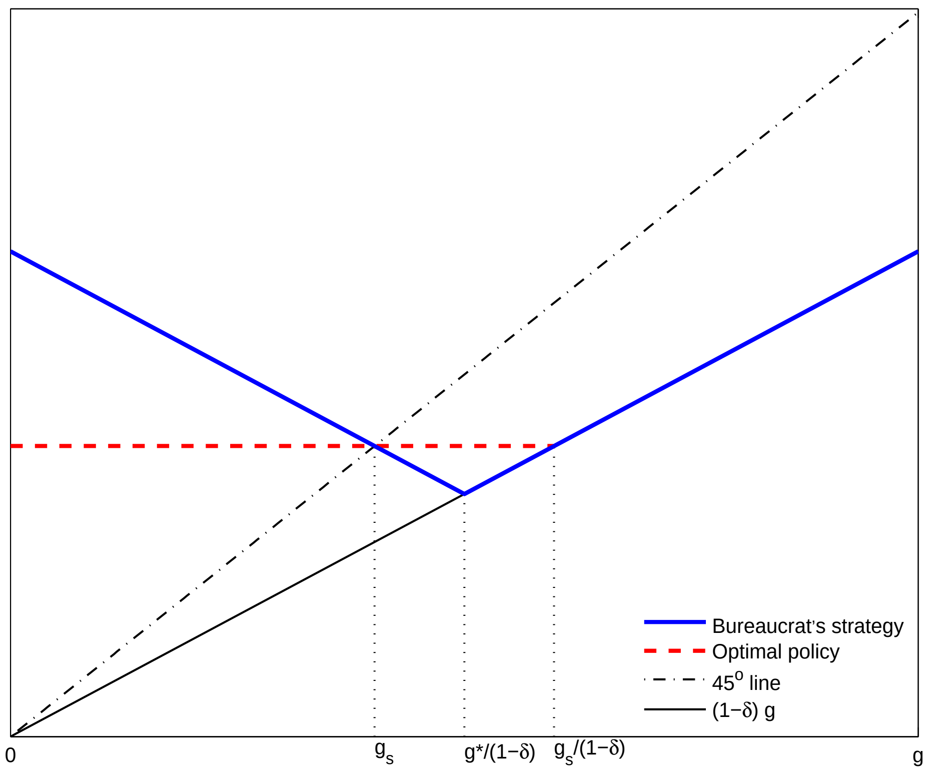

Figure 4: Equilibrium vs. optimum

Now suppose that the welfare weight on the citizen is such as to make the optimal steady state $g_s^o(\lambda)$ equal to the equilibrium steady state g_s.[22] This ensures that the equilibrium yields the efficient level of the public good eventually. Nonetheless, the transition to the steady state differs in the equilibrium. The situation is illustrated in Figure 4 which compares the bureaucrat's equilibrium strategy and the optimal policy function under the assumption that $g_s^o(\lambda)$ equals g_s. The optimal policy function, which comes from (18), is the dashed horizontal line.

From Figure 4, it is clear that equilibrium exceeds optimal investment whenever g is less than g_s and is lower whenever g lies in the interval $(g_s, \frac{g_s}{1-\delta}]$. Thus, unless the initial level of the public good just happens to equal g_s, the equilibrium and optimal paths diverge. The over-shooting

[22] In the Appendix, we show that the welfare weight on the citizen that makes the equilibrium and optimal steady states coincide is

$$\lambda = \frac{(1-\beta(1-\delta))\,\beta\,(2-\delta)}{\delta\,(\beta(1-\delta)+1)\,(\beta b_0 - c\,(1-\beta(1-\delta))) + (1-\beta(1-\delta))\,\beta\,(2-\delta)}.$$

arises because the bureaucrat exploits the low reversion level by investing too much. The undershooting arises because the voter exploits the high reversion level by blocking efficient investment. Both parties would be better off with an investment path that followed the optimal rule in (18) but with a different steady state. This argument establishes:

Proposition 5 *With quadratic public good benefits, in the Romer-Rosenthal equilibrium with bureaucrat strategy given by (11), the equilibrium path of investment is Pareto inefficient provided only that the initial level of the public good the community begins with differs from the equilibrium steady state level.*

Proposition 5 tells us that the political equilibrium exhibits *political failure* in the sense defined by Besley and Coate (1998). The source of the inefficiency is lack of commitment. Starting with a low initial value of the public good, the voter would like to reassure the bureaucrat that he will support higher levels of investment in the future if the bureaucrat proposes a lower level today. However, this is not incentive compatible. Similarly, starting with a higher initial value of the public good, the bureaucrat would like to reassure the voter that he will propose lower levels of investment in the future if the voter supports a higher level today. Again, this is not incentive compatible. The model therefore provides a nice example of political failure arising because of lack of commitment in dynamic political interactions. While far from the first such example, it has the virtue of emerging simply in the obvious dynamic extension of a very well-known political economy model.[23]

5 Beyond quadratic public good benefits

The results from the quadratic case naturally raise the question of what might happen with different public good benefit functions. In particular, does the Romer-Rosenthal equilibrium exist more generally and, if so, does it exhibit similar dynamics and inefficiencies? This section sheds light on this question by analyzing what happens with a CRRA benefit function $B(g)$ equal to $bg^{1-\sigma}/(1-\sigma)$ where b and σ are positive parameters.[24] The CRRA specification allows the

[23] For classic examples see Alesina and Tabellini (1990) and Persson and Svensson (1989). For other examples involving durable public investments see Battaglini and Coate (2007), Battaglini, Nunnari, and Palfrey (2012), Besley and Coate (1998), and LeBlanc, Tripathi, and Snyder (2000). In all these examples, decision-making power fluctuates between different political actors. In our example, by contrast, the allocation of decision-making power remains constant. For a general discussion of commitment problems in political environments see Acemoglu (2003).

[24] As is standard, when $\sigma = 1$ and the CRRA benefit function is not defined, $B(g)$ is assumed to take the logarithmic form $b \ln g$.

concavity of the benefit function to be varied (by changing σ) and therefore permits a richer understanding of possible equilibrium outcomes than does the quadratic case.[25] With this benefit function, the first best level of the public good is easily shown to equal

$$g_s^o = \left(\frac{\beta b}{c\,(1-\beta\,(1-\delta))} \right)^{\frac{1}{\sigma}}. \tag{22}$$

Given that this depends only on the ratio b/c we henceforth set the price of the public good c equal to 1.

As shown in Section 2.2, if a Romer-Rosenthal equilibrium exists, the voter's value function satisfies equation (8). In the CRRA case, therefore, if a Romer-Rosenthal equilibrium exists, the voter's value function must equal

$$V(g) = b\frac{g^{1-\sigma}}{1-\sigma} \sum_{t=0}^{\infty} \left[\beta(1-\delta)^{1-\sigma}\right]^t. \tag{23}$$

Note immediately that in order for the sum on the right hand side to converge we require that $\beta(1-\delta)^{1-\sigma}$ is less than 1. If this condition is satisfied, then the voter's value function is

$$V(g) = \left(\frac{b}{1-\beta(1-\delta)^{1-\sigma}} \right) \frac{g^{1-\sigma}}{1-\sigma}. \tag{24}$$

This function is increasing and concave as required. If the condition is not satisfied, then the voter's value function is not defined. Thus, a necessary condition for the existence of a Romer-Rosenthal equilibrium with a CRRA benefit function is that $\beta(1-\delta)^{1-\sigma}$ is less than 1. The remaining questions are then does a Romer-Rosenthal equilibrium exist under this condition and, if so, what are its properties? In the special case in which σ equals 2 these questions can be answered analytically. More generally, numerical analysis is required.

5.1 The CRRA case with $\sigma = 2$

When σ equals 2, we can prove the existence of the Romer-Rosenthal equilibrium analytically under the assumption that β is less than $(1-\delta)$ and solve for the equilibrium path in closed form. Substituting (24) into (4) and solving, we find that the voter's optimal public good level g^* is given by:

$$g^* = \sqrt{\frac{\beta b}{1-\frac{\beta}{1-\delta}}}. \tag{25}$$

[25] The CRRA function has the disadvantage of implying that the marginal benefit of the public good becomes infinite as its level goes to zero, which may not be a reasonable assumption. While this disadvantage can be overcome by adding a positive constant m to the function so that $B(g)$ equals $(g+m)^{1-\sigma}/(1-\sigma)$, such an addition sacrifices much of the CRRA function's famed tractability.

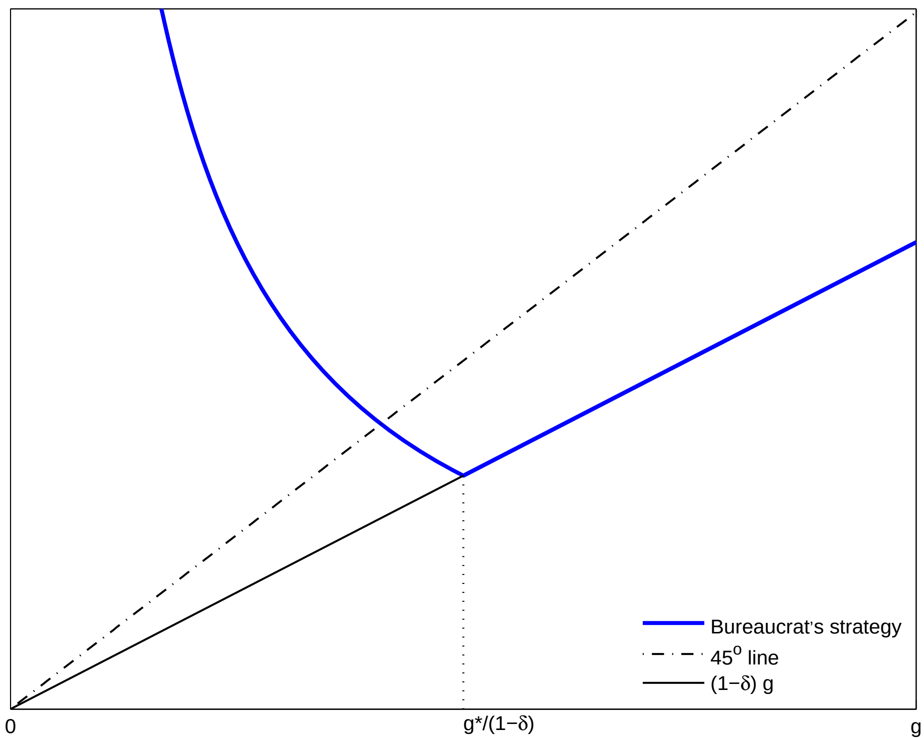

Figure 5: Bureaucrat's strategy, $\sigma = 2$

In addition, substituting (24) into (5), we can show that when g is less than $g^*/(1-\delta)$, the new public good level $g'(g)$ is equal to $(g^*)^2/\left[(1-\delta)g\right]$.[26] We may therefore conclude that in a Romer-Rosenthal equilibrium, the bureaucrat's strategy is given by

$$g'(g) = \begin{cases} \frac{(g^*)^2}{(1-\delta)g} & \text{if } g \leq g^*/(1-\delta) \\ (1-\delta)g & \text{if } g > g^*/(1-\delta) \end{cases} . \tag{26}$$

This strategy is illustrated in Figure 5. Comparing Figures 1 and 5, we see that what makes this different from the quadratic case is the convex shape of the bureaucrat's strategy. The amount of public good the bureaucrat is able to extract from the voter increases significantly as the existing level decreases because of the sharply increasing marginal utility of the public good.[27] Intuitively,

[26] The derivation can be found in the Appendix.

[27] Recall that our model implicitly assumes that the voter has sufficient lifetime resources to pay for equilibrium levels of investment. Since $lim_{g\searrow 0}g'(g) = \infty$, this assumption becomes problematic at very low levels of the public

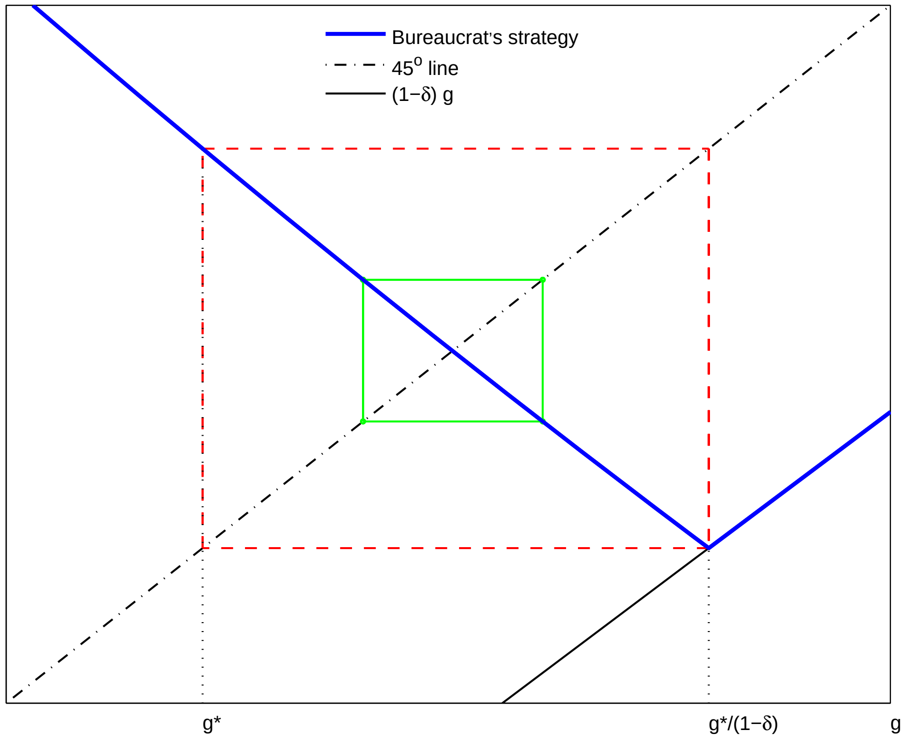

Figure 6: Different cycles, $\sigma = 2$

this would seem to raise the incentive for the bureaucrat to hold back investment and thus throws into doubt the existence of a Romer-Rosenthal equilibrium.

The dynamic evolution of the equilibrium public good level implied by the bureaucrat's strategy (26) is remarkably simple. If the public good level g lies in the interval $[g^*, g^*/(1-\delta)]$ it will immediately converge to a two period deterministic cycle with public good levels $(g^*)^2 / [(1-\delta)\, g]$ one period and g the next. Two such cyclic steady states are illustrated in Figure 6. The larger rectangle illustrates the case in which g is equal to g^* or $g^*/(1-\delta)$ and the public good alternates between g^* and $g^*/(1-\delta)$. What is particularly appealing about this case is that when the public good level is equal to $g^*/(1-\delta)$ there is no investment. Accordingly, periods of investment are followed by periods of non-investment. The smaller rectangle illustrates a case in which g lies between g^* and $g^*/(1-\delta)$. The volatility in public good levels is smaller in this case and there is

good. Nonetheless, in the equilibrium that we study, if the initial level of the public good is positive, equilibrium public good levels are bounded above and thus the voter's lifetime spending on the public good is bounded above.

investment in each period.

If the public good level starts out below g^*, the initial period's investment will put the level above $g^*/(1-\delta)$ and a number of periods with no investment will follow until it falls below $g^*/(1-\delta)$. After that point, the level is in the interval $[g^*, g^*/(1-\delta)]$ and will enter a two cycle. If the public good level starts out above $g^*/(1-\delta)$, a number of periods with no investment will follow until it falls below $g^*/(1-\delta)$. After that point, it will again begin a two cycle.

With these simple dynamics, it is relatively straightforward to compute the bureaucrat's value function. Moreover, it is possible to show that he cannot in fact gain by holding back investment. This allows us to prove:

Proposition 6 *With CRRA public good benefits, if $\sigma = 2$ there exists a Romer-Rosenthal equilibrium with bureaucrat strategy given by (26) where g^* is defined by (25). In this equilibrium, the public good converges to a deterministic two period cycle in which for some $g \in [g^*, g^*/(1-\delta)]$ the public good takes on the value $(g^*)^2 / [(1-\delta)\, g]$ one period and g the next. The exact value of g depends on the initial condition.*

Proof See the On-line Appendix. ■

In equilibrium, the public good level is always at least as big as g^*. Moreover, it is clear from comparing (22) and (25), that g^* exceeds the first best level.[28] Thus, the public good is over-provided in equilibrium as in the quadratic case. For this specification, the closer is the discount rate β to the depreciation rate $1-\delta$ the greater will be the degree of over-provision. Thus, *ceteris paribus*, raising β or δ will increase over-provision. In contrast to the quadratic case, however, there is no upper bound on the degree of over-provision.[29]

As in the quadratic case, it is easy to find parameter values such that $(1-\delta)\, g^*$ exceeds g_s^o, so that the voter is approving investment in every period even though the public good level that would arise in the next period if the investment were not undertaken exceeds his first best level.[30]

An important difference from the quadratic case is that the equilibrium displays a new type of inefficiency. Specifically, steady state public good provision is Pareto inefficient (as opposed to just the dynamic path) because of the volatility created by the cycle. The concavity of the voter's

[28] When comparing (22) and (25), recall that we have set $c = 1$.

[29] This is also true in the static agenda setter model with CRRA public good benefits.

[30] The necessary and sufficient condition is that $\beta[1-(1-\delta)^4]$ exceeds $(1-\delta)\,[1-(1-\delta)^2]$.

preferences means that he would be strictly better off with a constant level of the public good as arises in the steady state of the quadratic case.

The findings that the public good displays volatility in the long run and that the extent of this volatility depends on the initial condition strike us as surprising. They suggest that communities with very similar underlying environments could end up with quite different patterns of public good provision. The only commonality would be volatility and inefficiency. Naturally, these findings raise the question of what will happen for other values of σ.

5.2 The general CRRA case

We now study the model numerically for different values of σ. We consider values of σ in the set $\{1, 2, 3, 4, 5\}$. As a benchmark, we set the discount rate β equal to 0.95. We are constrained in our choice of δ by the required existence condition that $\beta(1-\delta)^{1-\sigma}$ be less than 1. With our choice of β, this condition requires that δ is less than 0.0127 when σ equals 5. Accordingly, we set δ equal to 0.0126.[31]

Note that as we vary σ, we will change the voter's optimal public good level g^*. Substituting (24) into (4) and solving, we have:

$$g^* = \left(\frac{\beta b}{1 - \beta(1-\delta)^{1-\sigma}} \right)^{\frac{1}{\sigma}}. \tag{27}$$

These changes in g^* shift the whole equilibrium and makes the cases difficult to compare. To circumvent this problem, as we vary σ, we will adjust the preference parameter b to keep g^* equal to 1. This assumption has the added bonus of simplifying equation (5) which becomes:

$$\frac{(g')^2 - ((1-\delta)g)^{1-\sigma}}{1-\sigma} = g' - (1-\delta)g. \tag{28}$$

5.2.1 Solution procedure

The solution procedure consists of three steps. The first is to solve equation (28) to obtain the bureaucrat's strategy in a Romer-Rosenthal equilibrium. The strategies for our five different values of σ are illustrated in Figure 7. As expected, the effect of raising σ is to make the bureaucrat's strategy more convex.

[31] We can raise δ by lowering β. When we do this, there is little qualitative change in the results.

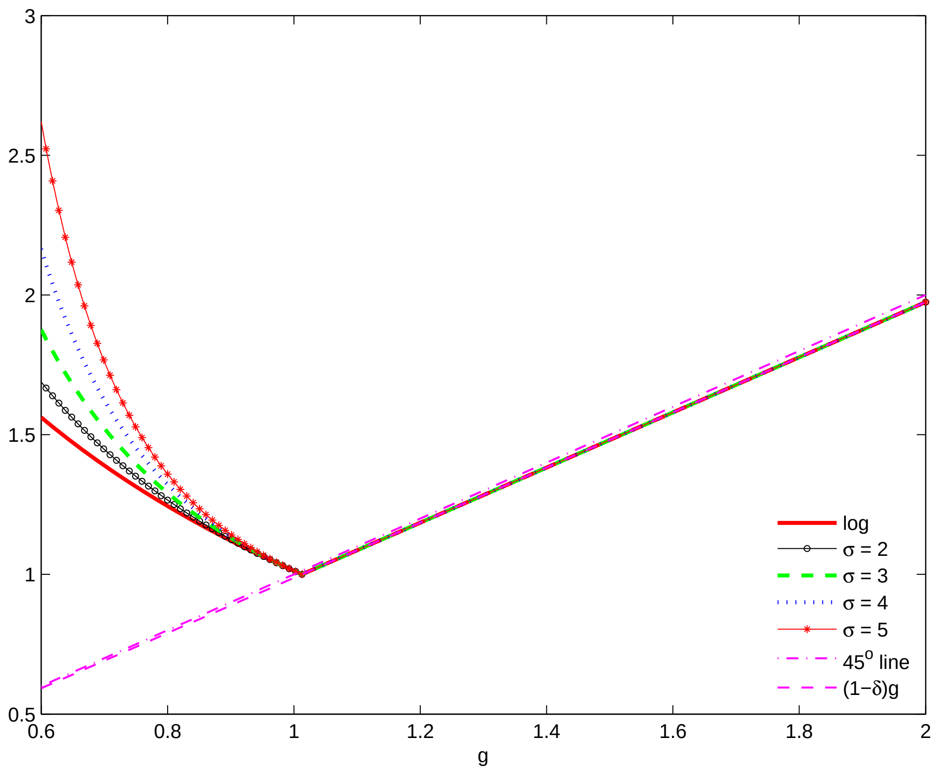

Figure 7: Bureaucrat's strategies, various σ

Equipped with the bureacrat's strategy, the second step is to construct the bureaucrat's value function. We do this by approximating the infinite sum in equation (12) by the finite sum

$$\widetilde{U}(g_0) = \sum_{t=0}^{T} \beta^t g_t,$$

where T is equal to $10,000$. These value functions are illustrated in Figure 8 for the cases σ equal to 1 and 2. The shape is similar for the higher values of σ but it is not helpful to plot the functions on the same diagram because of scaling issues. The key point to note is that the value functions are U-shaped. This reflects the fact that at low levels of the public good, increasing the level reduces the amount of investment the bureaucrat can successfully propose by an amount sufficient to offset the immediate consumption benefits.

The final step is to see whether when g is less than $g^*/(1-\delta)$, the bureaucrat is indeed better off choosing a public good level g' that satisfies (28) rather than a smaller level. This requires seeing

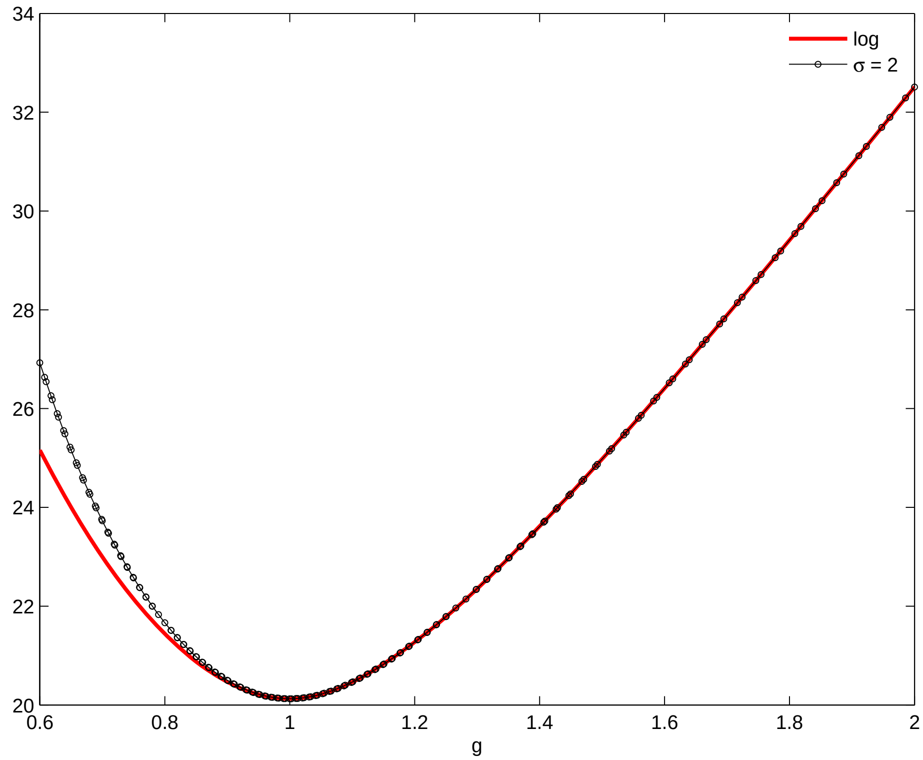

Figure 8: Bureaucrat's value functions, $\sigma = 1$ & 2

if the bureaucrat's value function takes on a value at $g'(g)$ at least as large as the value for any g' in the interval $[(1-\delta)g, g'(g)]$. Given the shape of the bureaucrat's value function as illustrated in Figure 8, it suffices to compare the function at the two endpoints of the interval $(1-\delta)g$ and $g'(g)$.[32] To do this, we construct a large grid of points in the interval $(0, g^*/(1-\delta)]$[33] and, for each point in this grid, compare $\widetilde{U}(g'(g))$ and $\widetilde{U}((1-\delta)g)$. We conclude that a Romer-Rosenthal equilibrium exists if and only if at all points this difference is non-negative.

[32] For those readers uncomfortable with this diagrammatic argument, we have checked numerically that the bureaucrat's value function $\widetilde{U}(g)$ is convex.

[33] We use the grid

$$\left\{10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, \left\{\frac{i}{(1000)(1-\delta)}\right\}_{i=1}^{1000}\right\}.$$

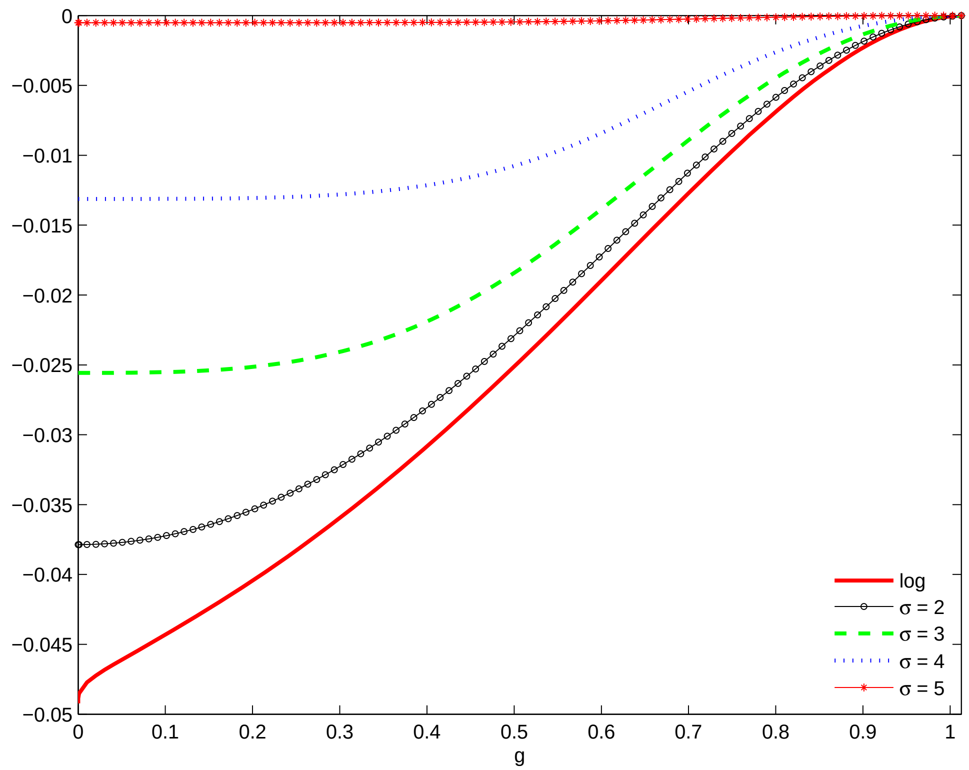

Figure 9: Gains from deviation, % of $U(g)$

5.2.2 Results

Our first question is does the Romer-Rosenthal equilibrium exist whenever $\beta(1-\delta)^{1-\sigma}$ is less than 1? The answer is yes. Figure 9 graphs the normalized gain from deviating

$$\frac{\beta\left(\widetilde{U}((1-\delta)g)-\widetilde{U}(g'(g))\right)}{\widetilde{U}(g)}, \tag{29}$$

for the different σ values. As is clear, these gains are always non-negative. Interestingly, the losses from deviating are higher at smaller levels of the public good. A similar picture emerges for every example we have tried which satisfies the necessary condition that $\beta(1-\delta)^{1-\sigma}$ is less than 1. This result suggests that the Romer-Rosenthal equilibrium concept is, in general, a useful one for our dynamic agenda setter model.

Our second question is what steady state dynamics arise under the different values of σ? Our findings here are more nuanced. Proposition 6 already establishes that when σ is equal to 2 the public good converges to a deterministic two period cycle which depends on the initial condition. By contrast, when σ is equal to 1, the public good level converges to a constant level as in the quadratic case.[34] When σ is equal to 3, 4, or 5, public good levels "almost converge" to a two cycle in which the public good alternates between g^* and $g^*/(1-\delta)$. However, convergence is not complete: even in the last periods of our simulations, small differences remain comparing across two period blocks. Moreover, the nature of these differences depend upon the initial condition.

Figures 10 and 11 illustrate these findings. They display the public good levels arising in the last seven periods of a 100,000 period simulation for our five values of σ for two different initial conditions. The public good level is measured on the vertical axis and periods are measured on the horizontal. The dashed horizontal line represents $g^*/(1-\delta)$ (i.e., $1/(1-0.0126)$). Above this line, no investment takes place. The solid lines connect the equilibrium public good levels and thus illustrate the dynamic path. Figure 10 assumes the initial public good level is g^* and Figure 11 assumes that for each σ the initial level is just below the level at which the bureaucrat's strategy crosses the 45^o line (i.e., $g'(g) = g$).

In the case illustrated in Figure 10, the differences between the long run dynamics when $\sigma \in \{2, 3, 4, 5\}$ are so small as to be imperceptible. It looks as if in all cases the public good level follows a two cycle with a value of 1 one period and $1/(1-\delta)$ the next. This is roughly speaking true, but the Figure obscures some details. In particular, when $\sigma \in \{3, 4, 5\}$, in periods $99,994$ and $99,998$ the public good level slightly exceeds $1/(1-\delta)$, while in periods $99,996$ and $10,000$ it is slightly below $1/(1-\delta)$. Moreover, the public good levels in periods $99,994$ and $99,998$ and periods $99,996$ and $100,000$ very slightly differ, so that the system has not fully converged. Finally, comparing across the different values of σ, the public good levels in each period differ slightly so that σ matters.

In the case illustrated in Figure 11, the difference between the case $\sigma = 2$ and the cases $\sigma \in \{3, 4, 5\}$ is more apparent. This is because our choice of initial condition is such as to make the two cycle almost collapse when $\sigma = 2$. In the other cases, the paths again look as if the public good level reaches a two cycle with a value of 1 one period and $1/(1-\delta)$ the next. Again, the Figure obscures some details. In particular, when $\sigma \in \{3, 4, 5\}$, in periods $99,994$ and $99,998$ the public good level slightly exceeds $1/(1-\delta)$, while in periods $99,996$ and $10,000$ it is slightly below $1/(1-\delta)$. Moreover, the public good levels in periods $99,994$ and $99,998$ and periods $99,996$ and

[34] We have also verified that this is the case for values of σ lying between 1 and 2.

100,000 very slightly differ, so that the system has not fully converged. Finally, comparing across the two Figures, there are slight differences in the public good levels in each period illustrating that the initial condition matters.

Insight into these findings can be obtained by understanding the bureaucrat strategies for the different values of σ. The first point to note is that these strategies are almost identical on the interval $[1, 1/(1-\delta)]$ (see Figure 7). For all values of σ, $g'(1/(1-\delta))$ is equal to 1 and $g'(1)$ is approximately equal to $1/(1-\delta)$, the value it takes on when σ is equal to 2. When σ is less than 2, $g'(1)$ is slightly less than $1/(1-\delta)$, and when σ exceeds 2, $g'(1)$ is slightly greater than $1/(1-\delta)$. The second point to note is that when $\sigma = 2$ the slope of the bureaucrat's strategy at the point at which it crosses the 45^o line is exactly -1 (this can be verified from (26)). This means that when σ equals 1 the slope at the intersection point is greater than -1 and thus this point is a locally stable steady state. This in turn implies that the dynamics force the public good level towards the steady state point. By contrast, when $\sigma \in \{3, 4, 5\}$, the slope at the intersection point is less than -1 and therefore an unstable steady state. The public good level is therefore forced to the boundaries of the interval $[1, 1/(1-\delta)]$. However, since investment stops once the public good level exceeds $1/(1-\delta)$ and $g'(1)$ is only infinitessimally higher than $1/(1-\delta)$, it does not stray too far from these boundary points.

Our final question is how do the steady state public good levels compare with the optimal levels? Our findings here are in line with our analytical examples. Under the normalizing choice of the preference parameter b, the first best level of the public good described in equation (22) is decreasing in σ. It varies from 0.806 when σ is equal to 1 to 0.392 when σ is equal to 5. The lowest public good level arising in equilibrium is approximately 1. Thus, the public good level is always overprovided and the degree of overprovision is higher the greater is the concavity of the benefit function. Given that $(1-\delta)\, g^*$ is equal to 0.9874, the equilibrium reversion levels comfortably exceed the first best level of the public good in all cases.

6 Conclusion

This paper has explored the provision of a durable public good in Romer and Rosenthal's agenda setter model. It has identified a type of equilibrium, called a Romer-Rosenthal equilibrium, in which in every period the bureaucrat proposes the maximum level of public investment the voter will support. This is distinct from an equilibrium in which the bureaucrat holds back investment

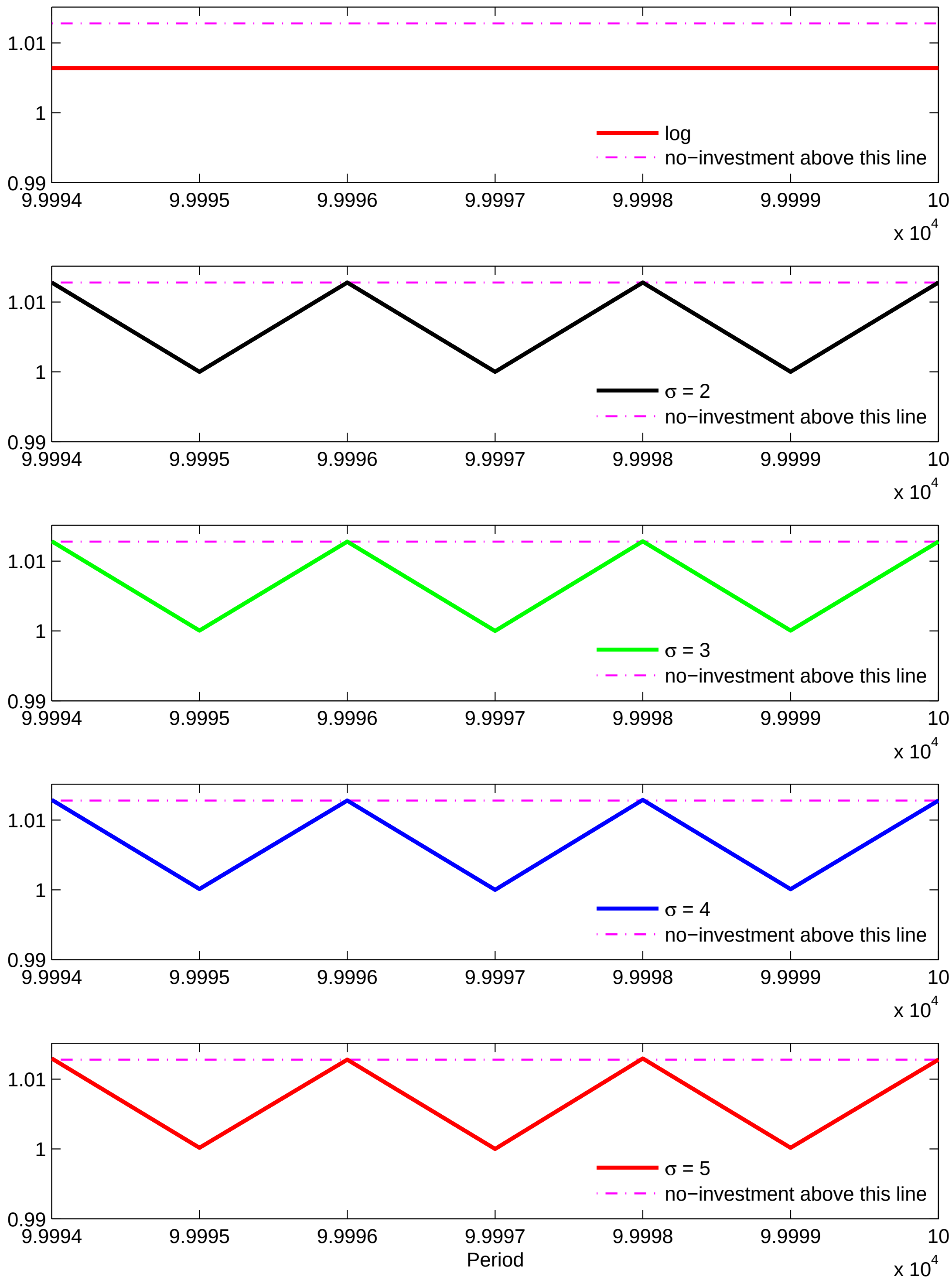

Figure 10: Convergence, starting at g^*

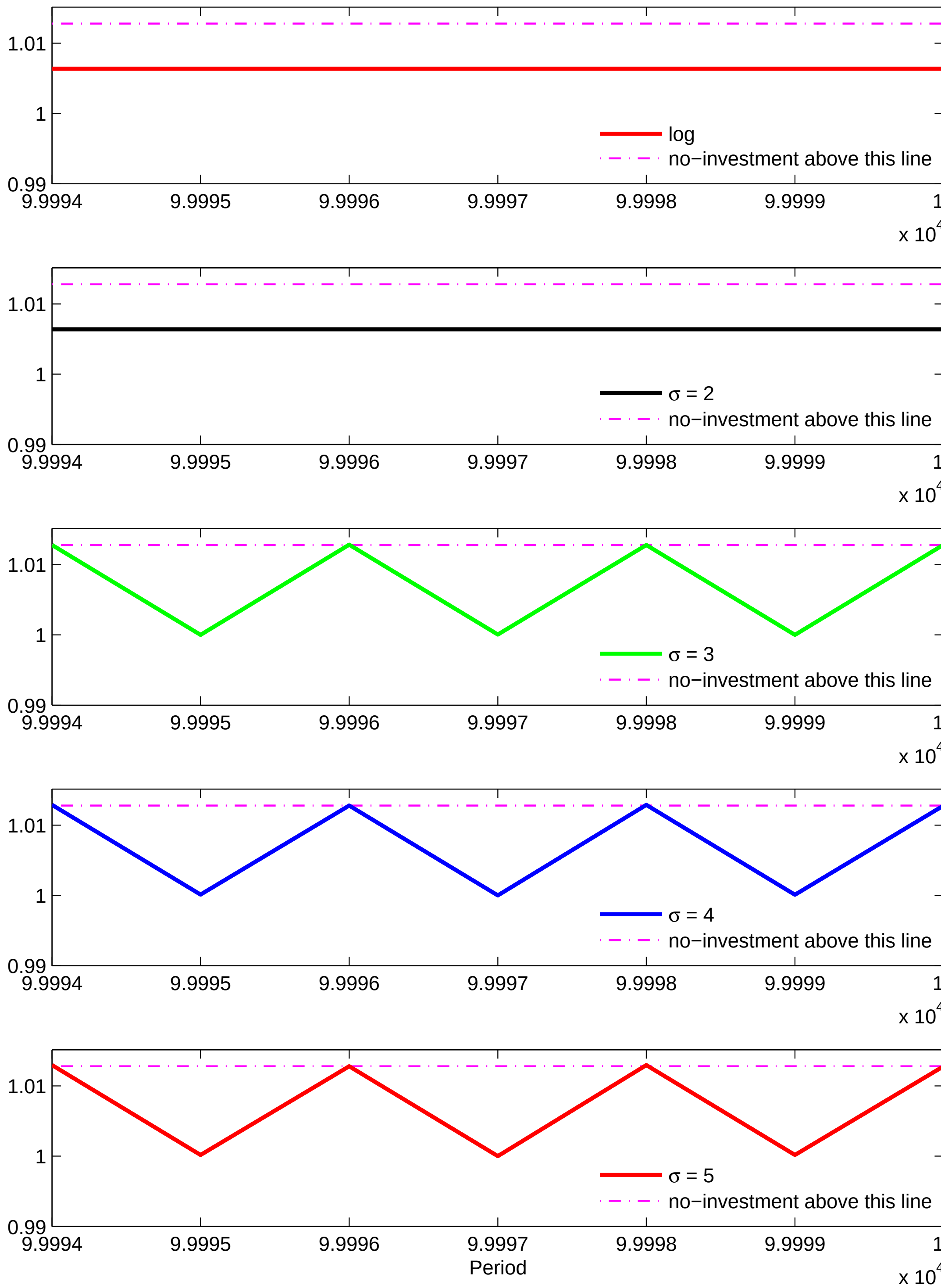

Figure 11: Convergence, starting just below $g'(g) = g$

today in order to obtain support for a higher level tomorrow. The paper has established that such an equilibrium exists for a variety of public good benefit functions. Equilibrium public good levels converge or almost converge to a steady state. These steady states can involve a unique public good level being provided each period or may exhibit a two period cycle. Steady state public good levels exceed the voter's optimal level.

The dynamic equilibrium exhibits two interesting features that distinguish it from the familiar lessons of the static model. The first feature is that equilibrium reversion levels in the steady state can actually be higher than the voter's first best steady state level. Thus, the voter repeatedly approves the bureaucrat's investment proposals, even though the public good levels that would arise without them exceed the voter's first best level! This reflects the fact that the marginal value of public goods is higher in equilibrium because they reduce exploitation by the bureaucrat. The important practical implication of this is that equilibrium reversion levels cannot be used as lower bounds for the voter's first best level. The second distinctive feature is that the equilibrium path of public investment is Pareto inefficient in the sense that there exists an alternative investment path under which both the bureaucrat and the voter are better off. The inefficiency reflects the inability of the agents to commit to their future proposing and voting behavior.

There is much more that could be done with this model. It would be worthwhile adding uncertainty into voters' preferences so that sometimes the bureaucrat's proposal would fail. This is a useful step in moving the model towards something that might guide empirical work. Persistent preference shocks would be interesting because then failure would signal a permanent shift in preferences. This may result in the scaling back of future proposals, which seems consistent with what happens when school bond issues fail. Another important extension would be to allow for more general preferences on the part of the bureaucrat than budget maximization. One could then explore how different degrees of agency problems between voters and bureaucrats would be manifested in the dynamics of equilibrium policies. Finally, to capture the idea that new investment might be financed from sources other than bonds, it might be interesting to allow two different ways of financing new investment only one of which requires voter approval.

References

Acemoglu, Daron, 2003, "Why not a Political Coase Theorem? Social Conflict, Commitment and Politics," *Journal of Comparative Economics*, 31, 620-652.

Alesina, Alberto and Guido Tabellini, 1990, "A Positive Theory of Fiscal Deficits and Government Debt," *Review of Economic Studies*, 57, 403-414.

Baron, David, 1996, "A Dynamic Theory of Collective Goods Programs," *American Political Science Review*, 90, 316-330.

Baron, David and Michael Herron, 2003, "A Dynamic Model of Multidimensional Collective Choice," in *Computational Models in Political Economy*, ed. Ken Kollman, John Miller and Scott Page, Cambridge, MA: MIT Press.

Battaglini, Marco and Stephen Coate, 2007, "Inefficiency in Legislative Policy-Making: A Dynamic Analysis," *American Economic Review*, 97, 118-149.

Battaglini, Marco and Thomas Palfrey, 2012, "The Dynamics of Distributive Politics," *Economic Theory*, 49, 739-777.

Battaglini, Marco, Nunnari, Salvatore and Thomas Palfrey, 2012, "Legislative Bargaining and the Dynamics of Public Investment," *American Political Science Review*, 106, 407-429.

Bergstrom, Ted, 1979, "When Does Majority Rule Supply Public Goods Efficiently?" *Scandinavian Journal of Economics*, 81, 216-226.

Besley, Timothy and Stephen Coate, 1998, "Sources of Inefficiency in a Representative Democracy: A Dynamic Analysis," *American Economic Review*, 88, 139-156.

Bowen, Howard, 1943, "The Interpretation of Voting in the Allocation of Economic Resources," *Quarterly Journal of Economics*, 58, 27-48.

Bowen, Renee, Chen, Ying and Hulya Eraslan, 2014, "Mandatory Versus Discretionary Spending: the Status Quo Effect," *American Economic Review*, forthcoming.

Bowen, Renee and Zaki Zahran, 2012, "On Dynamic Compromise," *Games and Economic Behavior*, 76, 391-419.

Cellini, Stephanie, Ferreira, Fernando and Jesse Rothstein, 2010, "The Value of School Facility Investments: Evidence from a Dynamic Discontinuity Design," *Quarterly Journal of Economics*, 125, 215-261.

Coate, Stephen, 2013, "Evaluating Durable Public Good Provision using Housing Prices," unpublished manuscript.

Diermeier, Daniel and Pohan Fong, 2011, "Legislative Bargaining with Reconsideration," *Quarterly Journal of Economics*, 126, 947-985.

Hamilton, Howard and Sylvan Cohen, 1974, *Policy Making by Plebiscite: School Referenda*, Lexington Books: Lexington, MA.

Ingberman, Daniel, 1985, "Running Against the Status Quo: Institutions for Direct Democracy Referenda and Allocations over Time," *Public Choice*, 46, 19-43.

Kalandrakis, Anastassios, 2004, “A Three-Player Dynamic Majoritarian Bargaining Game,” *Journal of Economic Theory*, 116, 294-322.

Leblanc, William, Snyder, James and Mickey Tripathi, 2000, “Majority-Rule Bargaining and the Under Provision of Public Investment Goods,” *Journal of Public Economics*, 75, 21-47.

Lizzeri, Alessandro and Nicola Persico, 2001, “The Provision of Public Goods under Alternative Electoral Incentives,” *American Economic Review*, 91, 225-239.

Lizzeri, Alessandro and Nicola Persico, 2005, “A Drawback of Electoral Competition,” *Journal of the European Economic Association*, 3, 1318-1348.

Niskanen, William, 1971, *Bureacracy and Representative Government*, Chicago: Aldine-Atherton.

Persson, Torsten and Lars Svensson, 1989, “Why a Stubborn Conservative would Run a Deficit: Policy with Time-Inconsistent Preferences,” *Quarterly Journal of Economics*, 104, 325-345.

Riboni, Alessandro, 2010, “Committees as Substitutes for Commitment,” *International Economic Review*, 51, 213-236.

Romer, Thomas and Howard Rosenthal, 1978, “Political Resource Allocation, Controlled Agendas, and the Status Quo,” *Public Choice*, 33, 27-43.

Romer, Thomas and Howard Rosenthal, 1979a, “The Elusive Median Voter,” *Journal of Public Economics*, 12, 143-170.

Romer, Thomas and Howard Rosenthal, 1979b, “Bureacrats versus Voters: On the Political Economy of Resource Allocation by Direct Democracy,” *Quarterly Journal of Economics*, 93, 563-587.

Romer, Thomas and Howard Rosenthal, 1982, “Median Voters or Budget Maximizers: Evidence from School Expenditure Referenda,” *Economic Inquiry*, 20, 556-578.

Romer, Thomas, Rosenthal, Howard and Vincent Munley, 1992, “Economic Incentives and Political Institutions: Spending and Voting in School Budget Referenda,” *Journal of Public Economics*, 49, 1-33.

Sydsaeter, Knut, 1981, *Topics in Mathematical Analysis for Economists*, London: Harcourt Brace Jovanovich.

Volden, Craig and Alan Wiseman, 2007, “Bargaining in Legislatures over Particularistic and Collective Goods,” *American Political Science Review*, 101, 79-92.

7 Appendix

7.1 Bureaucrat's strategy with quadratic benefits

For all $g \leq g^*/(1-\delta)$ we need to find the solution to (5) in the quadratic benefit case. From (9), we know that

$$\beta\left[V(g') - V(g(1-\delta))\right] = \left(\frac{\beta b_0}{1-\beta(1-\delta)}\right)(g' - g(1-\delta)) - \left(\frac{\beta b_1}{1-\beta(1-\delta)^2}\right)\left(g'^2 - g^2(1-\delta)^2\right).$$

Substituting this into (5) and rearranging, we find that

$$\begin{aligned}\left(\frac{\beta b_0 - c\,(1-\beta(1-\delta))}{1-\beta(1-\delta)}\right)(g' - g(1-\delta)) &= \left(\frac{\beta b_1}{1-\beta(1-\delta)^2}\right)\left[(g')^2 - g^2(1-\delta)^2\right] \\ &= \left(\frac{\beta b_1}{1-\beta(1-\delta)^2}\right)(g' - g(1-\delta))(g' + g(1-\delta)).\end{aligned}$$

Dividing both sides through by $g' - g(1-\delta)$ we obtain

$$\frac{\beta b_0 - c\,(1-\beta(1-\delta))}{1-\beta(1-\delta)} = \left(\frac{\beta b_1}{1-\beta(1-\delta)^2}\right)(g' + g(1-\delta)).$$

Solving this equation yields

$$g'(g) = \left(\frac{\frac{\beta b_0 - c(1-\beta(1-\delta))}{1-\beta(1-\delta)}}{\frac{\beta b_1}{1-\beta(1-\delta)^2}}\right) - g(1-\delta).$$

Using (10), we can write this as

$$g'(g) = 2g^* - (1-\delta)g.$$

We can therefore conclude that

$$g'(g) = \begin{cases} 2g^* - (1-\delta)g & \text{if } g \leq g^*/(1-\delta) \\ g(1-\delta) & \text{if } g > g^*/(1-\delta) \end{cases},$$

which is (11). ■

7.2 Proof of Proposition 3

Given the discussion in the text, it sufficies to show that the over-provision magnitude in equation (19) is increasing in β and δ. The over-provision magnitude is

$$\frac{\delta/2(1+\beta(1-\delta))}{(1-\delta/2)\,(1-\beta(1-\delta))}.$$

It is clear from inspection that this is increasing in β. Differentiating with respect to δ, we obtain

$$\frac{\partial\left(\frac{\delta/2(1+\beta(1-\delta))}{(1-\delta/2)(1-\beta(1-\delta))}\right)}{\partial\delta}=\frac{2\left(1-\beta\right)}{\left[\left(2-\delta\right)\left(1-\beta(1-\delta)\right)\right]^{2}}.$$

This is positive as required. ■

7.3 Proof of Proposition 4

The equilibrium reversion level is $(1-\delta)g_s$ and the first best level is g_s^o. We first show that $(1-\delta)g_s$ exceeds g_s^o when (20) is satisfied. Using (17) and (13), we see that the equilibrium and optimal steady states are related by the following equation

$$g_s=g_s^o\left(\frac{1-\beta(1-\delta)^2}{\left(1-\delta/2\right)\left(1-\beta(1-\delta)\right)}\right).$$

It follows that

$$(1-\delta)g_s=g_s^o\left(\frac{\left(1-\delta\right)\left(1-\beta(1-\delta)^2\right)}{\left(1-\delta/2\right)\left(1-\beta(1-\delta)\right)}\right).$$

Thus, $(1-\delta)g_s$ exceeds g_s^o when

$$\left(1-\delta\right)\left(1-\beta(1-\delta)^2\right)>\left(1-\delta/2\right)\left(1-\beta(1-\delta)\right)$$

or equivalently when

$$\left(1-\delta\right)\left(1-\beta(1-\delta)^2\right)>\left(1-\delta\right)\left(1-\beta(1-\delta)\right)+\delta/2\left(1-\beta(1-\delta)\right)$$

This in turn is equivalent to

$$\left(1-\delta\right)\left(\beta(1-\delta)\delta\right)>\delta/2\left(1-\beta(1-\delta)\right)$$

which simplifies to

$$\left[2(1-\delta)+1\right]\beta(1-\delta)>1,$$

as required.

We next show that for all $\beta>1/3$, there exists $\delta(\beta)\in(0,1)$ such that (20) is satisfied whenever $\delta\in(0,\delta(\beta))$. For given β, define the function

$$\varphi_\beta(\delta)=\left(2(1-\delta)+1\right)\beta(1-\delta)-1.$$

Clearly, (20) is satisfied whenever $\varphi_\beta(\delta)>0$. Observe first that

$$\varphi_\beta(0)=3\beta-1>0>-1=\varphi_\beta(1),$$

where the first inequality follows from the assumption that $\beta > 1/3$. Next observe by inspection that $\varphi_\beta(\delta)$ is decreasing. By continuity, there exists a unique $\delta(\beta) \in (0,1)$ such that $\varphi_\beta(\delta(\beta)) = 0$. It follows that $\varphi_\beta(\delta) > 0$ for all $\delta \in (0, \delta(\beta))$. The result follows. ■

7.4 Equilibrium welfare weight

Using (13) and (21), we see that the welfare weight on the citizen that makes the optimal steady state $g_s^o(\lambda)$ equal to the equilibrium steady state g_s satisfies the equation

$$\frac{(\beta b_0 - c(1-\beta(1-\delta)))\left(1-\beta(1-\delta)^2\right)}{(2-\delta)\beta b_1(1-\beta(1-\delta))} = \frac{\beta(\lambda b_0 + (1-\lambda)) - \lambda c(1-\beta(1-\delta))}{\lambda 2\beta b_1}.$$

Solving this for λ implies that

$$\lambda = \frac{\beta(2-\delta)(1-\beta(1-\delta))}{\left[\begin{array}{c} 2(\beta b_0 - c(1-\beta(1-\delta)))\left(1-\beta(1-\delta)^2\right) \\ +(2-\delta)(1-\beta(1-\delta))c(1-\beta(1-\delta)) + (2-\delta)(1-\beta(1-\delta))\beta(1-b_0) \end{array}\right]}$$

Collecting terms in the denominator, we can write this more simply as

$$\lambda = \frac{\beta(2-\delta)(1-\beta(1-\delta))}{\delta(\beta(1-\delta)+1)(\beta b_0 - c(1-\beta(1-\delta))) + \beta(2-\delta)(1-\beta(1-\delta))},$$

which is the expression in footnote 20. ■

7.5 Bureaucrat's strategy with CRRA benefits when $\sigma = 2$

When $\sigma = 2$, equation (24) tells us that the voter's value function is

$$V(g) = -\frac{b}{g\left(1-\frac{\beta}{1-\delta}\right)}.$$

From (6) when g is less than $g^*/(1-\delta)$, the strategy $g'(g)$ is such that

$$-(g'(g)-(1-\delta)g) - \frac{\beta b}{g'(g)(1-\frac{\beta}{1-\delta})} = -\frac{\beta b}{(1-\delta)g(1-\frac{\beta}{1-\delta})}.$$

Multiplying through by $g'(g)$ and rearranging yields the following quadratic equation

$$g'(g)^2 - g'(g)\left[(1-\delta)g + \frac{\beta b}{(1-\delta)g(1-\frac{\beta}{1-\delta})}\right] + \frac{\beta b}{(1-\frac{\beta}{1-\delta})} = 0.$$

This has solution

$$g'(g) = \frac{\beta b}{(1-\delta)g(1-\frac{\beta}{1-\delta})} = \frac{(g^*)^2}{(1-\delta)g},$$

which implies (26).