Fiscal Policy over the Real Business Cycle: A Positive Theory[1]

Levon Barseghyan
Department of Economics
Cornell University
Ithaca NY 14853
lb247@cornell.edu
607-255-6284

Marco Battaglini
Department of Economics
Princeton University
Princeton NJ 08544
mbattagl@princeton.edu
609-258-4002

Stephen Coate
Department of Economics
Cornell University
Ithaca NY 14853
sc163@cornell.edu
607-255-1912

Revised May 2013

[1]Corresponding author: Marco Battaglini. We thank Andrea Civelli and Woong Yong Park for outstanding research assistance. For helpful comments, we also thank two anonymous referees, Christian Hellwig, Ulrich Muller, Christopher Sims, Aleh Tsyvinski, Ivan Werning and seminar participants at the 5th Banco de Portugal Conference on Monetary Economics, University of Copenhagen, CORE, ECARES, Einaudi Institute for Economics and Finance, Erasmus Universiteit Rotterdam, Georgetown, Johns Hopkins, ITAM, University of Miami, NBER, Princeton, the LAEF conference at UCSB, Simon Fraser, the X Workshop in International Economics and Finance at Universidad Torcuato Di Tella, and York University. Battaglini gratefully acknowledges the hospitality of the Einaudi Institute for Economics and Finance while working on this paper.

Abstract

This paper explores the implications of the political economy model of Battaglini and Coate (2008) for the behavior of fiscal policy over the business cycle. The model predicts that fiscal policy is counter-cyclical with debt increasing in recessions and decreasing in booms. Public spending increases in booms and decreases during recessions, while tax rates decrease during booms and increase in recessions. In both booms and recessions, fiscal policies are set so that the marginal cost of public funds obeys a submartingale. When calibrated to the U.S. economy, the model broadly matches the empirical distribution of debt and also its negative correlation with output. However, the predictions of pro-cyclical spending and counter-cyclical taxation do not find empirical support. The calibrated model generates the same fit of the data as a benevolent government model in which the government faces an exogenous lower bound on debt. Nonetheless, the two models have very different comparative static implications.

JEL classification: E62, H60, H62

Keywords: fiscal policy, business cycle, budget deficit

1 Introduction

Real business cycle theory develops the idea that business cycles can be generated by random fluctuations in productivity. At the core of this research program, the fundamental issues are how individuals react to shocks and how these reactions affect the macro economy. While the issue of reaction to shocks is typically studied at the individual level, it can also be raised at the societal level. How do individuals, through their political institutions, collectively decide to adjust fiscal policies in response to shocks? Moreover, what is the role of changes in fiscal policy in amplifying or dampening shocks? Though understanding individual responses to shocks can be addressed with the tools of basic microeconomics, understanding societal responses requires a study of how collective choices are made in complex dynamic environments.

In the last two decades, political economy has made important progress, both theoretically and empirically, in understanding how governments function and the type of distortions that the political process generates in an economy. This *first generation* of research, however, has largely focused on static or two period models that are not well suited to answer the questions raised by real business cycle theory. When longer time horizons are considered, other important elements of the environment (such as shocks, rational forward-looking agents, etc) are muted. Thus, the basic question as to how governments react to business cycles is not well understood. Because of this, empirical analysis of the cyclical behavior of fiscal policy remains largely guided by normative models of policy making.

As part of a *second generation* of political economy research analyzing more general dynamic models, Battaglini and Coate (2008) propose a positive theory of fiscal policy.[1] Their framework begins with a tax smoothing model of fiscal policy of the form studied by Barro (1979), Lucas and Stokey (1983), and Aiyagari, Marcet, Sargent and Seppala (2002). The need for tax smoothing is generated by shocks in the benefits of public spending created by events like wars and natural disasters. Politics is introduced by assuming that policy choices are made by a legislature rather than a benevolent social planner. Moreover, the framework incorporates the friction that legislators can redistribute tax revenues back to their districts via pork-barrel spending. The theory yields clean predictions on how fiscal policy responds to public spending shocks and provides a sharp account of how politics distorts economic policy-making.

This paper explores the implications of the Battaglini-Coate theory for the behavior of fiscal policy over the business cycle. The paper has three parts. The first develops the theoretical predictions of the model. This involves replacing public spending shocks with revenue shocks generated by random fluctuations in the economy's productivity. These productivity shocks are assumed persistent as opposed to independent and identically distributed. Persistent shocks are

[1] Other examples of this type of work include Acemoglu, Golosov, and Tsyvinski (2008), Azzimonti (2011), Hassler et al (2003), Hassler et al (2005), Krusell and Rios-Rull (1999), Song, Zilibotti, and Storesletten (2012), and Yared (2010).

essential to capture the implications of cyclical fluctuations. When an economy enters a boom or a recession, legislators' expectations about future tax revenues will clearly be influenced and these changed expectations will impact current taxing, spending, and borrowing decisions. The second part of the paper evaluates the implications of the theory by calibrating the model to the U.S. economy. The performance of the model in explaining the debt distribution and the cyclical behavior of fiscal variables is analyzed. The third and final part compares the performance of the model with a benevolent government model in which, following the approach of Aiyagari, Marcet, Sargent and Seppala (2002), the social planner is assumed to face an exogenous lower bound on debt. The purpose of this exercise is to assess the importance of micro-founded modeling of political decision-making.

The specific environment analyzed assumes that a single good is produced using labor. This good can be consumed or used to produce a public good. Labor productivity follows a two state, serially-correlated Markov process. When productivity is high, the economy is in a "boom" and, when it is low, a "recession". Policy choices in each period are made by a legislature comprised of representatives elected by single-member, geographically-defined districts. The legislature can raise revenues by taxing labor income and by issuing one period risk-free bonds. Public revenues are used to finance public good provision and pork-barrel spending. The legislature makes policy decisions by majority (or super-majority) rule and legislative policy-making is modelled as non-cooperative bargaining.

While the incorporation of persistent shocks complicates the characterization of equilibrium, the model remains tractable. Equilibrium fiscal policies converge to a stochastic steady state in which they vary predictably over the business cycle. Upon entering a boom, public spending will increase and tax rates will fall. Over the course of the boom, public spending will continue to increase until it reaches a ceiling level, and tax rates will decrease until they reach floor levels. When the economy enters a recession, public spending will decrease and tax rates will increase. As the recession progresses, public spending will continue to decrease and tax rates will continue to increase. The overall fiscal stance as measured by the long run pattern of debt is, however, counter-cyclical: government debt decreases in booms and increases in recessions.[2]

[2] There are a number of definitions of "counter-cyclical" fiscal policy in the literature. Consistent with a Keynesian perspective, Kaminsky, Reinhart, and Vegh (2004) and Talvi and Vegh (2005) define fiscal policy to be counter-cyclical if government spending rises in recessions and tax rates fall. Adopting a neoclassical perspective, Alesina, Campante, and Tabellini (2008) define as counter-cyclical "a policy that follows the tax smoothing principle of holding constant tax rates and discretionary spending as a fraction of GDP over the cycle". Our definition is that fiscal policy is counter-cyclical if debt falls in booms and rises in recessions. Like Alesina, Campante, and Tabellini, our definition is motivated by tax smoothing principles. However, it recognizes the fact that in a world with incomplete markets and unanticipated productivity shocks, these principles do not imply constant tax rates or government spending over the cycle. While reflecting a neoclassical perspective, our definition does not discriminate between a neoclassical and

Perhaps the most interesting feature of the cyclical behavior of fiscal policy is that debt *falls* when the economy enters a boom. Intuitively, one might have guessed just the opposite. A boom will increase the expectation of future tax revenues and this may lead legislators to increase borrowing so they can appropriate these extra revenues for their districts. Indeed, this is precisely the logic of the well-known voracity effect of Tornell and Lane (1999). This intuition is correct, but ignores the fact that any increase in debt will have permanent effects. Thus, such a voracity effect-style debt expansion can arise the first time the economy moves from recession to boom, but, once this happens, the level of debt is too high for it to occur again.

In addition, the paper identifies an interesting implication of the theory concerning the dynamic evolution of the so-called marginal cost of public funds (MCPF). The MCPF, a basic concept in public finance, is the social marginal cost of raising an additional unit of tax revenue. It takes into account the distortionary costs of taxation for the economy. In the model, it depends upon the tax rate and the elasticity of labor supply. The theory implies that, at each point in time and over all phases of the cycle, the equilibrium choice of fiscal policies is such that the MCPF obeys a submartingale.[3] This means the expected MCPF next period is always at least as large as the current MCPF and is sometimes strictly larger. This prediction contrasts with that emerging from a planning model which implies that the MCPF obeys a martingale. Political distortions therefore create a wedge between the current MCPF and the future MCPF.[4]

The quantitive assessment of the model in the second part of the paper uses U.S. data from 1948 to 2011. The model is calibrated to match the empirically observed variation in output, the frequency and length of recessions, and the average debt/GDP and spending/GDP ratios. The model produces a distribution of debt which is similar to the empirically observed one. This means that debt averages around 42% of GDP, is volatile, and strongly negatively correlated with output. Both in the model and the data, the tax rate and public spending are much less volatile than debt. However, in the model both taxes and public spending are less volatile than in the data. Moreover, the predictions of a counter-cyclical tax rate and pro-cyclical public spending are not supported by the data.

The comparison with the Aiyagari, Marcet, Sargent and Seppala-style model (hereafter AMSS-style model) in the third part of the paper, begins by calibrating an exogenously constrained benevolent government model to the same data. The exogenous constraint is treated as a free

Keynesian view of optimal fiscal policy over the cycle: in both cases, government debt will rise in recessions and fall in booms. As suggested by Kaminsky, Reinhart, and Vegh (2004), the way to discriminate between these views is to look at the behavior of tax rates and public spending. We will discuss this point in greater detail below.

[3] In our model the assumptions of the standard submartingale convergence theorem are not satisfied, so the MCPF does not converge to a constant or to infinity as $t \to \infty$. Indeed, we show that in the long run the MCPF will have a non degenerate stationary distribution.

[4] This submartingale result is also true in the model of Battaglini and Coate (2008), although this has not previously been noted.

parameter which is determined as part of the calibration. While the two models are by no means theoretically equivalent, the calibrated models deliver the same fit of the data. However, the two models have starkly different comparative static implications. These differences highlight the importance of modelling the underlying political decision-making when predicting the implications of changes in the underlying economic environment.

The organization of the remainder of the paper is as follows. The next section places the paper in the context of the prior literature on the cyclical behavior of fiscal policy. Section 3 outlines the model and establishes a benchmark by describing socially optimal fiscal policies. Section 4 presents the theoretical results. Section 5 provides the quantitative assessment of the model. Section 6 compares the performance of the model with an AMSS-style model and Section 7 concludes.

2 Related literature

There is a large literature on the cyclical behavior of fiscal policy, with both theoretical and empirical branches. The benchmark theoretical model used in the literature is the tax smoothing model with perfect foresight (Barro (1979)). In this model, deterministic and perfectly anticipated cyclical variation in the economy generates fluctuations in tax revenues. The government smooths tax rates and public spending by borrowing in recessions and repaying in booms (see, for example, Talvi and Vegh (2005)). Thus, debt is negatively correlated with changes in GDP, while public spending and tax rates are uncorrelated with changes in GDP.

Support for the predictions of this model comes from Barro (1986) who studies the correlation between debt and income changes for the U.S. federal government. Using data from the period 1916-1982, he finds a negative correlation between changes in debt and changes in GNP.[5] Studies of the correlation between public spending and GDP provide more mixed support.[6] The basic findings are that public spending tends to be slightly pro-cyclical for developed economies, and much more

[5] Barro runs regressions of the form $(b_t - b_{t-1})/y_t = \alpha \cdot X_t + \beta yvar_t + \varepsilon_t$,where b_t is debt, y_t is GNP, X_t is a vector of control variables, $yvar_t$ is a business cycle indicator, and ε_t is a shock. The business cycle indicator takes on negative values during a boom and positive values during a recession. He finds that the coefficient β is positive, suggesting that debt behaves counter-cyclically.

[6] The correlation between government consumption (which excludes transfers and debt interest payments) and changes in GDP has been studied extensively for the U.S. both at the federal and state level, and for different groups of countries aggregated according to geographical location and stage of economic development. Gavin and Perotti (1997) compare a sample of Latin American countries with a sample of industrialized countries. Sorensen, Wu, and Yosha (2001) study the U.S. states. Lane (2003) looks at all the OECD countries. Alesina, Campante, and Tabellini (2008), Kaminsky, Reinhart, and Vegh (2004), Talvi and Vegh (2005), and Woo (2009) look at data sets containing a broad sample of developed and developing countries.

pro-cyclical for developing countries.[7] These findings have been interpreted as suggesting that fiscal policy is basically consistent with the perfect foresight tax smoothing model in developed countries and inconsistent in developing countries.

A variety of theories have been advanced to explain the stronger pro-cyclical behavior of government spending in developing countries. In an early attempt to explain the phenomenon, Gavin and Perotti (1997) note that pro-cyclical policies may be induced by tighter debt constraints in recessions. Borrowing limits in recessions would force contractionary policies; as the limits are relaxed in booms, we would observe expansionary policies. Other authors point to the dysfunctional political systems that pervade developing countries. In a dynamic common pool framework in which multiple groups compete for a share of the national pie, Lane and Tornell (1998) and Tornell and Lane (1999) suggest that group competition can increase following a positive income shock which may lead spending to increase more than proportionally to the increase in income - the *voracity effect*. In the context of a perfect foresight tax smoothing model, Talvi and Vegh (2005) show that if spending pressures increase with the size of the primary surplus, then optimal fiscal policy will imply a pro-cyclical pattern of spending. In a political agency framework, Alesina, Campante and Tabellini (2008) show that when faced with corrupt governments whose debt and consumption choices are hard to observe, citizens may rationally demand higher public spending in a boom.

The theory developed here is complementary to the political economy theories of Lane and Tornell and Alesina, Campante, and Tabellini. They are interested in modelling different, and much more dysfunctional, political systems than modelled by Battaglini and Coate. As noted in the introduction, this paper's theory predicts that a voracity effect-style debt expansion can arise the first time the economy moves from recession to boom. However, it differs from Lane and Tornell's work in that the economy is subject to recurrent cyclical shocks rather than a one time permanent shock that is either unforeseen (in the sense that it has zero ex ante probability) or perfectly anticipated at time zero. This accounts for the conclusion that the voracity effect does not arise in the long run.

A deeper problem with models of tax smoothing in which shocks are deterministic and perfectly anticipated is that its predictions are not robust to relaxing the assumption of perfect foresight. Under the more palatable assumption that cyclical variations are stochastic and therefore not perfectly foreseen, the tax smoothing approach can have trouble explaining cyclical fiscal policy in the long run. Specifically, in environments with incomplete markets, the approach can imply that the government should self-insure, eventually accumulating sufficient assets to finance government spending out of the interest earnings from these assets (Aiyagari, Marcet, Sargent and Seppala (2002)).[8] In this case, the model predicts no long run cyclical pattern in debt, taxes, or public

[7] See, in particular, Alesina, Campante, and Tabellini (2008), Gavin and Perotti (1997), Kaminsky, Reinhart, and Vegh (2004), Talvi and Vegh (2005), and Woo (2009).

[8] Different conclusions arise when there are complete markets and the government can issue state-contingent debt.

spending. As we explain in Section 3, this is the case in our environment.

In a world in which spending shocks drive fiscal policy, Aiyagari, Marcet, Sargent and Seppala (2002) show that the tax smoothing model can generate plausible fiscal dynamics if the government faces an upper bound on how many assets it can accumulate. Their paper therefore suggests that the tax smoothing model may be revived by simply adding on an exogenous lower bound on debt. Battaglini and Coate (2008) link their theory to this idea by showing that the political equilibrium solves a constrained planning problem in which the planner faces a lower bound on debt, a lower bound on taxes, and an upper bound on public good spending. However, these bounds are endogenous and depend on the economic and political environment. This paper shows that the lessons from Battaglini and Coate carry over to the business cycle context. The only difference is that with persistent shocks, the lower bound on debt depends on the shock. This naturally raises the question of how the political economy model performs relative to a tax smoothing model with an exogenous lower bound and motivates the comparison presented in Section 6.

3 Preliminaries

3.1 The model

3.1.1 The economic environment

A continuum of infinitely-lived citizens live in n identical districts indexed by $i = 1, ..., n$. The size of the population in each district is normalized to be one. There is a single (nonstorable) consumption good, denoted by z, that is produced using a single factor, labor, denoted by l, with the linear technology $z = wl$. There is also a public good, denoted by g, that can be produced from the consumption good according to the linear technology $g = z/p$.

Citizens consume the consumption good, benefit from the public good, and supply labor. Each citizen's per period utility function is

$$z + A\frac{g^{1-\sigma}}{1-\sigma} - \frac{l^{(1+1/\varepsilon)}}{\varepsilon + 1},$$

where $\sigma > 0$ and $\varepsilon > 0$.[9] The parameter A measures the utility from the public good relative to the utility from consumption and the parameter σ controls the elasticity of the citizens' utility with respect to the public good. Citizens discount future per period utilities at rate β.

We focus on the incomplete markets assumption here because we feel that it is the most appropriate for a positive analysis. We refer the reader to Chari, Christiano and Kehoe (1994) for a comprehensive analysis of optimal fiscal policy in a real business cycle model with complete markets and to Marcet and Scott (2009) for an interesting effort to empirically test between the complete and incomplete market assumptions.

[9] When $\sigma = 1$, the utility from the public good becomes $A\log(g)$.

The productivity of labor w varies across periods in a random way, reflecting the business cycle. Specifically, the economy can either be in a *boom* or a *recession*. Labor productivity is w_H in a boom and w_L in a recession, where $w_L < w_H$. The state of the economy follows a first order Markov process, with transition matrix

$$\begin{bmatrix} \alpha_{LL} & \alpha_{LH} \\ \alpha_{HL} & \alpha_{HH} \end{bmatrix}.$$

Thus, conditional on the economy being in a recession, the probability of remaining in a recession is α_{LL} and the probability of transitioning to a boom is α_{LH}. Similarly, conditional on being in a boom, the probability of remaining in a boom is α_{HH} and the probability of transitioning to a recession is α_{HL}. Though in many environments it is natural to assume that states are persistent, this assumption is not necessary for our results. However, we do require that α_{HH} exceeds α_{LH}, so that the economy is more likely to be in a boom if it was in a boom the previous period.[10]

There is a competitive labor market and competitive production of the public good. Thus, the wage rate is equal to w_H in a boom and w_L in a recession and the price of the public good is p. There is also a market in risk-free one period bonds. The assumption of a constant marginal utility of consumption implies that the equilibrium interest rate on these bonds must be $\rho = 1/\beta - 1$. At this interest rate, citizens will be indifferent as to their allocation of consumption across time.

3.1.2 Government policies

The public good is provided by the government. The government can raise revenue by levying a proportional tax on labor income. It can also borrow and lend by selling and buying bonds. Revenues can not only be used to finance the provision of the public good but can also be diverted to finance targeted district-specific transfers which are interpreted as (non-distortionary) pork-barrel spending.

Government policy in any period is described by an $n+3$-tuple $\{\tau, g, x, s_1, ..., s_n\}$, where τ is the income tax rate, g is the amount of the public good provided, x is the amount of bonds sold, and s_i is the proposed transfer to district i's residents. When x is negative, the government is buying bonds. In each period, the government must also repay any bonds that it sold in the

[10] Our basic model assumes that in the "up-part" of the business cycle there is a single productivity level w_H, and in the "down-part" a single productivity level w_L. Thus, within booms and recessions, there is no variation in productivity. While this is a rather spartan conception of a business cycle, the model can be extended to incorporate within state productivity shocks by assuming that productivity in state θ is given by $w_\theta + \omega$ where ω is an i.i.d "shock" with mean zero, range $[-\overline{\omega}, \overline{\omega}]$. Though the introduction of i.i.d shocks makes the distinction between booms and recessions less clear-cut, the equilibrium of the extended model has the same structure as the equilibrium of the simpler model described in the text and produces the same predictions of the key correlation between macro variables.

previous period. Thus, if it sold b bonds in the previous period, it must repay $(1+\rho)b$ in the current period. The government's initial debt level in period 1 is given exogenously and is denoted by b_0.

In a period in which government policy is $\{\tau, g, x, s_1, ..., s_n\}$ and the state of the economy (i.e., boom or recession) is $\theta \in \{L, H\}$, each citizen will supply an amount of labor

$$l_\theta^*(\tau) = \arg\max_l \{w_\theta(1-\tau)l - \frac{l^{(1+1/\varepsilon)}}{\varepsilon+1}\}.$$

It is straightforward to show that $l_\theta^*(\tau) = (\varepsilon w_\theta(1-\tau))^\varepsilon$, so that ε is the elasticity of labor supply. A citizen in district i who simply consumes his net of tax earnings and his transfer will obtain a per period utility of $u_\theta(\tau, g) + s_i$, where

$$u_\theta(\tau, g) = \frac{\varepsilon^\varepsilon (w_\theta(1-\tau))^{\varepsilon+1}}{\varepsilon+1} + A\frac{g^{1-\sigma}}{1-\sigma}.$$

Since citizens are indifferent as to their allocation of consumption across time, their lifetime expected utility will equal the value of their initial bond holdings plus the payoff they would obtain if they simply consumed their net earnings and transfers in each period.

Government policies must satisfy three feasibility constraints. The first is that revenues must be sufficient to cover expenditures. To see what this implies, consider a period in which the initial level of government debt is b, the policy choice is $\{\tau, g, x, s_1, ..., s_n\}$, and the state of the economy is θ. Expenditure on public goods and debt repayment is $pg + (1+\rho)b$, tax revenue is

$$R_\theta(\tau) = n\tau w_\theta l_\theta^*(\tau) = n\tau w_\theta(\varepsilon w_\theta(1-\tau))^\varepsilon,$$

and revenue from bond sales is x. Letting the *net of transfer surplus* (i.e., the difference between revenues and spending on public goods and debt repayment) be denoted by

$$B_\theta(\tau, g, x; b) = R_\theta(\tau) - pg + x - (1+\rho)b,$$

the constraint requires that $B_\theta(\tau, g, x; b) \geq \sum_i s_i$.

The second constraint is that the district-specific transfers must be non-negative (i.e., $s_i \geq 0$ for all i). This rules out financing public spending via district-specific lump sum taxes. With lump sum taxes, there would be no need to impose the distortionary labor tax and hence no tax smoothing problem.

The third and final constraint is that the amount of government borrowing must be feasible. In particular, there is an upper limit $\overline{x}$ on the amount of bonds the government can sell. This limit is motivated by the unwillingness of borrowers to hold bonds that they know will not be repaid. If the government were borrowing an amount x such that the interest payments exceeded the maximum possible tax revenues in a recession; i.e., $\rho x > \max_\tau R_L(\tau)$, then, if the economy were in recession, it would be unable to repay the debt *even if it provided no public goods or transfers.* Thus, the maximum level of debt is $\overline{x} = \max_\tau R_L(\tau)/\rho$.

We avoid assuming that there is any "ad hoc" limit on the amount of bonds that the government can purchase (see Aiyagari et al (2002)). In particular, the government is allowed to hold sufficient bonds to permit it to always finance the Samuelson level of the public good from the interest earnings. This level of bonds is given by $\underline{x} = -pg_S/\rho$, where g_S is the level of the public good that satisfies the *Samuelson Rule*.[11] Since the government will never want to hold more bonds than this, there is no loss of generality in constraining the choice of debt to the interval $[\underline{x}, \overline{x}]$ and we will do this below.[12] We also assume that the initial level of government debt, b_0, belongs to the interval $(\underline{x}, \overline{x})$.

3.1.3 The political process

Government policy decisions are made by a legislature consisting of representatives from each of the n districts. One citizen from each district is selected to be that district's representative. Since all citizens have the same policy preferences, the identity of the representative is immaterial and hence the selection process can be ignored.[13] The legislature meets at the beginning of each period. These meetings take only an insignificant amount of time, and representatives undertake private sector work in the rest of the period just like everybody else. The affirmative votes of $q < n$ representatives are required to enact any legislation.

To describe how legislative decision-making works, suppose the legislature is meeting at the beginning of a period in which the current level of public debt is b and the state of the economy is θ. One of the legislators is randomly selected to make the first proposal, with each representative having an equal chance of being recognized. A proposal is a policy $\{\tau, g, x, s_1, ..., s_n\}$ that satisfies the feasibility constraints. If the first proposal is accepted by q legislators, then it is implemented and the legislature adjourns until the beginning of the next period. At that time, the legislature meets again with the difference being that the initial level of public debt is x and that the state of the economy may have changed. If, on the other hand, the first proposal is not accepted, another legislator is chosen to make a proposal. There are $T \geq 2$ such proposal rounds, each of which takes a negligible amount of time. If the process continues until proposal round T, and the proposal made at that stage is rejected, then a legislator is appointed to choose a default policy. The only restrictions on the choice of a default policy are that it be feasible and that it involve a uniform district-specific transfer (i.e., $s_i = s_j$ for all i, j).

[11] The Samuelson Rule is that the sum of marginal benefits equal the marginal cost, which means that g_S satisfies the first order condition that $nAg^{-\sigma} = p$.

[12] By assuming that the government can choose to borrow any amount in the interval $[\underline{x}, \overline{x}]$, we are implicitly assuming that labor productivity is sufficiently high that the amount spent on public goods is never higher than national income. A sufficient condition for this is that $nw_L(\varepsilon w_L(\frac{\varepsilon}{1+\varepsilon}))^{\varepsilon} > pg_S$ (see Battaglini and Coate (2008) for details).

[13] While citizens may differ in their bond holdings, this has no impact on their policy preferences.

3.1.4 Discussion

Before turning to the analysis, it is worthwhile highlighting the key differences between the model just presented and that studied in Battaglini and Coate (2008). First, in Battaglini and Coate the production function is constant over time and shocks affect the preferences of the citizens for the public good. In the model just presented, citizens' preferences are constant, but the productivity of the economy is stochastic. A consequence of this is that in Battaglini and Coate shocks leave the set of feasible policies constant, while in this model low shocks reduce expected tax revenues. This, as we will see, has important implications for the analysis and seems an essential feature of any model intended to study how fiscal policy reacts to the business cycle. Second, in the model just presented shocks are persistent. In contrast to Battaglini and Coate, there are two state variables b and θ. From a technical point of view, this makes the analysis more complicated since the shocks change the citizens' expectations. To analyze these effects it will be essential to characterize how the shocks affect the expected marginal cost of taxation.

3.2 The social planner's solution

To create a normative benchmark with which to compare the political equilibrium, we begin by describing what fiscal policy would look like if policies were chosen by a social planner who wished to maximize aggregate utility. The planner's problem can be formulated recursively.[14] In a period in which the current level of public debt is b and the state of the economy is θ, the problem is to choose a policy $\{\tau, g, x, s_1, ..., s_n\}$ to solve:

$$\begin{gathered} \max\ u_\theta(\tau, g) + \frac{\sum_i s_i}{n} + \beta[\alpha_{\theta H} v_H^\circ(x) + \alpha_{\theta L} v_L^\circ(x)] \\ s.t.\ \ s_i \geq 0 \ \text{ for all } i,\ \sum_i s_i \leq B_\theta(\tau, g, x; b),\ \&\ x \in [\underline{x}, \overline{x}], \end{gathered}$$

where $v_\theta^\circ(x)$ denotes the representative citizen's value function in state θ (net of bond holdings).

Surplus revenues will optimally be rebated back to citizens and hence $\sum_i s_i = B_\theta(\tau, g, x; b)$. Thus, we can reformulate the problem as choosing a tax-public good-debt triple (τ, g, x) to solve:

$$\begin{gathered} \max\ u_\theta(\tau, g) + \tfrac{B_\theta(\tau, g, x; b)}{n} + \beta[\alpha_{\theta H} v_H^\circ(x) + \alpha_{\theta L} v_L^\circ(x)] \\ s.t.\ \ B_\theta(\tau, g, x; b) \geq 0\ \&\ x \in [\underline{x}, \overline{x}]. \end{gathered}$$

The problem in this form is fairly standard. The citizen's value functions v_L° and v_H° solve the

[14] Because the interest rate is constant, there is no time inconsistency problem in this model. Thus, assuming that the planner chooses policies period-by-period yields the same results as assuming that he is a *Ramsey planner*, choosing a time path of policies at the beginning of period 1.

functional equations

$$v_\theta^\circ(b) = \max_{(\tau,g,x)} \left\{ \begin{array}{c} u_\theta(\tau,g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta[\alpha_{\theta H} v_H^\circ(x) + \alpha_{\theta L} v_L^\circ(x)] \\ s.t. \;\; B_\theta(\tau,g,x;b) \geq 0 \;\&\; x \in [\underline{x}, \overline{x}] \end{array} \right\} \quad \theta \in \{L, H\} \tag{3.1}$$

and the planner's policies in state θ, $\{\tau_\theta^\circ(b),\, g_\theta^\circ(b),\, x(b)\}$, are the optimal policy functions for this program.

In any given state (b, θ) the planner's optimal policies $\{\tau_\theta^\circ(b),\, g_\theta^\circ(b),\, x_\theta^\circ(b)\}$ are implicitly defined by three conditions. The first is that the social marginal benefit of the public good is equal to the social marginal cost of financing it; that is,

$$nAg^{-\sigma} = p(\frac{1-\tau}{1-\tau\,(1+\varepsilon)}). \tag{3.2}$$

To interpret this, note that $(1-\tau)/(1-\tau(1+\varepsilon))$ measures the *marginal cost of public funds* (MCPF) - the social cost of raising an additional unit of revenue via a tax increase. The term on the right hand side therefore represents the cost of financing an additional unit of the public good. The condition is just the Samuelson Rule modified to take account of the fact that taxation is distortionary and it determines the optimal public good level for any given tax rate. The second condition is that the marginal cost of public funds today equals the expected marginal cost of debt tomorrow; that is,[15]

$$\frac{1-\tau}{1-\tau\,(1+\varepsilon)} = -n\beta[\alpha_{\theta H} v_H^{\circ\prime}(x) + \alpha_{\theta L} v_L^{\circ\prime}(x)]. \tag{3.3}$$

This ensures that, on the margin, the cost of financing public goods via taxation equals that of financing them by issuing debt. The final condition is that the net of transfer surplus be zero; that is,

$$B_\theta(\tau, g, x; b) = 0. \tag{3.4}$$

This implies that the planner raises no more revenues than are necessary to finance public good spending.

Using these conditions, it is possible to show that for each state θ the optimal tax rate and debt level are increasing in b and the optimal public good level is decreasing in b. Using the *Envelope Theorem*, it is also straightforward to show that the marginal cost of debt tomorrow in state θ is just the marginal cost of public funds tomorrow in state θ; that is,

$$-n\beta v_\theta^{\circ\prime}(x) = (\frac{1-\tau_\theta^\circ(x)}{1-\tau_\theta^\circ(x)\,(1+\varepsilon)}).$$

Substituting this into (3.3), yields the Euler equation for the planner's problem:

$$\frac{1-\tau_\theta^\circ(b)}{1-\tau_\theta^\circ(b)\,(1+\varepsilon)} = \alpha_{\theta H}(\frac{1-\tau_H^\circ(x(b))}{1-\tau_H^\circ(x(b))\,(1+\varepsilon)}) + \alpha_{\theta L}(\frac{1-\tau_L^\circ(x(b))}{1-\tau_L^\circ(x(b))\,(1+\varepsilon)}). \tag{3.5}$$

[15] Note that in deriving (3.3) we are ignoring the upperbound $x \leq \overline{x}$. We show in the Appendix (Section 8.5) that this is without loss of generality.

This equation tells us that the optimal debt level equalizes the current MCPF with the corresponding expected MCPF and implies that *the MCPF obeys a martingale*.[16] The condition illustrates the planner's desire to smooth taxation between periods.

The Euler equation (3.5) is the key to understanding the dynamic evolution of the system. It implies that the planner raises debt in a recession and lowers it in a boom. He raises debt in a recession because he anticipates that the economic environment can only improve in the future. If it does improve, the MCPF will be lower since tax rates are lower in booms than in recessions.[17] Thus, debt must increase to maintain equation (3.5). Likewise, when the economy is in a boom, the planner anticipates that the economic environment can only get worse in the future and thus decreases debt. The upshot is that debt behaves counter-cyclically. On the other hand, public good spending behaves pro-cyclically with spending increasing in booms and falling in recessions.

What happens in the long run? Since the MCPF is a convex function of the tax rate τ, the martingale property implies that the current tax rate exceeds the expected tax rate. Thus, the tax rate behaves as a supermartingale.[18] The *Martingale Convergence Theorem* therefore implies that the tax rate converges to a constant with probability one. The only steady state compatible with a constant tax rate, is a steady state in which the government has accumulated such a large pool of assets that spending needs can be financed out of the interest earned, and taxation is zero. Indeed, if this were not true (and taxation were positive), then the tax rate would have to depend on θ. We can therefore conclude that the social planner's solution converges to a steady state in which the debt level is $\underline{x}$, the tax rate is 0, and the public good level is g_S.[19]

The key take away point is that, while in the short run debt displays the counter-cyclical pattern usually associated with the tax smoothing approach, this disappears in the long run. Moreover,

[16] Bohn (1990) establishes this result for a stochastic version of the tax smoothing model studied by Barro (1979). Aiyagari et al (2002) show a similar result for the planner's solution in a model very similar to ours. To ease the comparison, however, note that the negative of their Lagrangian multiplier ψ_t corresponds to our MCPF minus one. It should also be noted that in their model the planner's MCPF follows a supermartingale because the upper bound on debt will bind with positive probability. This however depends on the fact that g_t is an exogenous process. This can not happen in our framework because g_t is endogenous.

[17] While tax rates being lower in booms than in recessions (i.e., $\tau_H^\circ(b) < \tau_L^\circ(b)$) seems natural, it may not be immediate how to prove it. Since the planner's solution is a special case of the political equilibrium when $q = n$, the result will follow from Lemma 2 in Section 4.3.1.

[18] If the MCPF is linear in the tax rate, as assumed in Bohn (1990), the tax rate behaves as a martingale as was conjectured by Barro (1979).

[19] A similar conclusion holds when public spending shocks rather than revenue shocks are the driver of fiscal policy (see Aiyagari et al (2002) and Battaglini and Coate (2008)). However, with public spending shocks, optimal public good spending is uncertain and the government accumulates sufficient assets to finance the highest level of such spending. Interest earnings in excess of optimal public good spending are rebated back to the citizens via a uniform transfer. In this model, the planner does not need to use transfers since optimal public good spending is constant.

all other fiscal policy variables are also constant. This observation underscores the point noted in Section 2: when cyclical variations are not perfectly anticipated, the tax smoothing approach has difficulty explaining cyclical fiscal policy in the long run.

4 Theory

4.1 Political equilibrium

4.1.1 Definition

To characterize behavior when policies are chosen by a legislature, we look for a symmetric Markov-perfect equilibrium. In this type of equilibrium any representative selected to propose at round $r \in \{1, ..., T\}$ of the meeting at some time t makes essentially the same proposal and this depends only on the current level of public debt (b) and the state of the economy (θ). Similarly, at the voting stage of a round r, the probability a legislator votes for a proposal depends only on the proposal itself and the state (b, θ). As is standard in the theory of legislative voting, we focus on weakly stage undominated strategies, which implies that legislators vote for a proposal if they prefer it (weakly) to continuing on to the next proposal round.

An equilibrium can be described by a collection of proposal functions $\{\tau^r_\theta(b),\ g^r_\theta(b),\ x^r_\theta(b),\ s^r_\theta(b)\}^T_{r=1}$ which specify the proposal made by the proposer in round r of the meeting in a period in which the state is (b, θ). Here $\tau^r_\theta(b)$ is the proposed tax rate, $g^r_\theta(b)$ is the public good level, $x^r_\theta(b)$ is the new level of public debt, and $s^r_\theta(b)$ is a transfer offered to the districts of $q-1$ randomly selected representatives. The proposer's district receives the surplus revenues $B_\theta(\tau^r_\theta(b), g^r_\theta(b), x^r_\theta(b); b) - (q-1)s^r_\theta(b)$. Associated with any equilibrium are a collection of value functions $\{v^r_\theta(b)\}^{T+1}_{r=1}$. For $r \in \{1, ..., T\}$ the value function $v^r_\theta(b)$ specifies the expected future payoff of a legislator at the beginning of proposal round r in a period in which the state is (b, θ). By contrast, the value function $v^{T+1}_\theta(b)$ represents the expected payoff of a legislator after the round T proposal has been rejected.

We focus, without loss of generality, on equilibria in which at each round r, proposals are immediately accepted by at least q legislators, so that on the equilibrium path, no meeting lasts more than one proposal round. Accordingly, the policies that are actually implemented in equilibrium are those proposed in the first round. In what follows, we will drop the superscript and refer to the round 1 value function as $v_\theta(b)$ and the round 1 policy proposal as $\{\tau_\theta(b),\ g_\theta(b),\ x_\theta(b),\ s_\theta(b)\}$.

In equilibrium, there is a reciprocal feedback between the policy proposals $\{\tau^r_\theta(b),\ g^r_\theta(b),\ x^r_\theta(b),\ s^r_\theta(b)\}^T_{r=1}$ and the associated value functions $\{v^r_\theta(b)\}^{T+1}_{r=1}$. On the one hand, given that future payoffs are described by the value functions, the prescribed policy proposals must maximize the proposer's payoff subject to the incentive constraint of getting the required number of affirmative votes and the appropriate feasibility constraints. Formally, given $\{v^r_\theta(b)\}^{T+1}_{r=1}$, for each proposal round r and state (b,θ), the proposal $\{\tau_\theta^r(b),\, g_\theta^r(b),\, x_\theta^r(b),\, s_\theta^r(b)\}$ must solve the problem:

$$\begin{aligned}\max_{(\tau,g,x,s)} \; & u_\theta(\tau,g) + B_\theta(\tau,g,x;b) - (q-1)\,s + \beta[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)] \\ s.t.\quad & u_\theta(\tau,g) + s + \beta[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)] \geq v_\theta^{\tau+1}(b), \\ & B_\theta(\tau,g,x;b) \geq (q-1)s, \quad s \geq 0 \;\&\; x \in [\underline{x}, \overline{x}].\end{aligned}$$

The first constraint is the incentive constraint and the remainder are feasibility constraints. The formulation reflects the assumption that on the equilibrium path, the proposal made in round 1 is accepted.

On the other hand, the value functions $\{v_\theta^r(b)\}_{r=1}^{T+1}$ are themselves determined by the equilibrium policy proposals. The legislators' round 1 value functions $v_L(b)$ and $v_H(b)$ are determined recursively using $\{\tau_\theta(b),\, g_\theta(b),\, x_\theta(b),\, s_\theta(b)\}$ by the system:

$$v_\theta(b) = u_\theta(\tau_\theta(b), g_\theta(b)) + \frac{B_\theta(\tau_\theta(b), g_\theta(b), x_\theta(b); b)}{n} + \beta[\alpha_{\theta H} v_H(x_\theta(b)) + \alpha_{\theta L} v_L(x_\theta(b))] \qquad \theta \in \{L, H\}. \tag{4.1}$$

To understand this recall that a legislator is chosen to propose in round 1 with probability $1/n$. If chosen to propose, he obtains a payoff in that period of

$$u_\theta(\tau_\theta(b), g_\theta(b)) + B_\theta(\tau_\theta(b), g_\theta(b), x_\theta(b); b) - (q-1)s_\theta(b).$$

If he is not chosen to propose, but is included in the coalition of legislators whose districts receive a transfer, he obtains $u_\theta(\tau_\theta(b), g_\theta(b)) + s_\theta(b)$, and, if he is not included, he obtains just $u_\theta(\tau_\theta(b), g_\theta(b))$. The probability that his district will receive a transfer, conditional on not being chosen to propose, is $(q-1)/(n-1)$. Taking expectations, the pork barrel transfers $s_\theta(b)$ cancel and the period payoff is as described in (4.1).

The value functions for rounds 2 and beyond are determined by the associated policy proposals and the round 1 value functions. For all proposal rounds $r = 1, .., T-1$ the expected future payoff of a legislator if the round r proposal is rejected is

$$v_\theta^{r+1}(b) = u_\theta\left(\tau_\theta^{r+1}(b), g_\theta^{r+1}(b)\right) + \frac{B_\theta(\tau_\theta^{r+1}(b), g_\theta^{r+1}(b), x_\theta^{r+1}(b); b)}{n} + \beta[\alpha_{\theta H} v_H(x_\theta^{r+1}(b)) + \alpha_{\theta L} v_L(x_\theta^{r+1}(b))].$$

This reflects the assumption that, in the out-of-equilibrium event that play reaches proposal round $r+1$, the proposal made at that point will be immediately accepted. Recall that if the round T proposal is rejected, the assumption is that a legislator is appointed to choose a default tax rate, public good level, level of debt and a uniform transfer. Thus,

$$v_\theta^{T+1}(b) = \max_{(\tau,g,x)} \left\{ u_\theta(\tau,g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)] : B_\theta(\tau,g,x;b) \geq 0 \;\&\; x \in [\underline{x}, \overline{x}] \right\}.$$

We say that an equilibrium is *well-behaved* if the associated round 1 legislators' value functions v_L and v_H are continuous and concave on $[\underline{x}, \overline{x}]$. In what follows, we will first characterize a well-behaved equilibrium and then establish the existence of such an equilibrium. Henceforth, when we refer to an "equilibrium", it is to be understood that it is well-behaved.

4.1.2 Characterization

To understand equilibrium behavior, note that to get support for his proposal the proposer must obtain the votes of $q-1$ other representatives. Accordingly, given that utility is transferable, he is effectively making decisions to maximize the utility of q legislators. It is therefore *as if* a randomly chosen *minimum winning coalition* (mwc) of q representatives is selected in each period and this coalition chooses a policy choice to maximize its aggregate utility. Formally, this means that, when the state is (b, θ), the tax-public good-debt triple (τ, g, x) solves the problem

$$\begin{aligned} &\max u_\theta(\tau, g) + \tfrac{B_\theta(\tau,g,x;b)}{q} + \beta[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)] \\ &s.t. \quad B_\theta(\tau, g, x; b) \geq 0 \ \& \ x \in [\underline{x}, \overline{x}] \,. \end{aligned} \tag{4.2}$$

In any given state (b, θ), there are two possibilities: either the mwc will provide pork to the districts of its members or it will not. Providing pork requires reducing public good spending or increasing taxation in the present or the future (if financed by issuing additional debt). When b is high and/or the economy is in a recession, the opportunity cost of revenues may be too high to make this attractive. In this case, the mwc will not provide pork, so $B_\theta(\tau, g, x; b) = 0$. From (4.2), it is clear that the outcome will then be *as if* the mwc is maximizing the utility of the legislature as a whole. Indeed, the policy choice will be identical to that a social planner would choose in the same state and with the same value function.

When b is low and/or the economy is in a boom, the opportunity cost of revenues is lower. Less tax revenues need to be devoted to debt repayment when b is low and both current and expected future tax revenues are more plentiful when the economy is in a boom. As a result, the mwc will allocate revenues to pork and policies will diverge from those that would be chosen by a planner. Interestingly, it turns out that this diversion of resources toward pork, effectively creates lower bounds on how low the tax rate and debt level can go, and an upper bound on how high the level of the public good can be.

To show this, we must first characterize the policy choices that the mwc selects when it provides pork. Consider again problem (4.2) and suppose that the constraint $B_\theta(\tau, g, x; b) \geq 0$ is not binding. Using the first-order conditions for this problem, we find that the optimal tax rate τ^* satisfies the condition that

$$\frac{1}{q} = \frac{\left[\frac{1-\tau^*}{1-\tau^*(1+\varepsilon)}\right]}{n}.$$

The condition says that the benefit of raising taxes in terms of increasing the per-coalition member transfer $(1/q)$ must equal the per-capita MPCF. Similarly, the optimal public good level g^* satisfies the condition that

$$A\,(g^*)^{-\sigma} = \frac{p}{q}.$$

This says that the per-capita benefit of increasing the public good must equal the per-coalition member reduction in transfers that providing the additional unit necessitates. The optimal public debt level x_θ^* satisfies the condition that

$$x_\theta^* = \arg\max\{\frac{x}{q} + \beta[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)] : x \in [\underline{x}, \overline{x}]\}. \tag{4.3}$$

The optimal level balances the benefit of increasing debt in terms of increasing the per-coalition member transfer with the expected per-capita cost of an increase in the debt level.

We can now make precise how the legislature's ability to divert resources toward pork-barrel spending effectively creates endogenous bounds on the policy choices.

Proposition 1. *The equilibrium value functions $v_H(b)$ and $v_L(b)$ solve the system of functional equations*

$$v_\theta(b) = \max_{(\tau,g,x)} \left\{ \begin{array}{c} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta[\alpha_{\theta L} v_L(x) + \alpha_{\theta H} v_H(x)] \\ s.t. \;\; B_\theta(\tau, g, x; b) \geq 0,\, \tau \geq \tau^*,\, g \leq g^* \;\&\; x \in [x_\theta^*, \overline{x}] \end{array} \right\} \quad \theta \in \{L, H\} \tag{4.4}$$

and the equilibrium policies $\{\tau_\theta(b),\, g_\theta(b),\, x_\theta(b)\}$ are the optimal policy functions for this program.

Thus, the equilibrium policy choices solve a *constrained planner's problem* in which the tax rate can not fall below τ^*, the public good level can not exceed g^*, and debt can not fall below the state contingent threshold x_θ^*.[20] However, there is a fundamental difference with the planner's problem (3.1). The thresholds that constrain the policies are endogenous because they depend on the economic fundamentals and, in the case of x_L^* and x_H^*, on the equilibrium: so rather than being constraints that *affect* the value function, they are determined simultaneously *with* the value function.

Given Proposition 1, the nature of the equilibrium policies in a given state θ is clear. For any equilibrium, define b_θ^* to be the value of debt such that the triple $(\tau^*, g^*, x_\theta^*)$ satisfies the constraint that $B_\theta(\tau^*, g^*, x_\theta^*; b) = 0$. This is given by:

$$b_\theta^* = \frac{R_\theta(\tau^*) + x_\theta^* - pg^*}{1+\rho}. \tag{4.5}$$

Then, if the debt level b is such that $b \leq b_\theta^*$ the tax-public good-debt triple is $(\tau^*, g^*, x_\theta^*)$ and the net of transfer surplus $B_\theta(\tau^*, g^*, x_\theta^*; b)$ is used to finance transfers. If $b > b_\theta^*$ the budget constraint binds so that no transfers are given. The tax rate and public debt level strictly exceed (τ^*, x_θ^*) and the public good level is strictly less than g^*. In this case, therefore, the solution can be characterized by obtaining the first order conditions for problem (4.4) with only the budget constraint binding. These are conditions (3.2), (3.3), and (3.4) except with the equilibrium value functions. It is easy to show that the tax rate and debt level are increasing in b, while the public good level is decreasing in b.[21]

[20] This result extends Proposition 4 of Battaglini and Coate (2008) by showing that when shocks are persistent the lower bound on debt in the constrained planning problem will be state-contingent.

[21] Details are available from the authors upon request.

4.1.3 Existence

To prove the existence of an equilibrium, we first establish that the conditions of Proposition 1 are not only necessary but also sufficient.

Proposition 2. *Suppose that the value functions $v_H(b)$ and $v_L(b)$ solve the system of functional equations (4.4) where x_L^* and x_H^* satisfy (4.3). Then, there exists an equilibrium in which the round 1 value functions are $v_H(b)$ and $v_L(b)$ and the round 1 policy choices $\{\tau_\theta(b), g_\theta(b), x_\theta(b)\}$ are the optimal policy functions for program (4.4).*

Using Proposition 2 we can now establish the existence of an equilibrium by showing that there must exist a pair of value functions $v_H(b)$ and $v_L(b)$ and a pair of debt thresholds x_L^* and x_H^* such that: (i) $v_H(b)$ and $v_L(b)$ solve (4.4) given x_L^* and x_H^*, and, (ii) x_L^* and x_H^* solve (4.3) given $v_H(b)$ and $v_L(b)$. In this way, we obtain:

Proposition 3. *There exists an equilibrium.*

4.2 Tax smoothing in political equilibrium

As discussed in Section 3, the social planner smooths taxation over time by equalizing the current MCPF with the expected MCPF next period. This implies that the MCPF behaves as a martingale and the tax rate as a supermartingale. In this sub-section, we explain how political decision making distorts tax smoothing.

Note first that in political equilibrium, whether the mwc is providing pork or not, the debt level must be such that the MCPF today equals the expected marginal cost of debt tomorrow; that is,[22]

$$\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)\,(1+\varepsilon)} = -n\beta[\alpha_{\theta H}v_H'(x_\theta(b)) + \alpha_{\theta L}v_L'(x_\theta(b))]. \tag{4.6}$$

If, for example, the MCPF exceeded the expected marginal cost of debt, the mwc could shift the financing of its spending program from taxation to debt and make each coalition member better off.

To develop the implications of equation (4.6), the next step is to develop an expression for the marginal cost of debt in each state.

Lemma 1. *For each state of the economy $\theta \in \{L, H\}$, the equilibrium value function $v_\theta(\cdot)$ is differentiable for all b such that $b \neq b_\theta^*$. Moreover:*

$$-v_\theta'(b) = \begin{cases} (\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\varepsilon)})(\frac{1+\rho}{n}) & \text{if } \ b > b_\theta^* \\ (\frac{1+\rho}{n}) & \text{if } \ b < b_\theta^* \end{cases}.$$

[22] Again, in deriving (4.6) we are ignoring the upperbound $x \leq \overline{x}$. In the Appendix (Section 8.5) we prove that this is without loss of generality.

To understand this, recall that when the initial debt level b exceeds b^*_θ, there is no pork, so to pay back an additional unit of debt requires an increase in taxes. This means that the cost of an additional unit of debt is equal to the repayment amount $1+\rho$ multiplied by the per capita MCPF. By contrast, when b is less than b^*_θ, pork will be reduced to pay back additional debt since that is the marginal use of resources. The cost of an additional unit of debt is thus equal to $1+\rho$ multiplied by the expected per capita reduction in pork which is $1/n$. Notice that the value function is not differentiable at $b = b^*_\theta$. The left hand derivative at $b = b^*_\theta$ is equal to $(1+\rho)/n$ and the right hand derivative is equal to $(1+\rho)/q$ (since the tax rate $\tau_\theta(x)$ equals τ^* at $b = b^*_\theta$).[23] This discontinuity reflects the fact that increasing taxes is more costly than reducing pork because the marginal cost of taxation exceeds 1.

Using Lemma 1, we can rewrite equation (4.6) as follows:

$$\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\varepsilon)} = \Pr\left(\theta' \text{ s.t. } x_\theta(b) \le b^*_{\theta'} \,|\theta\right) + \sum_{\theta' \text{ s.t. } x_\theta(b) > b^*_{\theta'}} \alpha_{\theta\theta'} \frac{1-\tau_{\theta'}(x_\theta(b))}{1-\tau_{\theta'}(x_\theta(b))(1+\varepsilon)}. \tag{4.7}$$

Now recall from the characterization that when $x_\theta(b)$ is less than or equal to $b^*_{\theta'}$, the tax rate $\tau_{\theta'}(x_\theta(b))$ will equal τ^*. Thus, equation (4.7) can be rewritten as:[24]

$$\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\varepsilon)} = E\left[\frac{1-\tau_{\theta'}(x_\theta(b))}{1-\tau_{\theta'}(x_\theta(b))(1+\varepsilon)} \,|\theta\right] - \Pr\left(\theta' \text{ s.t. } x_\theta(b) \le b^*_{\theta'} \,|\theta\right)\left[\frac{\varepsilon\tau^*}{1-\tau^*(1+\varepsilon)}\right]. \tag{4.8}$$

The current MCPF is therefore less than or equal to the expected future MCPF. This yields:

Proposition 4. *In any equilibrium, the marginal cost of public funds is a submartingale; that is,*

$$\frac{1-\tau_\theta(b)}{1-\tau_\theta(b)(1+\varepsilon)} \le \alpha_{\theta H}\left[\frac{1-\tau_H(x_\theta(b))}{1-\tau_H(x_\theta(b))(1+\varepsilon)}\right] + \alpha_{\theta L}\left[\frac{1-\tau_L(x_\theta(b))}{1-\tau_L(x_\theta(b))(1+\varepsilon)}\right], \tag{4.9}$$

with the inequality strict when b is sufficiently low.

Why when the inequality in equation (4.9) is strict does the mwc not find it optimal to raise taxes and reduce debt in order to equalize the current MCPF with the expected future MCPF? The answer is that if next period's mwc is providing pork, the correspondent increase in revenues will simply be diverted toward pork. This creates a wedge between the current MCPF and the expected future MCPF. The generality of this intuition indeed suggests that a similar result would be true in any dynamic political economy model of debt.

What can we say about the evolution of the tax rate? As noted in the discussion of the planner's solution, when the MCPF obeys a martingale, the tax rate behaves as a supermartingale. In states (b,θ) such that $x_\theta(b)$ is less than or equal to $b^*_{\theta'}$ for some θ' however, two forces push the difference

[23] The set of sub-gradients of the value function v_θ at $x = b^*_\theta$ is $[-(\frac{1+\rho}{q}), -(\frac{1+\rho}{n})]$.

[24] Equation (4.8) is obtained by adding and subtracting $\Pr\left(\theta' \text{ s.t. } x_\theta(b) \le b^*_{\theta'} \,|\theta\right)\left[\frac{1-\tau^*}{1-\tau^*(1+\varepsilon)}\right]$ from the right hand side of (4.7).

between the current and expected tax rate in opposite directions: the convexity of the MCPF pushes the difference up, and the submartingale property of the MCPF pushes it down. As we prove in the following proposition, there are states in which the first force dominates, implying that the current tax rate is strictly less than the expected future tax rate. This yields:

Proposition 5. *The tax rate is not a martingale of any type; that is, there exist states such that next period's expected tax rate exceeds the current tax rate and states such that the opposite is true.*

By the same logic, it is easy to prove that debt and the public good level will not be martingales of any type as well. In Section 4.3 we will show that the distribution of the MCPF, the tax rate, the public good level and debt all converge to a unique stationary distribution.

4.3 Cyclical behavior of fiscal policies

From the characterization in Section 4.1, we understand the nature of the equilibrium policies in a given state θ. This sub-section first explains how policies compare across states. It then use this understanding to explore the behavior of fiscal policies over the business cycle.

4.3.1 Comparing policies in booms and recessions

To compare policies across states, the key step is to understand how the political constraints change over the cycle, i.e. the relationship between x_L^* and x_H^*. Using Lemma 1 and the first order conditions for problem (4.3), we can show that:

Lemma 2. *In any equilibrium:* $b_L^* < x_L^* \leq x_H^* \leq b_H^*$.

The proof of this result also implies that if x_L^* is less than b_H^*, then x_L^* is less than x_H^*. Thus, the only circumstance in which x_L^* equals x_H^* is when both equal b_H^*. While this is possible, it only arises when α_{LH} is sufficiently close to α_{HH} to make a recession barely persistent. Under these circumstances, legislators would not find it optimal to borrow less when providing pork during a recession than during a boom because the recession is sufficiently likely to revert to a boom. From here on, we will assume that the transition probabilities are such that x_L^* is less than x_H^* which we see as the most interesting case.[25]

With Lemma 2 in hand, we can provide a complete picture of how fiscal policy compares across booms and recessions for any level of debt b. Along with public spending, taxes and debt, we are interested in the *primary surplus* which is the difference between tax revenues and public spending other than interest payments. In our model, it is the difference between tax revenues and spending on the public good and pork. Using the budget constraint, we may write the primary surplus when the state of the economy is θ and the current debt level is b as $PS_\theta(b) = (1+\rho)b - x_\theta(b)$.

[25] A sufficient condition for this to be true is that recessions are sufficiently persistent, that is α_{LL} is sufficiently high.

When b is less than b_L^* the mwc provides pork in both booms and recessions (since $b_L^* < b_H^*$ by Lemma 2). In this case, the tax rate and public good provision are constant across states, respectively at τ^* and g^*, while debt will be higher in a boom than in a recession (respectively, x_H^* versus x_L^*). Tax revenues will be higher in a boom and these extra revenues, together with the extra borrowing, will be used to finance higher levels of pork-barrel spending. The primary surplus will be lower in a boom because borrowing is higher. When b is between b_L^* and b_H^* the mwc provides pork in a boom but not in a recession. In this case, taxes will be higher in a recession and public good provision will be lower. Over this interval of initial debt levels, the new level of debt will be constant in a boom, but increasing in a recession. We show in the Appendix that there will be a threshold debt level $\widehat{b}$ between b_L^* and b_H^* such that new debt will be higher in a recession if and only if b exceeds $\widehat{b}$. Tax revenues will be higher in a boom when b exceeds $\widehat{b}$, while the primary surplus will be higher in a boom if and only if b exceeds $\widehat{b}$. Finally, when b exceeds b_H^* the mwc does not provide pork in either state. In this range, public good levels will be lower in a recession ($g_L(b) < g_H(b)$), tax rates will be higher ($\tau_L(b) > \tau_H(b)$), and public borrowing will be higher ($x_L(b) > x_H(b)$). Tax revenues and the primary surplus will be higher in a boom.

4.3.2 Policy dynamics

With this understanding of how policies compare across states, we can now turn to the dynamic evolution of policy. Clearly, the key to understanding the dynamics is to understand how debt behaves. The cyclical behavior of all the remaining fiscal policies will follow from the behavior of debt given the results we already have.

The fundamental result concerning the dynamic evolution of debt is the following:

Lemma 3. *In any equilibrium: (i) $x_L(b)$ exceeds b for all $b \in [\underline{x}, \overline{x})$, and, (ii) $x_H(b)$ exceeds b for all $b \in (\underline{x}, x_H^*)$ and $x_H(b)$ is less than b for all $b \in (x_H^*, \overline{x}]$.*

Part (i) implies that the debt level always increases in a recession. Intuitively, if we are in a recession today, the economic environment can only improve in the future. This makes it worthwhile for the legislature to increase debt. Part (ii) implies that the debt level decreases in a boom if the initial debt level exceeds x_H^* and increases otherwise. Figure 1 graphs the functions $x_L(b)$ and $x_H(b)$.

We can now infer the cyclical behavior of debt. Note first that, in the short run, it is possible for debt to behave pro-cyclically - jumping up when the economy enters a boom. To see this, suppose that the economy's initial level of debt (b_0) is less than x_H^* and the economy starts out in a recession. Then, once the first boom arrives, if the level of accumulated debt remains less than x_H^*, debt will increase to x_H^* upon entering the boom. The boom increases both current and expected future productivity, which reduces the expected marginal cost of debt. Debt-financed pork instantaneously becomes more attractive for the mwc because of the downward shift in the expected marginal cost of borrowing. Debt jumps to a level at which equality between the marginal benefit

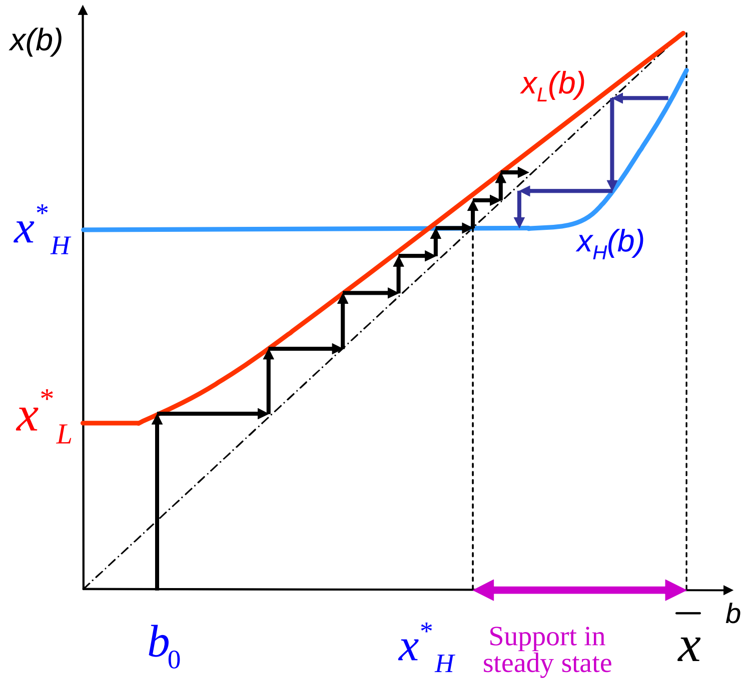

Figure 1: Equilibrium dynamics

of pork and the expected marginal cost of borrowing is reestablished and, during this process, a "pork-fest" occurs. This is very similar to the logic underlying Lane and Tornell's voracity effect.

In the long run, however, debt must behave counter-cyclically - decreasing when the economy enters a boom and increasing when it enters a recession.[26] For once such a pro-cyclical debt expansion has occurred it can never happen again. The damage of the pork-fest to public finances is permanent. This is clear from Figure 1. The debt level is bounded below by x_H^* in a boom and, as demonstrated in Lemma 3, it is increasing in a recession. In the long-run, therefore, once the first boom has occurred and debt has jumped up to x_H^*, fiscal policy will behave counter-cyclically: in a recession, debt will increase and, in a boom, debt will decrease down to x_H^* and then remain constant. Moreover, we can show that no matter what the economy's initial debt level, the same distribution of debt emerges in the long run. To summarize:

Proposition 6. *In any equilibrium, the debt distribution strongly converges to a unique, non degenerate, invariant distribution with support on* $[x_H^*, \overline{x}]$. *The dynamic pattern of debt is counter-cyclical. When the economy enters a recession, debt will increase and will continue to increase as long as the recession persists. When the economy enters a boom, debt decreases and, during the boom, continues to decline until it reaches* x_H^*.

[26] As noted earlier, the voracity effect papers just consider the implications of a one time positive income shock.

Why can we not have recurrent episodes of pro-cyclical fiscal policy ("pork fests") in the long term? As we said, these episodes occur only after the arrival of an unexpected increase in productivity that increases politicians' expectation of future revenues and triggers a permanent increase in debt. In our economy there is no permanent growth, so there is a limit to these positive productivity "surprises". Specifically, such a surprise only occurs the first time the economy moves from a recession to a boom. Once this has happened, the level of debt already incorporates the effects of potential productivity growth. In an economy with permanent growth, positive technological surprises may lead to constant (though stochastic) increases in productivity. We conjecture that in such an economy pro-cyclical "pork fests" will occur even in the long run whenever the upperbound on productivity is increased.[27] The result in Proposition 6 is therefore best interpreted as applying to a mature economy in which these growth effects are not a dominant force.

Since the remaining fiscal policies are all functions of debt, Proposition 6 implies that the distribution of these policies will also be invariant in the long term. Combining Proposition 6 with our understanding of equilibrium policies from the previous section, allows us to predict their long-run cyclical behavior. We deal first with taxes and public good spending.

Proposition 7. *In any equilibrium, in the long run, when the economy enters a recession, the tax rate increases and public good provision decreases. Moreover, the tax rate will continue to increase and public good provision will continue to decrease as long as the recession persists. When the economy enters a boom, the tax rate decreases and public good provision increases. During the boom, the tax rate continues to decline and public good provision continues to increase until they reach, respectively,* τ^* *and* g^**.*

The cyclical behavior of the tax rate determines the dynamics of the MCPF. Proposition 7 implies that the MCPF will increase when the economy enters a recession and continue to increase as long as the recession persists. At any point in time, the MCPF is finite because the tax rate is always lower than the revenue maximizing level (which is $1/(1+\varepsilon)$). In a sufficiently long recession, however, the tax rate may become arbitrarily close to $1/(1+\varepsilon)$, and so the MCPF may become arbitrarily large.[28] When the MCPF is large, however, it must behave as a martingale. For, by then, b will be bigger than b^*_H. Proposition 7 also implies that the MCPF will decrease when the economy enters a boom and continue to decline until it reaches its floor level (which is n/q). Along this decreasing path, the MCPF will eventually start to behave as a strict submartingale. After a

[27] The behavior of fiscal policy in such an economy is an interesting subject for further research.

[28] It is perhaps worthwhile to point out that the fact the MCPF is arbitrarily large when τ is close to the peak of the Laffer curve only means that at that point tax revenues can not be further increased by increasing τ. Moreover, since it can be shown that $\sup_t E(MCPF_t)$ is unbounded, the standard submartingale convergence theorem does not apply and so the MCPF does not converge to a constant (see for example Shiryayev (1996)); and this does not imply that the MCPF converges to an arbitrarily large number. Indeed it is clear that the MCPF must recurrently drop to its floor level $\frac{n}{q}$ in the long run.

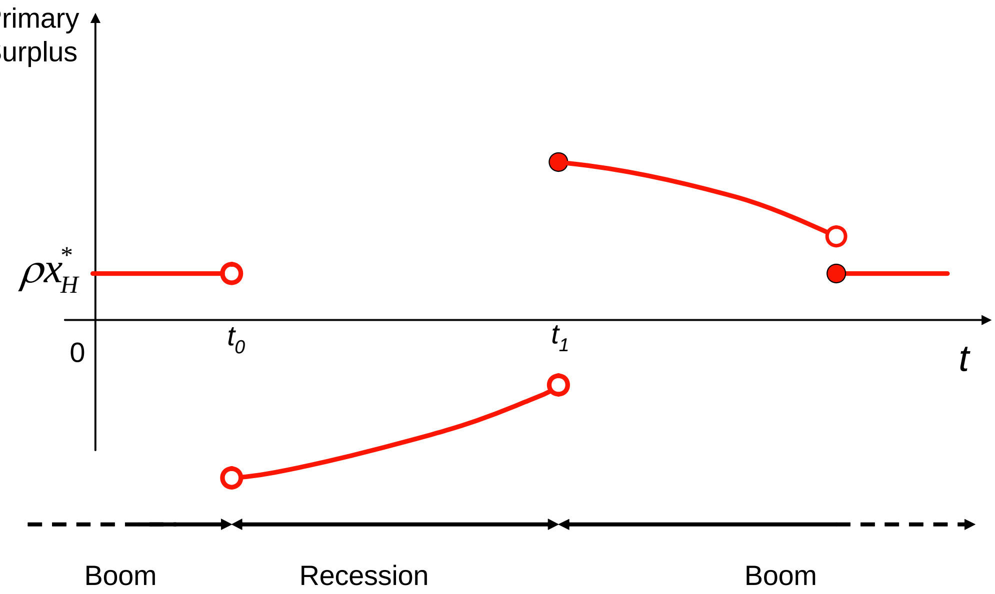

Figure 2: The long run behavior of primary surplus

sufficiently long recession, however, the MCPF will temporarily continue to behave as a martingale even when the economy returns to a boom because it will take time for debt to reduce to a level such that the probability of the event $\{(b,\theta)\,|x_\theta(b) < b_H^*\}$ is positive.

We turn next to the cyclical pattern of pork-barrel spending.

Proposition 8. *In any equilibrium, in the long run, pork-barrel spending will not occur during a recession. Moreover, it will only occur during a boom when debt has fallen below b_H^*. If $x_H^* = b_H^*$, there will be no pork-barrel spending in the long run.*

Pork-barrel spending will not occur during a recession in the long-run because the debt level is at least as large x_H^* and this exceeds b_L^*. There will be no pork in the long-run if $x_H^* = b_H^*$ because then it is not possible for debt to fall below b_H^*.

When combined with the dynamic pattern of public good spending described in Proposition 7, an important implication of Proposition 8 is that total public expenditure (which includes pork-barrel spending and public good provision) is pro-cyclical. The equilibrium changes in public spending and taxes therefore serve to amplify the business cycle. These predictions are distinctive and serve to nicely differentiate the predictions of our neoclassical theory from what would be expected if government were following a Keynesian counter-cyclical fiscal policy. For, in a recession, a Keynesian government would reduce taxes and increase public spending to bolster aggregate demand.

Our final fiscal policy of interest is the primary surplus.

Proposition 9. *In any equilibrium, in the long run, when the economy enters a recession, the primary surplus jumps down and then starts gradually increasing. When the economy enters a boom, the primary surplus jumps up and then starts gradually declining until it reaches a minimal level of* ρx_H^*.

This long run behavior is illustrated in Figure 2, which is drawn under the assumption that x_H^* is positive. Notice that because in long run equilibrium debt always exceeds x_H^*, the primary surplus will be larger in a boom than a recession for any given level of observed debt.[29]

5 Quantitative analysis

We now turn to the quantitative assessment of the theoretical model.[30] The model is calibrated to the U.S. economy and its predictions compared to the data. We assess the performance of the model in two main areas. First, explaining the distribution of debt. Second, explaining the cyclical behavior of debt, taxes, and public spending, which includes their volatility, autocorrelation, and correlation with output.

5.1 Empirical facts

We study the U.S. economy over the post-war period: from 1948 to 2011. Figure 3 and Table 1 present an overview of the behavior of debt, taxes, and public spending during the relevant time period. We measure debt as total outstanding federal debt not held by government accounts, taxes as the total federal revenue/GDP ratio, and public spending as total federal expenditures less net interest on debt.[31] Two features emerge. First, the debt/GDP ratio is persistently higher than 24% and, on average, is equal to 42.4%. Second, there is high volatility and strong countercyclicality of debt. As reported in Table 1, the correlation between debt and GDP is negative and statistically significant. The revenue/GDP ratio is strongly procyclical and less volatile than the debt/GDP ratio. Revenue is also strongly procyclical–its correlation with GDP is 0.77. Public spending is nearly acyclical, and has a lower volatility than debt and revenue.

[29] This follows from the results in section 4.3.1 once it is observed that $\widehat{b}$ is smaller than x_H^*.

[30] Our quantitative analysis complements the work of Azzimonti, Battaglini, and Coate (2009) who calibrate the Battaglini and Coate (2008) model. The focus of the Azzimonti et al paper is to provide a quantitative assessment of the case for imposing a balanced budget rule in the Battaglini and Coate model. Since the driver of fiscal policy is independent and identically distributed spending shocks, Azzimonti et al calibrate their model by matching moments of peacetime and wartime spending, as well as moments of the debt distribution. By contrast, the purpose of this paper is to explore the cyclical behavior of fiscal policies and the driver of fiscal policy is persistent productivity shocks. Thus, the key to our calibration exercise is matching the cyclical properties of GDP.

[31] Data sources and detailed definitions of fiscal variables are provided in the Appendix.

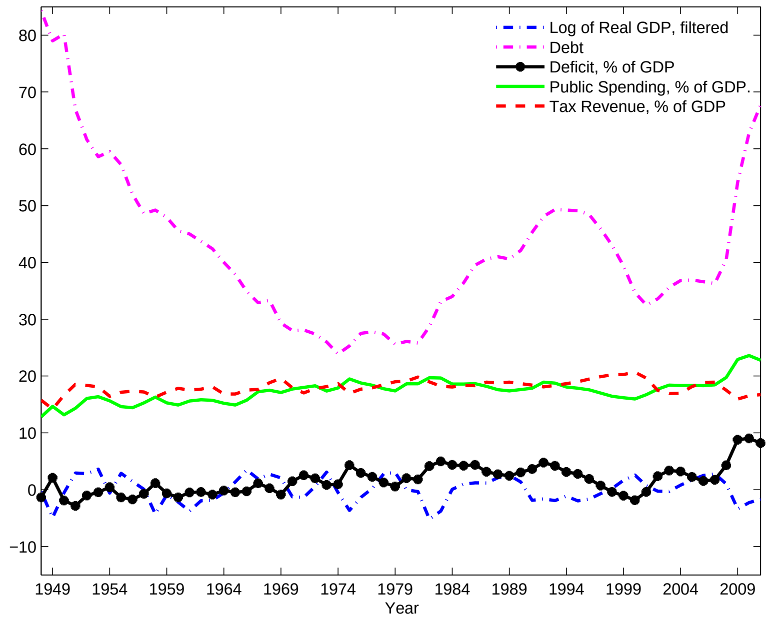

Figure 3: US fiscal policy over the business cycle, 1948-2011

Std			Correlation with output		
$\frac{\text{Debt}}{\text{GDP}}$	$\frac{\text{Spending}}{\text{GDP}}$	$\frac{\text{Revenue}}{\text{GDP}}$	$\frac{\text{Debt}}{\text{GDP}}$	$\frac{\text{Spending}}{\text{GDP}}$	$\frac{\text{Revenue}}{\text{GDP}}$
3.10	0.73	0.80	-0.44	-0.52	0.51
Debt	Spending	Revenue	Debt	Spending	Revenue
6.67	3.74	5.97	-0.35	0.05	0.77

Table 1. Empirical facts about fiscal variables[32]

5.2 Model parameterization

We set the discount factor ρ to 1/1.06, which is a common choice in the real business cycle literature (Jaimovich and Rebelo (2009)). This implies that the annual interest rate on bonds ρ is 6%. The elasticity of labor supply, ε, is set to 2.25, which is in a mid-range of parameters used in the literature (see, for example, Greenwood, Hercowitz and Huffman (1988) and Jaimovich and Rebelo (2009)). The parameters w_L, p, and n are scale parameters that do not directly affect any of our

[32] All data are in logs and HP-filtered. All correlations are statistically significant at the 1% level, except for the correlation of spending and GDP. The latter correlation is not statistically significant.

results. We set the first two to 1, and the third to 100. The parameter q is set to 51, implying that the legislature operates by simple majority rule.[33]

	std(GDP)	$\frac{\text{Debt}}{\text{GDP}}$	$\frac{\text{Spending}}{\text{GDP}}$	$\frac{\text{Revenue}}{\text{GDP}}$	Avg. length of a downturn	% time in a downturn
Data	2.22	42.4	17.4	18.0	2.2	1/3
Model	2.22	43.2	17.5	20.1	2.2	1/3

Table 2. Calibration: Matching Moments

We calibrate five parameters that are specific to the model: the persistence of a boom, α_{HH}, and a recession, α_{LL}; the productivity of the economy in a boom w_H, and the two parameters governing the relative value and elasticity of the public good, A and σ. The persistence parameters are chosen to match the average frequency and length of downturns in the data. To this end, the probability of transiting from a boom to a recession and from a recession to a boom are respectively chosen to be 22% and 48%.[34] The remaining parameters w_H, A, and σ are jointly chosen to minimize the distance between the model generated and empirical values of three variables: the standard deviation of (HP-filtered) output, the average debt/GDP ratio, and the average public spending/GDP ratio. Our search, performed over a fine grid, yields the following parameter values: $w_H = 1.0162$, $A = .684$, and $\sigma = 4.0$. As Table 2 reports, the model comes close to matching the targeted moments. In addition, though not explicitly targeted in the calibration, the average tax rate in the model (as measured by the ratio of revenue to GDP) is 20.1% which is close to the 18.0% average in the data.

5.3 Results

We begin by discussing the distribution of debt. Figure 4 compares the long run distribution of the debt/GDP ratio as well as its empirical counterpart. The average debt/GDP ratio, explicitly targeted in the calibration, is close to that in the data. Though not explicitly targeted, the long run distribution of this variable is broadly consistent with the empirically observed one. Finally, the lowest level of debt in the model is 17% of GDP. In the data, the lowest level of the debt/GDP ratio is 24%.

[33] Our results are similar for choices of q in the range 51 to 60.

[34] The model's predictions would be very similar if we used an alternative calibration for the technology process, e.g., discretized version of the Fernald's (2012) TFP series.

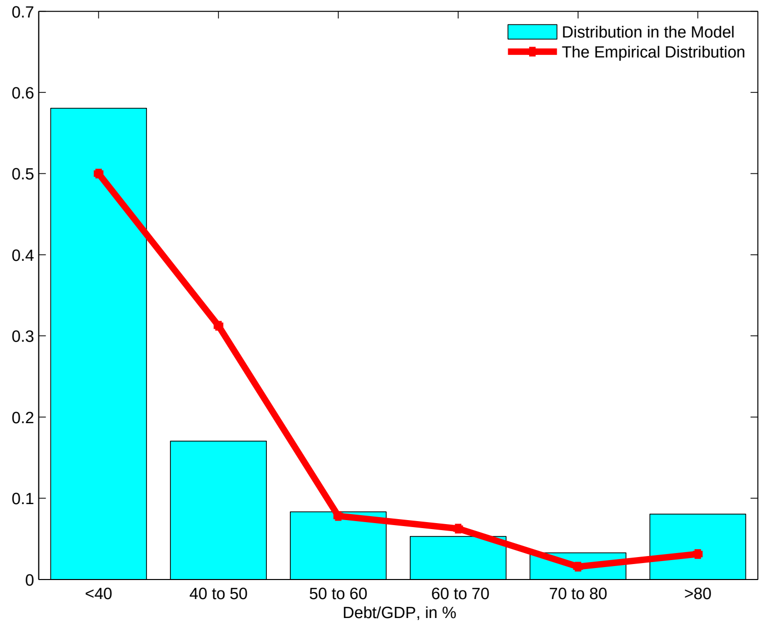

Figure 4: The distribution of the debt/GDP ratio

Std			Correlation with output		
$\frac{\text{Debt}}{\text{GDP}}$	$\frac{\text{Spending}}{\text{GDP}}$	$\frac{\text{Revenue}}{\text{GDP}}$	$\frac{\text{Debt}}{\text{GDP}}$	$\frac{\text{Spending}}{\text{GDP}}$	$\frac{\text{Revenue}}{\text{GDP}}$
1.45	0.38	0.05	-0.85	-0.999	-0.67
Debt	Spending	Revenue	Debt	Spending	Revenue
1.74	0.10	2.08	-0.46	0.65	0.997

Table 3. Model Generated Moments

We next turn to the cyclical behavior of fiscal policy variables. Table 3 reports the model generated second moments of fiscal variables. Comparing with the data in Table 1, we see that the volatility of all fiscal policy variables are smaller than their empirical counterparts. In both the model and the data, debt is strongly counter-cyclical. For the debt/GDP ratio, the strength of correlation is higher in the model, with the model picking up twice the empirical correlation. The correlation of debt generated by the model is more closely aligned with the data. Public spending is positively correlated with GDP in both the model and the data, but the correlation is much weaker in the data, and in fact is not statistically significant. Thus, the model's prediction of pro-cyclical public spending is not supported. The tax rate (i.e., revenue/GDP) is strongly negatively correlated with GDP in the model, but positively correlated in the data, so the model's prediction of a counter-cyclical tax rate is also not supported.

5.4 Discussion

The model's main failures lie in its predictions of pro-cyclical public spending and counter-cyclical tax rates. As shown in Propositions 7 and 8, these predictions are unambiguous theoretical results. The spending prediction is perhaps the less troubling of the two since, as discussed in Section 2, the literature finds that, on average, public spending in developed countries is indeed slightly pro-cyclical. The tax prediction, by contrast, appears to also be at odds with the data from other developed countries. Furceri and Karras (2011) in a study of 26 OECD countries over the period 1965-2003 report finding no statistically significant correlations between effective tax rates and GDP.[35]

One response to these failures, is to try to downplay them by noting that the tax revenue and spending data reflect considerations outside the model.[36] On the spending front, outlays on certain government programs (such as unemployment insurance, food stamps, and welfare) automatically go up during a recession. Thus, counter-cyclical outlays on these programs reflect their basic design rather than a decision by legislators to spend more. On the revenue side, the receipts from some non-labor income taxes (such as corporate taxes) may be expected to rise more than proportionately with GDP during a boom. This could even be true for labor income taxes, since rates are progressive. This effect could create the impression that rates were higher in a boom, even if legislators were actually reducing rates.

Another response is to see the failures as reflecting more basic problems with the model. For example, the model's simplifying assumption that utility is linear in consumption may be partially responsible for the prediction of a counter-cyclical tax rate. With concave utility over private consumption, consumption smoothing over time is welfare improving. To smooth consumption, policy-makers might want to reduce taxes to offset the effects of negative productivity shocks rather than increasing them to generate revenue. More fundamentally, one may point to the model's simplistic view of the labor market which precludes a role for activist fiscal policy. Recent experience around the world suggests that policy makers and politicians are generally optimistic about the potential fiscal policy has to mitigate unemployment. To be successful, a positive model may need

[35] Furceri and Karras (2011) was written in response to an earlier version of this paper. The authors interpret their evidence as supportive of the basic tax smoothing model with constant tax rates. They distinguish this from our political economy theory and a Keynesian theory which would suggest a pro-cyclical tax rate. As noted in Section 2, the predictions of the basic tax smoothing model are not robust to introducing unanticipated cyclical variation which would seem to disqualify it on *a priori* grounds. Moreover, as we have seen, in the U.S. tax rates are indeed pro-cyclical.

[36] We thank Christian Hellwig for making this point.

to reflect this. This suggests that it would be interesting to look at the political economy of fiscal policy in a model with labor market frictions.

6 Comparison with an AMSS-style model

As discussed in Section 2, in a world in which spending shocks drive fiscal policy, Aiyagari, Marcet, Sargent and Seppala (2002) show that the tax smoothing model can generate plausible fiscal dynamics if the government faces a lower bound on debt. This suggests that an acceptable positive model of the behavior of fiscal policy over the business cycle might be obtained by simply adding a debt constraint to the planning problem studied in Section 3.2. Indeed, at first glance, Propositions 1 and 2 might be seen as providing a theoretical underpinning for this practice. However, there are in fact substantial differences between the constrained planning problem associated with the political equilibrium and the problem suggested by Aiyagari, Marcet, Sargent and Seppala. In particular, the constraints in Propositions 1 and 2 apply to all policy variables (debt, taxes, and public good provision), depend on the state of the economy, and are endogenous.

A natural question to ask is how these differences actually matter? To address this question, we added a debt constraint to the planning problem studied in Section 3.2 and calibrated the model to the data discussed in Section 5. The interesting point to note is that this calibration exercise produces the same results as for the political economy model. In particular, the debt lower bound chosen by the calibration is exactly the x_H^* generated by the political economy model and the parameters α_{HH}, α_{LL}, w_H, A and σ are the same. Moreover, the analogues to Tables 2 and 3 are identical. For this data, then the AMSS-style model fits identically to the political economy model.

This is a puzzling finding given the differences in the constrained planning problems associated with the political equilibrium and the AMSS-style model. The explanation is that in the calibration of the political economy model, $x_H^* = b_H^*$. This implies that there is no pork-barrel spending in the long run. Indeed, all pork-barrel spending takes place in the initial period and, if the economy starts off in recession, the first time the economy enters a boom. What this means is that the constraints on taxes and the public good are never binding. All that is relevant is the x_H^* constraint. The lack of pork in long run equilibrium reflects the fact that there are only two productivity states in the model. The productivity difference between the two states is sufficiently low and the persistence of the high state sufficiently high, that legislators have no incentive to restrain their pork consumption in a boom in anticipation of a recession.

Importantly, however, even in this case, the two models have very different comparative static implications. To illustrate this, suppose we are interested in forecasting the effects of a change in labor regulations that implies a change in the elasticity of labor supply. The question of interest is how fiscal policy and debt react? To answer this question, we start from the calibrated parameter values which best fit our data. The model predictions for various levels of labor supply elasticity are presented in Table 4.

ε	2.15	2.2	2.25	2.3	2.35
Average Debt, % of GDP					
PE	-2.53	21.3	43.2	63.2	81.7
AMSS	52.8	47.7	43.2	39.1	35.5
Minumum Debt, % of GDP					
PE	-27.5	-4.00	16.7	35.6	52.3
AMSS	21.8	19.0	16.7	14.6	12.8

Table 4. Comparative Statics in PE versus AMSS-style models

What is striking is that the two models generate the exact opposite comparative statics predictions. Under the political economy model reducing labor supply elasticity from its baseline level of 2.25 to 2.2 reduces the average debt/GDP roughly by a half. By contrast, the AMSS-style model predicts a noticeable increase in debt/GDP. The same patterns hold for the lower bound on debt/GDP ratio: under the political economy model it decreases, but in the AMSS-style model it increases.

The reason why the two models predict differently is explained by the bottom panel of Table 4. As we reduce the elasticity of labor supply, the politicians find it easier to extract taxes from the citizens: hence taxes go up. However, GDP goes down, and hence spending (which does not directly depend on labor supply) as a share of GDP increases. These forces reduce debt/GDP, and in particular, its minimum value.[37] In the AMSS-style model the opposite occurs: as the elasticity is reduced, taxes go up and GDP down; but since the lower bound on debt is exogenous, the debt/GDP ratio goes up.

The comparative statics exercise nicely illustrates the dangers of using an AMSS-style model for policy analysis. While an AMSS-style model fits this particular data just as well as the political economy model, it provides completely different comparative static implications. If the political economy model captures the true decision-making process, the AMSS-style model may, therefore, lead to inaccurate predictions. The point is that a theory that can help rationalize a given "data point", may not be well suited for comparative statics, making it less useful for policy predictions. By providing a microfoundation for the political distortions, we have a theory of how the lower bound of debt changes with the fundamentals.

[37] When $x_H^* = b_H^*$, which is the case in our calibration, the lower bound on debt can be expressed as $x_H^* = \left(\tau^* w_H (\varepsilon w_H (1-\tau^*))^{\varepsilon} - pg^*\right)/\rho$. Defining $GDP_H^* = w_H(\varepsilon w_H(1-\tau^*))^{\varepsilon}$, we have that: $x_H^*/GDP_H^* = \left(\tau^* - \frac{pg^*}{GDP_H^*}\right)/\rho$. As ε increases, the impact of the decline in GDP_H^* outweighs the increase in τ^* and hence x_H^*/GDP_H^* declines.

7 Conclusion

This paper has explored the implications of the political economy theory proposed in Battaglini and Coate (2008) for the cyclical behavior of fiscal policy. This has required replacing public spending shocks with productivity shocks and making these shocks persistent as opposed to independent and identically distributed. While persistent shocks complicate the characterization of equilibrium, the model remains tractable. In particular, equilibrium policy choices continue to solve a constrained planning problem. The difference is that persistence makes the lower bound constraint on debt state-contingent and this is key to understanding the cyclical behavior of fiscal policy.

The theoretical analysis yields three central predictions. First, in the long run, debt displays a counter-cyclical pattern, increasing in recessions and decreasing in booms. A pro-cyclical debt expansion can only arise the first time the economy moves from recession to boom. This is because any increase in debt has permanent effects on public finances. Second, public spending displays a pro-cyclical pattern, with spending increasing in booms and decreasing in recessions, while tax rates display a counter-cyclical pattern decreasing in booms and increasing in recessions. The equilibrium changes in public spending and taxes therefore serve to amplify the business cycle. Third, equilibrium fiscal policies are such that the marginal cost of public funds obeys a submartingale.

The paper has assessed the quantitative implications of the theory by calibrating the model to the U.S. economy using post WWII data. The model broadly matches the empirical distribution of debt and also its high volatility and negative correlation with output. The success of the model in explaining the cyclical behavior of public spending and the tax rate is much more limited. Consistent with the data, the model implies that these fiscal variables are persistent and not very volatile. However, the predictions of pro-cyclical spending and counter-cyclical taxation do not find empirical support.

Finally, to assess the importance of micro-founded modeling of political decision-making, the paper has compared the model with a benevolent government model in which, following the approach of Aiyagari, Marcet, Sargent and Seppala (2002), the government is assumed to face an exogenous lower bound on debt. Interestingly, while the two models are far from theoretically equivalent, the calibrated models deliver the same fit of the data. This reflects the fact that the calibrated political economy model predicts no pork-barrel spending in the long run. However, the two models have starkly different comparative static implications. These differences highlight the importance of modelling the underlying political decision-making when predicting the implications of changes in the underlying economic environment.

The paper shows that the Battaglini and Coate model produces a theory of the cyclical behavior of fiscal policy that is only partially successful in explaining the data. Thus, this paper represents only a step on the path toward a satisfactory positive theory of fiscal policy. There are a number of directions in which future research on this topic might usefully go. Working within the general framework of this paper, the counter-factual predictions concerning taxation may be resolved in a model with concave utility over consumption. More radically, the underlying economic model might usefully be changed to incorporate a role for activist fiscal policy.

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A Appendix

A.1 Proof of Proposition 1

Let $\{\tau_\theta^r(b),\ g_\theta^r(b),\ x_\theta^r(b),\ s_\theta^r(b)\}_{r=1}^T$ be an equilibrium with associated value functions $v_L(b)$ and $v_H(b)$. It is enough to show that for $\theta \in \{L, H\}$, $\{\tau_\theta(b),\ g_\theta(b),\ x_\theta(b)\}$ solves the problem

$$\begin{array}{c} \max_{(\tau,g,x)} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta[\alpha_{\theta L} v_L(x) + \alpha_{\theta H} v_H(x)] \\ s.t.\ \ B_\theta(\tau, g, x; b) \geq 0,\ \tau \geq \tau^*,\ g \leq g^*\ \&\ x \in [x_\theta^*, \overline{x}], \end{array} \tag{A.1}$$

where x_H^* and x_L^* satisfy (4.3). For then it would follow immediately from (4.1) that the value functions $v_L(b)$ and $v_H(b)$ have the required properties.

We begin by making precise the claim made at the beginning of Section 4.1.2 that, given transferable utility, the proposer is effectively making decisions to maximize the collective utility of q legislators under the assumption that they get to divide any surplus revenues among their districts.

Lemma A.1: *Let $\{\tau_\theta^r(b),\ g_\theta^r(b),\ x_\theta^r(b),\ s_\theta^r(b)\}_{r=1}^T$ be an equilibrium with associated value functions $v_L(b)$ and $v_H(b)$. Then, for all states (b, θ), the tax rate-public good-public debt triple $(\tau_\theta^r(b), g_\theta^r(b), x_\theta^r(b))$ proposed in any round τ solves the problem*

$$\begin{array}{c} \max_{(\tau,g,x)} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{q} + \beta[\alpha_{\theta L} v_L(x) + \alpha_{\theta H} v_H(x)] \\ s.t.\quad B_\theta(\tau, g, x; b) \geq 0\ \&\ x \in [\underline{x}, \overline{x}]\,. \end{array} \tag{A.2}$$

Moreover, the transfer to coalition members is given by

$$s_\theta^r(b) = v_\theta^{r+1}(b) - u_\theta(\tau_\theta^r(b), g_\theta^r(b)) - \beta E v_{\theta'}(x_\theta^r(b)).$$

Proof: The proof of this result is similar to the proof of an analogous result in Battaglini and Coate (2008) and thus is omitted. A proof is available from the authors upon request.

As we argued in the text, if the constraint $B_\theta(\tau, g, x; b) \geq 0$, is not binding, then the solution to problem (A.2) is $(\tau^*, g^*, x_\theta^*)$. On the other hand, if the constraint is binding, then the solution to this problem solves the problem

$$\begin{array}{c} \max_{(\tau,g,x)} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta[\alpha_{\theta L} v_L(x) + \alpha_{\theta H} v_H(x)] \\ s.t.\quad B_\theta(\tau, g, x; b) \geq 0\ \&\ x \in [\underline{x}, \overline{x}]\,. \end{array} \tag{A.3}$$

Letting b_θ^* be as defined in (4.5), we conclude that $\{\tau_\theta(b),\ g_\theta(b),\ x_\theta(b)\} = (\tau^*, g^*, x_\theta^*)$ when $b \leq b_\theta^*$ and solves (A.3) when $b > b_\theta^*$. Thus, we need to show (i) that when $b \leq b_\theta^*$ the solution to problem (A.1) is $(\tau^*, g^*, x_\theta^*)$, and, (ii) that when $b > b_\theta^*$ the constraints $\tau \geq \tau^*$, $g \leq g^*$ and $x \geq x_\theta^*$ will not be binding in problem (A.1). For (ii), note first that the solution to (A.3) when $b = b_\theta^*$ is $(\tau^*, g^*, x_\theta^*)$

and second that the optimal tax rate and debt level for problem (A.3) are non decreasing in b and the public good is non increasing in b. For (i), note that when $b \leq b_\theta^*$ the budget constraint cannot be binding in problem (A.1) and, if the budget constraint is not binding, the individual constraints $\tau \geq \tau^*$, $g \leq g^*$ and $x \geq x_\theta^*$ must all bind. ■

A.2 Proof of Proposition 2

Let $\widetilde{v}_H$ and $\widetilde{v}_L$ be a pair of value functions and $\widetilde{x}_H$ and $\widetilde{x}_L$ a pair of debt levels such that (i) $\widetilde{v}_H$ and $\widetilde{v}_L$ solve (4.4) given $\widetilde{x}_H$ and $\widetilde{x}_L$, and, (ii) $\widetilde{x}_H$ and $\widetilde{x}_L$ solve (4.3) given $\widetilde{v}_H$ and $\widetilde{v}_L$. Let $(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b), \widetilde{x}_\theta(b))$ be the corresponding optimal policies that solve the program in (4.4). For each proposal round r and state of the economy $\theta = H, L$ define the following strategies:

$$(\tau_\theta^r(b), g_\theta^r(b), x_\theta^r(b)) = (\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b), \widetilde{x}_\theta(b));$$

for proposal rounds $r = 1, ..., T-1$

$$s_\theta^r(b) = B_\theta(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b), \widetilde{x}_\theta(b); b)/n;$$

and for proposal round T

$$s_\theta^T(b) = v_\theta^{T+1}(b) - u_\theta(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b)) - \beta\left[\alpha_{\theta H}\widetilde{v}_H(\widetilde{x}_\theta(b)) + \alpha_{\theta L}\widetilde{v}_L(\widetilde{x}_\theta(b))\right];$$

where

$$v_\theta^{T+1}(b) = \max_{(\tau,g,x)} \left\{ \begin{array}{c} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta\left[\alpha_{\theta H}\widetilde{v}_H(x) + \alpha_{\theta L}\widetilde{v}_L(x)\right] \\ \text{s.t. } B_\theta(\tau, g, x; b) \geq 0 \ \&\ x \in [\underline{x}, \overline{x}] \end{array} \right\}.$$

Given these proposals, the legislators' round one value functions are given by $\widetilde{v}_H$ and $\widetilde{v}_L$. This follows from the fact that

$$\begin{aligned} v_\theta^1(b) &= u_\theta(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b)) + \frac{B_\theta(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b), \widetilde{x}_\theta(b); b)}{n} + \\ &\quad \beta[\alpha_{\theta H}\widetilde{v}_H(\widetilde{x}_\theta(b)) + \alpha_{\theta L}\widetilde{v}_L(\widetilde{x}_\theta(b))] = \widetilde{v}_\theta(b). \end{aligned}$$

Similarly, the round $\tau = 2, ..., T$ legislators' value functions are given by $\widetilde{v}_H$ and $\widetilde{v}_L$.

To show that $\{\tau_\theta^r(b), g_\theta^r(b), x_\theta^r(b), s_\theta^r(b)\}_{r=1}^T$ together with the associated value functions $\{v_\theta^r(b)\}_{r=1}^{T+1}$ describe an equilibrium, we need only show that for proposal rounds $r = 1, .., T$ the proposal $(\tau_\theta^r(b), g_\theta^r(b), x_\theta^r(b), s_\theta^r(b))$ solves the problem

$$\begin{aligned} \max_{(\tau,g,x,s)} \ & u_\theta(\tau, g) + B_\theta(\tau, g, x; b) - (q-1)\, s + \beta[\alpha_{\theta H}\widetilde{v}_H(x) + \alpha_{\theta L}\widetilde{v}_L(x)] \\ s.t. \quad & u_\theta(\tau, g) + s + \beta[\alpha_{\theta H}\widetilde{v}_H(x) + \alpha_{\theta L}\widetilde{v}_L(x)] \geq \Upsilon_\theta^{r+1}(b) \\ & B_\theta(\tau, g, x; b) \geq (q-1)s, \quad s \geq 0 \ \&\ x \in [\underline{x}, \overline{x}], \end{aligned}$$

where $\Upsilon_\theta^{r+1}(b) = \widetilde{v}_\theta(b)$ for $r = 1..., T-1$, and $\Upsilon_\theta^{T+1}(b) = v_\theta^{T+1}(b)$. We show this result only for $r = 1, ..., T-1$ – the argument for $r = T$ being analogous.

Consider some proposal round $r = 1, ..., T-1$. Let (b, θ) be given. To simplify notation, let

$$(\widehat{\tau}, \widehat{g}, \widehat{x}, \widehat{s}) = (\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b), \widetilde{x}_\theta(b), \frac{B_\theta(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b), \widetilde{x}_\theta(b); b)}{n}).$$

Since $\widetilde{x}_\theta$ solves (4.3), it follows from the discussion in Section 4.1.2 (and it can easily be formally verified) that $(\widehat{\tau}, \widehat{g}, \widehat{x})$ solves the problem:

$$\max_{(\tau,g,x)} u_\theta(\tau, g) + \tfrac{B_\theta(\tau,g,x;b)}{q} + \beta[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)]$$
$$s.t. \quad B_\theta(\tau, g, x; b) \geq 0 \ \& \ x \in [\underline{x}, \overline{x}],$$

and that

$$\widehat{s} = \widetilde{v}_\theta(b) - u_\theta(\widehat{\tau}, \widehat{g}) - \beta[\alpha_{\theta H}\widetilde{v}_H(\widehat{x}) + \alpha_{\theta L}\widetilde{v}_L(\widehat{x})].$$

Suppose that $(\widehat{\tau}, \widehat{g}, \widehat{x}, \widehat{s})$ does not solve the round τ proposer's problem. Then there exist some (τ', g', x', s') which achieves a higher value of the proposer's objective function. We know that $s' \geq \widetilde{v}_\theta(b) - u_\theta(\tau', g') - \beta[\alpha_{\theta H}\widetilde{v}_H(x') + \alpha_{\theta L}\widetilde{v}_L(x')]$. Thus, we have that the value of the proposer's objective function satisfies

$$\begin{aligned} & u_\theta(\tau', g') + B_\theta(\tau', g', x'; b) - (q-1)\, s' + \beta[\alpha_{\theta H}\widetilde{v}_H(x') + \alpha_{\theta L}\widetilde{v}_L(x')] \\ \leq \; & q\{u_\theta(\tau', g') + \beta[\alpha_{\theta H}\widetilde{v}_H(x') + \alpha_{\theta L}\widetilde{v}_L(x')]\} + B_\theta(\tau', g', x'; b). \end{aligned}$$

But since $B_\theta(\tau', g', x'; b) \geq 0$, we know that

$$\begin{aligned} & q\{u_\theta(\tau', g') + \beta[\alpha_{\theta H}\widetilde{v}_H(x') + \alpha_{\theta L}\widetilde{v}_L(x')]\} + B_\theta(\tau', g', x'; b) \\ \leq \; & q\{u_\theta(\widehat{\tau}, \widehat{g}) + \beta[\alpha_{\theta H}\widetilde{v}_H(\widehat{x}) + \alpha_{\theta L}\widetilde{v}_L(\widehat{x})]\} + B_\theta(\widehat{\tau}, \widehat{g}, \widehat{x}; b). \end{aligned}$$

But the right hand side of the inequality is the value of the proposer's objective function under the proposal $(\widehat{\tau}, \widehat{g}, \widehat{x}, \widehat{s})$. This therefore contradicts the assumption that (τ', g', x', s') achieves a higher value for the proposer's problem. ■

A.3 Proof of Proposition 3

By Proposition 2, we can establish the existence of an equilibrium by showing that we can find a pair of value functions $v_H(b)$ and $v_L(b)$ and a pair of debt thresholds x_L^* and x_H^* such that (i) $v_H(b)$ and $v_L(b)$ solve (4.4) given x_L^* and x_H^*, and, (ii) x_L^* and x_H^* solve (4.3) given $v_H(b)$ and $v_L(b)$. We simply sketch how to do this here, the details are available on request.

Let F denote the set of all real valued functions $v(\cdot)$ defined over the set $[\underline{x}, \overline{x}]$ that are continuous and concave. For each $\theta \in \{H, L\}$ and any $z_\theta \in [(R_L(\tau^*) - pg^*)/\rho, \overline{x}]$, define the operator $T_{z_\theta}^\theta$:

$F \times F \to F$ as follows:

$$T^{\theta}_{z_\theta}(v_H, v_L)(b) = \left\{ \begin{array}{c} \max_{(\tau,g,x)} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta\left[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)\right] \\ s.t. \quad B_\theta(\tau, g, x; b) \geq 0,\ \tau \geq \tau^*,\ g \leq g^* \ \&\ x \in [z_\theta, \overline{x}] \end{array} \right\}.$$

Let $\mathbf{z} = (z_H, z_L)$ and let $T_{\mathbf{z}}(v_H, v_L)(b) = (T^H_{z_H}(v_H, v_L)(b), T^L_{z_L}(v_H, v_L)(b))$ be the corresponding function from $F \times F$ to itself. For any $\mathbf{z} \in [(R_L(\tau^*) - pg^*)/\rho, \overline{x}]^2$, it can be verified that $T_{\mathbf{z}}$ is a contraction and admits a unique fixed point $v_{\mathbf{z}}$ (where we use the subscript $\mathbf{z}$ to indicate that this fixed point depends on $\mathbf{z}$). Given $v_{\mathbf{z}}$, let

$$M_\theta(\mathbf{z}) = \arg\max\{\frac{x}{q} + \beta\left[\alpha_{\theta H} v_{H\mathbf{z}}(x) + \alpha_{\theta L} v_{L\mathbf{z}}(x)\right] : x \in [z_\theta, \overline{x}]\}$$

and let $M(\mathbf{z}) = M_H(\mathbf{z}) \times M_L(\mathbf{z})$. Then, we have an equilibrium if we can find a fixed point of this correspondence, $\mathbf{z} \in M(\mathbf{z})$. This can be proven by showing that M satisfies the conditions of *Kakutani's Fixed Point Theorem.* ■

A.4 Proof of Lemma 1

From Proposition 1, we know that

$$v_\theta(b) = \max_{(\tau,g,x)} \left\{ \begin{array}{c} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta[\alpha_{\theta L} v_L(x) + \alpha_{\theta H} v_H(x)] \\ s.t. \ \ B_\theta(\tau, g, x; b) \geq 0,\ \tau \geq \tau^*,\ g \leq g^* \ \&\ x \in [x^*_\theta, \overline{x}] \end{array} \right\}.$$

Moreover, from the discussion in the text, we know that if $b \leq b^*_\theta$ the optimal policies are $(\tau^*, g^*, x^*_\theta)$, and, if $b > b^*_\theta$ the constraints $\tau \geq \tau^*$, $g \leq g^*$ and $x \geq x^*_\theta$ in the maximization problem will not be binding, but the budget constraint will be binding.

Suppose first that $b_o < b^*_\theta$. Then, we know that in a neighborhood of b_o it must be the case that

$$v_\theta(b) = u_\theta(\tau^*, g^*) + \frac{B_\theta(\tau^*, g^*, x^*_\theta; b)}{n} + \beta[\alpha_{\theta H} v_H(x^*_\theta) + \alpha_{\theta L} v_L(x^*_\theta)].$$

Thus, it is immediate that the value function $v_\theta(b)$ is differentiable at b_o and that

$$v'_\theta(b_o) = -(\frac{1+\rho}{n}).$$

Now suppose that $b_o > b^*_\theta$. Then, we know that in a neighborhood of b_o it must be the case that

$$v_\theta(b) = \max_{(\tau,g,x)} \left\{ \begin{array}{c} u_\theta(\tau, g) + \frac{B_\theta(\tau,g,x;b)}{n} + \beta\left[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)\right] \\ B_\theta(\tau, g, x; b) \geq 0 \ \&\ x \in [x^*_\theta, \overline{x}] \end{array} \right\}.$$

Define the function

$$g(b) = \frac{R_\theta(\tau_\theta(b_o)) + x_\theta(b_o) - (1+\rho)b}{p}$$

and let

$$\eta(b) = u_\theta(\tau_\theta(b_o), g(b)) + \frac{B_\theta(\tau_\theta(b_o), g(b), x_\theta(b_o); b)}{n} + \beta\left[\alpha_{\theta H} v_H(x_\theta(b_o)) + \alpha_{\theta L} v_L(x_\theta(b_o))\right].$$

Notice that $(\tau_\theta(b_o), g(b), x_\theta(b_o))$ is a feasible policy when the initial debt level is b so that in a neighborhood of b_o we must have that $v_\theta(b) \geq \eta(b)$. Moreover, $\eta(b)$ is twice continuously differentiable with derivatives

$$\eta'(b) = -Ag(b)^{-\sigma}(\tfrac{1+\rho}{p})$$
$$\eta''(b) = -\sigma Ag(b)^{-(\sigma+1)}(\tfrac{1+\rho}{p})^2 < 0$$

The second derivative property implies that $\eta(b)$ is strictly concave. It follows from Theorem 4.10 of Stokey, Lucas and Prescott (1989) that $v_\theta(b)$ is differentiable at b_o with derivative $v_\theta'(b_0) = \eta'(b_o) = -Ag(b_o)^{-\sigma}(\frac{1+\rho}{p})$. To complete the proof note that $(\tau_\theta(b_o), g_\theta(b_o))$ must solve the problem:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} u_\theta(\tau, g) + \frac{B_\theta(\tau, g, x_\theta(b_o); b_o)}{n} \\ B_\theta(\tau, g, x_\theta(b_o); b_o) \geq 0 \end{array} \right\},$$

which implies that $nAg_\theta(b_o)^{-\sigma} = p[\frac{1-\tau_\theta(b_o)}{1-\tau_\theta(b_o)(1+\varepsilon)}]$. Thus, we have that

$$v_\theta'(b) = -[\frac{1-\tau_\theta(b_o)}{1-\tau_\theta(b_o)(1+\varepsilon)}](\frac{1+\rho}{n}).$$

■

A.5 Proof of Proposition 4

We proceed in two steps. First we prove that both in a political equilibrium and in the planner's solution, the upperbound on debt $x \leq \overline{x}$ does not bind for any $b < \overline{x}$. This establishes equation (3.3) in Section 3.2 and equation (4.6) in Section 4.2. Then we prove the statement of Proposition 4 and equation (3.5) of Section 3.2.

Step 1. Consider a particular political equilibrium and let $\{\tau_\theta(b), g_\theta(b), x_\theta(b)\}$ be the associated equilibrium policies. We wish to prove that for any state (b, θ) there is an $\epsilon(b, \theta) > 0$ such that $x_\theta(b) < \overline{x} - \epsilon(b, \theta)$. Since the planner's solution arises as a special case of the political equilibrium when $q = n$, this would imply that the same property holds for the planner's solution. Assume that there is a state (b, θ) such that $x_\theta(b)$ is arbitrarily close to $\overline{x}$; that is, $x_\theta(b) = \overline{x} - \eta$, where η is arbitrarily small. We can write $g_\theta(b) = \phi(\tau_\theta(b))$ where $\phi(\tau)$ is a continuous function implicitly defined by the solution of the equation $nAg^{-\sigma} = [\frac{1-\tau}{1-\tau(1+\varepsilon)}]p$. Since $x_\theta(b) > x_\theta^*$, we must have

$$B_\theta(\tau_\theta(b), \phi(\tau_\theta(b)), x_\theta(b); b) = 0. \tag{A.4}$$

Thus, we can express all the policy choices as a function of η, where $x_\theta(b) = \overline{x} - \eta = x(\eta)$, $\tau_\theta(b) = \tau(\eta)$ solves (A.4) and $g_\theta(b) = \phi(\tau(\eta)) = g(\eta)$. Note that as $\eta \to 0$, we have $\tau(\eta) \to \widetilde{\tau} < 1/(1+\varepsilon)$.

For if $\tau(\eta) \to 1/(1+\varepsilon)$, then $g(\eta) \to 0$ and (A.4) would not be satisfied since $b < \overline{x}$. Moreover, $\tau(\eta) \to \widetilde{\tau}$ implies $g(\eta) \to \widetilde{g} > 0$.

From the first order condition on debt, we have that:

$$\begin{aligned}(\frac{1-\tau(\eta)}{1-\tau(\eta)(1+\varepsilon)}) \quad &\geq \quad -\beta[\alpha_{\theta H} v_H'(x(\eta)) + \alpha_{\theta L} v_L'(x(\eta))] \qquad \text{(A.5)} \\ &\geq \quad -\beta\alpha_{\theta L} v_L'(x(\eta)) = -\beta\alpha_{\theta L}\left(\frac{1-\tau_L(x(\eta))}{1-\tau_L(x(\eta))(1+\varepsilon)}\right).\end{aligned}$$

It is easy to see that $\tau_L(x(\eta)) \to 1/(1+\varepsilon)$ as $\eta \to 0$. This implies that the right hand side of (A.5) diverges to infinity, while the left hand side converges to a finite value: a contradiction. ■

Step 2. We now prove that the deadweight loss of taxation is a submartingale when $q < n$, with strict inequality for some states (b, θ). The argument in Section 4.2 establishes that the MCPF is a submartingale (equation (4.8)). To complete the statement of the proposition, note that if $q < n$, then $\tau^* > 0$, and $b_\theta^* > \underline{x}$. It is also easy to show that if $q < n$, there is a $b' > x_H^*$ such that for any $b \leq b'$ we have $\Pr\left(\theta' \text{ s.t. } x_\theta(b) \leq b_{\theta'}^* \,|\theta\right) > 0$ for any θ. For these states (4.9) holds as a strict inequality. To prove (3.5), note that if $q = n$, then $\tau^* = 0$. In this case, $\Pr\left(\theta' \text{ s.t. } x_\theta(b) \leq b_{\theta'}^* \,|\theta\right) \frac{\tau^*\varepsilon}{1-\tau^*(1+\varepsilon)} = 0$: which implies that both in an equilibrium with unanimity and in the planner's solution, the MCPF is a martingale. ■

A.6 Proof of Proposition 5

Clearly for all states (b, θ) such that $\tau_\theta^*(b) = \tau^*$, we have $\tau^* < E\left(\tau_{\theta'}\left(x_\theta(b)\right)|\theta\right)$. We now prove that there are states in which $\tau_\theta(b) > E\left(\tau_{\theta'}(x_\theta(b))\,|\theta\right)$. We know from Lemma 2 and 3 below that $\tau_H(x_\theta(b)) < \tau_\theta(b)$ for all states (b, θ) with $b \in [\underline{x}, \overline{x}]$ and that both $\lim_{b\to\overline{x}} \tau_L(x_L(b)) = 1/(1+\varepsilon)$ and $\lim_{b\to\overline{x}} \tau_L(b) = 1/(1+\varepsilon)$. So there is a η such that $\tau_H(x_L(b)) < \tau_L(b) - \eta$ for all $b \in [\underline{x}, \overline{x}]$, and there is a b' such that $\tau_L(x_L(b'')) < \tau_L(b'') + \frac{\eta}{2}$ for $b'' > b'$. This implies that for $b > b'$ we have

$$\begin{aligned}E\left(\tau_{\theta'}\left(x_L(b)\right)|\theta = L\right) \quad &= \quad \alpha_{LL}\tau_L(x_L(b)) + \alpha_{LH}\tau_H(x_L(b)) \\ &= \quad \alpha_{LL}\tau_L(b) + \alpha_{LH}\tau_L(b) - \frac{\eta}{2} \\ &< \quad \tau_L(b).\end{aligned}$$

This shows that for all states $(\widetilde{b}, \widetilde{\theta})$ such that $\widetilde{b} > b'$ and $\widetilde{\theta} = L$, we have $E\left(\tau_{\theta'}\left(x_{\widetilde{\theta}}(\widetilde{b})\right)\Big|\widetilde{\theta}\right) < \tau_{\widetilde{\theta}}(\widetilde{b})$. While for states $(\widetilde{b}, \widetilde{\theta})$ such that $\widetilde{b} \leq b_{\widetilde{\theta}}^*$, we have $E\left(\tau_{\theta'}\left(x_{\widetilde{\theta}}(\widetilde{b})\right)\Big|\widetilde{\theta}\right) > \tau_{\widetilde{\theta}}(\widetilde{b})$. So $\tau_\theta(b)$ is not a martingale of any type. ■

A.7 Proof of Lemma 2

We begin by showing that $x_H^* \geq x_L^*$. Suppose that, to the contrary, that $x_H^* < x_L^*$. There are two possibilities. The first is that $b_L^* < b_H^*$. Note that the first order condition for problem (4.3) is

$$\frac{1}{q} = -\beta[\alpha_{\theta H} v_H'(x_\theta^*) + \alpha_{\theta L} v_L'(x_\theta^*)]. \tag{A.6}$$

From this and Lemma 1, it follows that $b_L^* < x_H^* < x_L^* \leq b_H^*$ and that x_H^* and x_L^* satisfy the following two first order conditions:

$$\alpha_{HL}(\frac{1-\tau_L(x_H^*)}{1-\tau_L(x_H^*)(1+\varepsilon)}) + \alpha_{HH} = \frac{n}{q},$$

and

$$\alpha_{LL}(\frac{1-\tau_L(x_L^*)}{1-\tau_L(x_L^*)(1+\varepsilon)}) + \alpha_{LH} \leq \frac{n}{q} \ (= \text{if } x_L^* < b_H^*).$$

But since $x_H^* < x_L^*$ we know that

$$\frac{1-\tau_L(x_H^*)}{1-\tau_L(x_H^*)(1+\varepsilon)} < \frac{1-\tau_L(x_L^*)}{1-\tau_L(x_L^*)(1+\varepsilon)}.$$

In addition, $\alpha_{HL} \leq \alpha_{LL}$ and hence the above two first order conditions are clearly inconsistent.

The second possibility is that $b_L^* > b_H^*$. In this case, it follows from (A.6) and Lemma 1 that $b_H^* < x_H^* < x_L^* \leq b_L^*$. Since $x_H^* > b_H^*$, it must be that in a boom with debt level $b = x_H^*$ the policy is such that $\tau_H(x_H^*) > \tau^*$, $g_H(x_H^*) < g^*$, and $x_H(x_H^*) > x_H^*$. This implies that

$$\begin{aligned} 0 &= B_H(\tau_H(x_H^*), g_H(x_H^*), x_H(x_H^*); x_H^*) \\ &> B_H(\tau^*, g^*, x_H^*; x_H^*) = R_H(\tau^*) - pg^* - \rho x_H^* > R_H(\tau^*) - pg^* - \rho x_L^*. \end{aligned} \tag{A.7}$$

Since $x_L^* \leq b_L^*$, it must be that in a recession with debt level $b = x_L^*$, the policy is such that $\tau_L(x_L^*) = \tau^*$, $g_L(x_L^*) = g^*$, and $x_L(x_L^*) = x_L^*$. This implies:

$$0 \leq B_L(\tau^*, g^*, x_L^*; x_L^*) = R_L(\tau^*) - pg^* - \rho x_L^* < R_H(\tau^*) - pg^* - \rho x_H^*,$$

which is in contradiction with (A.7).

Given that $x_H^* \geq x_L^*$, it follows from (4.5) that $b_L^* < b_H^*$. Lemma 1 and the first order condition (A.6) imply that $x_H^* \leq b_H^*$ and that $b_L^* < x_L^*$. ■

A.8 Proof of the results of Section 4.3.1

In Section 4.3.1 we make the following three claims. (i) If $b \geq b_H^*$ then $\tau_L(b) > \tau_H(b)$, $g_L(b) < g_H(b)$ and $x(b) > x(b)$. (ii) There is a debt level $\widehat{b}$ between b_L^* and b_H^* such that new debt will be higher in a recession than a boom if and only if $b > \widehat{b}$. (iii) Tax revenues will be higher in a boom than a recession when $b \geq \widehat{b}$.

We begin with claim (i). Assume for now that we know that when $b \geq b_H^*$, $\tau_L(b) > \tau_H(b)$. Then the remaining components of part (i) follow easily. The first order conditions tell us that $\{\tau_\theta(b),g_\theta(b)\}$ must satisfy the following equality:

$$nAg^{-\sigma} = p[\frac{1-\tau}{1-\tau(1+\varepsilon)}],$$

which implies that $g_L(b) < g_H(b)$. In addition, since $x_H^* \leq b_H^* \leq b$, by Lemma 3 below we have that

$$x_H(b) \leq b < x_L(b).$$

Claim (ii) follows from the facts that $x_L(b)$ is increasing in b on the interval $(b_L^*, b_H^*]$, $x_H(b)$ is constant on the interval $(b_L^*, b_H^*]$, $x_H(b_L^*) > x_L(b_L^*)$, and $x_H(b_H^*) < x_L(b_H^*)$ (by claim (i)).

For claim (iii), note first that

$$R_H(\tau_H(b)) \geq pg_H(b) - x_H(b) + (1+\rho)b$$

and that

$$R_L(\tau_L(b)) = pg_L(b) - x_L(b) + (1+\rho)b.$$

Now note that $g_H(b) > g_L(b)$ and, for $b \geq \widehat{b}$, $x_L(b) \geq x_H(b)$.

It remains therefore to show that when $b \geq b_H^*$, it must be the case that $\tau_L(b) > \tau_H(b)$. When $b \geq b_H^*$, we know from the discussion following Proposition 1 that $\{\tau_\theta(b),\, g_\theta(b),\, x_\theta(b)\}$ satisfies the following three equations:

$$nAg^{-\sigma} = p[\frac{1-\tau}{1-\tau(1+\varepsilon)}],$$

$$[\frac{1-\tau}{1-\tau(1+\varepsilon)}] = -\beta n[\alpha_{\theta H} v_H'(x) + \alpha_{\theta L} v_L'(x)],$$

and

$$B_\theta(\tau, g, x; b) = 0.$$

Suppose, to the contrary, that $\tau_L(b) \leq \tau_H(b)$. Then it follows immediately that $g_L(b) \geq g_H(b)$. In addition, we have that

$$-\beta n[\alpha_{HH} v_H'(x_H(b)) + \alpha_{HL} v_L'(x_H(b))] \geq -\beta n[\alpha_{LH} v_H'(x_L(b)) + \alpha_{LL} v_L'(x_L(b))].$$

Suppose that it were the case that $-v_H'(x_H(b)) \leq -v_L'(x_H(b))$. Then, since $\alpha_{HH} > \alpha_{LH}$, we would have that

$$-\beta n[\alpha_{HH} v_H'(x_H(b)) + \alpha_{HL} v_L'(x_H(b))] \leq -\beta n[\alpha_{LH} v_H'(x_H(b)) + \alpha_{LL} v_L'(x_H(b))].$$

Combining these two inequalities we could conclude that $x_H(b) \geq x_L(b)$. But then we would have

$$0 = B_H(\tau_H(b), g_H(b), x_H(b); b) > B_L(\tau_L(b), g_L(b), x_L(b); b) = 0$$

a contradiction. Thus, we would have shown that $\tau_L(b) > \tau_H(b)$.

It follows that we can prove that $\tau_L(b) > \tau_H(b)$ by showing the following result:

Lemma A.2. *If v_H and v_L are differentiable at $b \in [\underline{x}, \overline{x}]$, then*

$$-v_H'(b) \leq -v_L'(b).$$

Proof of Lemma A.2: As in the proof of Proposition 3, let F denote the set of all real valued functions $v(\cdot)$ defined over the set $[\underline{x}, \overline{x}]$ that are continuous and concave. For $\theta \in \{H, L\}$, define the operator $T^{\theta} : F \times F \rightarrow F$ as follows:

$$T^{\theta}(v_H, v_L)(b) = \left\{ \begin{array}{c} \max_{(\tau,g,x)} u_{\theta}(\tau, g) + \frac{B_{\theta}(\tau,g,x;b)}{n} + \beta\left[\alpha_{\theta H} v_H(x) + \alpha_{\theta L} v_L(x)\right] \\ s.t. \quad B_{\theta}(\tau, g, x; b) \geq 0,\, \tau \geq \tau^*,\, g \leq g^* \ \&\ x \in [x^*_{\theta}, \overline{x}] \end{array} \right\}.$$

Let $T(v_H, v_L)(b) = (T^H(v_H, v_L)(b), T^L(v_H, v_L)(b))$ be the corresponding function from $F \times F$ to itself. From Proposition 2, we know that $(v_H, v_L) = T(v_H, v_L)$. Moreover, T is a contraction.

Now let $\widetilde{v}_H$ and $\widetilde{v}_L$ belong to F and assume that for any b if ξ_L and ξ_H are sub-gradients of $\widetilde{v}_L$ and $\widetilde{v}_H$ at b, then we have that: $-\xi_L \geq -\xi_H$. Define $\mathbf{v}_0 = (\widetilde{v}_H, \widetilde{v}_H)$ and consider the sequence of functions $\langle v_{\theta k}(b)\rangle_{k=1}^{\infty}$ for $\theta = H, L$, defined inductively as follows: $v_{\theta 1} = T^{\theta}(\mathbf{v}_0)$, and $v_{\theta k+1} = T^{\theta}(v_{Hk}, v_{Lk})$. Let $\mathbf{v}_k = (v_{Hk}, v_{Lk})$ and note that, since T is a contraction, $\langle \mathbf{v}_k \rangle_{k=1}^{\infty}$ must converge to (v_H, v_L).

Finally, for all $\mu > 0$, let

$$X^{\mu}_{\theta}(\mathbf{v}_k) = \arg\max_{x} \{\frac{x}{\mu} + \beta\left[\alpha_{\theta H} v_{Hk}(x) + \alpha_{\theta L} v_{Lk}(x)\right] : x \in [x^*_{\theta}, \overline{x}]\}$$

and let $x^{\mu}_{\theta}(\mathbf{v}_k)$ be the largest element of the compact set $X^{\mu}_{\theta}(\mathbf{v}_k)$. Notice that $x^{\mu}_{\theta}(\mathbf{v}_k)$ is non-increasing in μ. Also let

$$b^*_{\theta}(x) = \frac{R_{\theta}(\tau^*) + x - pg^*}{1+\rho}.$$

Then we have:

Claim: *For all k, for any $b \in [\underline{x}, \overline{x}]$ if ξ^k_L and ξ^k_H are sub-gradients of v_{Lk} and v_{Hk} at b then we have that: $-\xi^k_L \geq -\xi^k_H$. In addition, if $b \in (b^*_H(x^q_H(\mathbf{v}_{k-1})), \overline{x}]$, then v_{Hk} and v_{Lk} are differentiable at b and $-v'_{Lk}(b) > -v'_{Hk}(b)$.*

Proof: The proof proceeds via induction. Consider the claim for $k = 1$. In state θ if (τ, g, x) is a solution to the problem

$$\begin{array}{c} \max u_{\theta}(\tau, g) + \frac{B_{\theta}(\tau,g,x;b)}{n} + \beta[\alpha_{\theta H}\widetilde{v}_H(x) + \alpha_{\theta L}\widetilde{v}_L(x)] \\ B_{\theta}(\tau, g, x; b) \geq 0,\, g \leq g^*,\, \tau \geq \tau^* \ \&\ x \in [x^*_{\theta}, \overline{x}] \end{array},$$

then: (i) if $b \leq b^*_{\theta}(x^n_{\theta}(\mathbf{v}_0))$, $(\tau, g) = (\tau^*, g^*)$ and $x \in X^n_{\theta}(\mathbf{v}_0) \cap \{x : B_{\theta}(\tau^*, g^*, x; b) \geq 0\}$; (ii) if $b \in (b^*_{\theta}(x^n_{\theta}(\mathbf{v}_0)), b^*_{\theta}(x^q_{\theta}(\mathbf{v}_0))]$, $(\tau, g) = (\tau^*, g^*)$ and $B_{\theta}(\tau^*, g^*, x; b) = 0$; and, (iii) if $b > b^*_{\theta}(x^q_{\theta}(\mathbf{v}_0))$, (τ, g, x) is uniquely defined and the budget constraint is binding. Moreover, $\tau > \tau^*$ and $g < g^*$. Denote the solution in case (iii) as $(\tau_{\theta}(b; \mathbf{v}_0), g_{\theta}(b; \mathbf{v}_0), x_{\theta}(b; \mathbf{v}_0))$.

It follows from this that, if $b \leq b^*_{\theta}(x^n_{\theta}(\mathbf{v}_0))$

$$T^{\theta}(\mathbf{v}_0)(b) = u_{\theta}(\tau^*, g^*) + \frac{B_{\theta}(\tau^*, g^*, x^n_{\theta}(\mathbf{v}_0); b)}{n} + \beta[\alpha_{\theta H}\widetilde{v}_H(x^n_{\theta}(\mathbf{v}_0)) + \alpha_{\theta L}\widetilde{v}_L(x^n_{\theta}(\mathbf{v}_0))].$$

Thus, $T^{\theta}(\mathbf{v}_0)$ is differentiable and its derivative is

$$-\frac{dT^{\theta}(\mathbf{v}_0)(b)}{db} = \frac{1+\rho}{n}.$$

If $b \in (b^*_{\theta}(x^n_{\theta}(\mathbf{v}_0)), b^*_{\theta}(x^q_{\theta}(\mathbf{v}_0))]$, then

$$T^{\theta}(\mathbf{v}_0)(b) = u_{\theta}(\tau^*, g^*) + \beta[\alpha_{\theta H}\widetilde{v}_H(pg^* + (1+\rho)b - R_{\theta}(\tau^*)) + \alpha_{\theta L}\widetilde{v}_L(pg^* + (1+\rho)b - R_{\theta}(\tau^*))].$$

It follows that if μ_{θ} is a sub-gradient of $T^{\theta}(\mathbf{v}_0)$ at b there exist sub-gradients ξ_H and ξ_L of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $pg^* + (1+\rho)b - R_{\theta}(\tau^*)$ such that $\mu_{\theta} = \alpha_{\theta H}\xi_H + \alpha_{\theta L}\xi_L$. Notice that in this range, $b \in (b^*_{\theta}(x^n_{\theta}(\mathbf{v}_0)), b^*_{\theta}(x^q_{\theta}(\mathbf{v}_0))]$ and hence if μ_{θ} is a sub-gradient of $T^{\theta}(\mathbf{v}_0)$ at b

$$-\beta\mu_{\theta}(1+\rho) \in (\frac{1+\rho}{n}, \frac{1+\rho}{q}].$$

If $b > b^*_{\theta}(x^q_{\theta}(\mathbf{v}_0))$ then

$$T^{\theta}(\mathbf{v}_0)(b) = \max_{(\tau,g,x)} \left\{ \begin{array}{c} u_{\theta}(\tau, g) + \frac{B_{\theta}(\tau,g,x;b)}{n} + \beta[\alpha_{\theta H}\widetilde{v}_H(x) + \alpha_{\theta L}\widetilde{v}_L(x)] \\ B_{\theta}(\tau, g, x; b) \geq 0 \ \& \ x \in [x^*_{\theta}, \overline{x}] \end{array} \right\}.$$

By the same argument used to prove Lemma 1, $T^{\theta}(\mathbf{v}_0)$ is differentiable and its derivative is

$$-\frac{dT^{\theta}(\mathbf{v}_0)(b)}{db} = \frac{1 - \tau_{\theta}(b; \mathbf{v}_0)}{n(1 - \tau_{\theta}(b; \mathbf{v}_0)(1+\varepsilon))}(1+\rho).$$

Since $\tau_{\theta}(b; \mathbf{v}_0) > \tau^*$, in this range we have that

$$-\frac{dT^{\theta}(\mathbf{v}_0)(b)}{db} > \frac{(1+\rho)}{q}.$$

Given the expressions for the derivatives and subgradients derived above, the result would follow for $k = 1$ if: (i) $b^*_L(x^n_L(\mathbf{v}_0)) \leq b^*_H(x^n_H(\mathbf{v}_0))$; (ii) $b^*_L(x^q_L(\mathbf{v}_0)) \leq b^*_H(x^q_H(\mathbf{v}_0))$; (iii) for all $b \in (b^*_H(x^n_H(\mathbf{v}_0)), b^*_L(x^q_L(\mathbf{v}_0)))$ if ξ_H and ξ_L are subgradients of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $pg^* + (1+\rho)b - R_H(\tau^*)$ and ξ'_H and ξ'_L are subgradients of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $pg^* + (1+\rho)b - R_L(\tau^*)$, then

$$-\beta[\alpha_{HH}\xi_H + \alpha_{HL}\xi_L](1+\rho) \leq -\beta[\alpha_{LH}\xi'_H + \alpha_{LL}\xi'_L](1+\rho);$$

and, (iv) for all $b > b^*_H(x^q_L(\mathbf{v}_0))$

$$\frac{1 - \tau_L(b; \mathbf{v}_0)}{(1 - \tau_L(b; \mathbf{v}_0)(1+\varepsilon))} > \frac{1 - \tau_H(b; \mathbf{v}_0)}{(1 - \tau_H(b; \mathbf{v}_0)(1+\varepsilon))}.$$

We will now establish that these four conditions are satisfied. For the first, we will show that $x^n_H(\mathbf{v}_0) \geq x^n_L(\mathbf{v}_0)$. Recall that by definition $x^n_{\theta}(\mathbf{v}_0)$ is the largest element in the compact set

$$X^n_{\theta}(\mathbf{v}_0) = \arg\max_x \{\frac{x}{n} + \beta\left[\alpha_{\theta H}\widetilde{v}_H(x) + \alpha_{\theta L}\widetilde{v}_L(x)\right] : x \in [x^*_{\theta}, \overline{x}]\}.$$

As shown in Lemma 2, we have that $x_H^* \geq x_L^*$. We can assume wlog that $x_L^n(\mathbf{v}_0) > x_L^*$. Thus, there exists sub-gradients ξ_H and ξ_L of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $x_L^n(\mathbf{v}_0)$ such that

$$\frac{1}{n} = -\beta \left[\alpha_{LH} \xi_H + \alpha_{LL} \xi_L \right].$$

Suppose that $x \leq x_L^n(\mathbf{v}_0)$. Then, if ξ_H' and ξ_L' of $\widetilde{v}_H$ and $\widetilde{v}_L$ at x then since $-\xi_H \leq -\xi_L$, $\alpha_{HH} \geq \alpha_{LH}$, and $-\xi_\theta' \leq -\xi_\theta$, we know that:

$$-\beta \left[\alpha_{HH} \xi_H' + \alpha_{HL} \xi_L' \right] \leq -\beta \left[\alpha_{LH} \xi_H + \alpha_{LL} \xi_L \right] = \frac{1}{n}.$$

This implies that $x_H^n(\mathbf{v}_0) \geq x_L^n(\mathbf{v}_0)$. A similar argument establishes the second condition.

For the third condition, let $b \in (b_H^*(x_H^n(\mathbf{v}_0)), b_L^*(x_L^q(\mathbf{v}_0)))$, let ξ_H and ξ_L be subgradients of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $pg^* + (1+\rho)b - R_H(\tau^*)$, and let ξ_H' and ξ_L' be subgradients of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $pg^* + (1+\rho)b - R_L(\tau^*)$. Then we have

$$\begin{aligned} -\beta[\alpha_{HH}\xi_H + \alpha_{HL}\xi_L](1+\rho) &\leq -\beta[\alpha_{LH}\xi_H + \alpha_{LL}\xi_L](1+\rho) \\ &\leq -\beta[\alpha_{LH}\xi_H' + \alpha_{LL}\xi_L'](1+\rho), \end{aligned}$$

where the first inequality follows from the facts that $\alpha_{HH} \geq \alpha_{LH}$ and $-\xi_H \leq -\xi_L$, and the second inequality follows from the facts that $\widetilde{v}_H$ and $\widetilde{v}_L$ are concave and that $R_H(\tau^*) > R_L(\tau^*)$.

For the fourth condition, note that $(\tau_\theta(b;\mathbf{v}_0), g_\theta(b;\mathbf{v}_0), x_\theta(b;\mathbf{v}_0))$ is defined by the following three conditions:

$$nAg_\theta(b;\mathbf{v}_0)^{-\sigma} = p[\frac{1-\tau_\theta(b;\mathbf{v}_0)}{1-\tau_\theta(b;\mathbf{v}_0)(1+\varepsilon)}],$$

there exist subgradients ξ_H and ξ_L be subgradients of $\widetilde{v}_H$ and $\widetilde{v}_L$ at $x_\theta(b;\mathbf{v}_0)$ such that

$$[\frac{1-\tau_\theta(b;\mathbf{v}_0)}{1-\tau_\theta(b;\mathbf{v}_0)(1+\varepsilon)}] = -\beta n[\alpha_{\theta H}\xi_H + \alpha_{\theta L}\xi_L],$$

and

$$B_\theta(\tau_\theta(b;\mathbf{v}_0), g_\theta(b;\mathbf{v}_0), x_\theta(b;\mathbf{v}_0); b) = 0.$$

Suppose to the contrary that $\tau_H(b;\mathbf{v}_0) \geq \tau_L(b;\mathbf{v}_0)$. Then, $g_H(b;\mathbf{v}_0) \leq g_L(b;\mathbf{v}_0)$ and $x_H(b;\mathbf{v}_0) \geq x_L(b;\mathbf{v}_0)$. It follows that

$$\begin{aligned} 0 &= B_H(\tau_H(b;\mathbf{v}_0), g_H(b;\mathbf{v}_0), x_H(b;\mathbf{v}_0); b) \geq B_H(\tau_L(b;\mathbf{v}_0), g_L(b;\mathbf{v}_0), x_L(b;\mathbf{v}_0); b) \\ &> B_L(\tau_L(b;\mathbf{v}_0), g_L(b;\mathbf{v}_0), x_L(b;\mathbf{v}_0); b). \end{aligned}$$

This is a contradiction.

Now assume that the claim is true for $t = 1, ...k$ and consider it for $k+1$. By the induction step, for any $b \in [\underline{x}, \overline{x}]$ if ξ_{Lk} is a sub-gradient of v_{Lk} at b and ξ_{Hk} is a sub-gradient of v_{Hk} at b then we have that: $-\xi_{Lk} \geq -\xi_{Hk}$. It follows that v_{Hk} and v_{Lk} have the same properties as the functions $\widetilde{v}_H$ and $\widetilde{v}_L$ and the same argument as above applies to step $k+1$. ■

We can now prove Lemma A.2. Given Lemma 1, all we need to do is to establish that if $b \in (b_H^*, \overline{x}]$ then

$$-v_L'(b) \geq -v_H'(b).$$

Suppose, to the contrary, that there exists some $x \in (b_H^*, \overline{x}]$ such that $-v_L'(x) < -v_H'(x)$. Let $\varepsilon > 0$ be such that $x - \varepsilon > b_H^*$. Given that $\mathbf{v}_k$ converges to (v_H, v_L), it must be the case that $x_H^q(\mathbf{v}_k)$ converges to $x_H^q(v_H, v_L) = x_H^*$ as $k \to \infty$. Thus, for sufficiently large k, $b_H^*(x_H^q(\mathbf{v}_k)) < x - \varepsilon$. For any k sufficiently large, therefore, the Claim implies that v_{Hk} and v_{Lk} are differentiable on $(x - \varepsilon, \overline{x}]$ and $-v_{Lk}'(b) > -v_{Hk}'(b)$ for any $b \in (x - \varepsilon, \overline{x}]$. Thus, by Theorem 25.7 of Rockafellar (1970), we know that $\lim_{k\to\infty} v_{\theta k}'(b) = v_\theta'(b)$ for any $b \in (x - \varepsilon, \overline{x}]$, which includes x: a contradiction. ■

A.9 Proof of Lemma 3

(i) If $b \leq b_L^*$, we have that $x_L(b) = x_L^* > b_L^* \geq b$. Assume then that $b > b_L^*$. Suppose, contrary to the claim, that $x_L(b) \leq b$. By Lemma 1, we have that

$$-\beta n v_L'(b) = \frac{1 - \tau_L(b)}{1 - \tau_L(b)(1 + \varepsilon)}.$$

Since $x_L(b) < \overline{x}$ the first order conditions for $(\tau_L(b), g_L(b), x_L(b))$ imply that there must exist sub-gradients ξ_L and ξ_H of v_L and v_H at $x_L(b)$ such that

$$\frac{1 - \tau_L(b)}{1 - \tau_L(b)(1 + \varepsilon)} = -\beta n \left[\alpha_{LH}\xi_H + \alpha_{LL}\xi_L\right].$$

Since $\tau_L(b) > \tau^*$, for this equation to hold we must have that $x_L(b) > b_L^*$ and hence we know by Lemma 1 that

$$\xi_L = -\frac{1 - \tau_L(x_L(b))}{1 - \tau_L(x_L(b))(1 + \varepsilon)}(\frac{1 + \rho}{n}).$$

In addition, it must be the case that

$$-\beta\xi_H < -\beta\xi_L.$$

Clearly, this is case if $x_L(b) \leq b_H^*$. If $x_L(b) > b_H^*$, the inequality follows from the fact that $\tau_L(x_L(b)) > \tau_H(x_L(b))$. Thus, we have that

$$\begin{aligned} \frac{1 - \tau_L(b)}{1 - \tau_L(b)(1 + \varepsilon)} &= -\beta n \left[\alpha_{LH}\xi_H + \alpha_{LL}\xi_L\right] \\ &< -\beta n \xi_L \\ &= \frac{1 - \tau_L(x_L(b))}{1 - \tau_L(x_L(b))(1 + \varepsilon)}. \end{aligned}$$

But this is a contradiction because the facts that $\tau_L(\cdot)$ is increasing and that $b \geq x_L(b)$, imply that $\tau_L(b) \geq \tau_L(x_L(b))$.

(ii) If $b \leq b_H^*$, we have that $x_H(b) = x_H^*$. Thus, $x_H(b) > b$ if $b < x_H^*$ and $x_H(b) < b$ if $b \in (x_H^*, b_H^*]$. Assume then that $b > b_H^*$. Suppose, contrary to the claim, that $x_H(b) \geq b$. By Lemma 1, we have that

$$-\beta n v_H'(b) = \frac{1-\tau_H(b)}{1-\tau_H(b)(1+\varepsilon)}.$$

Since $x_H(b) \geq b > b_H^* > b_L^*$, then we know from the first order condition for $x_H(b)$ and Lemma 1 that

$$\frac{1-\tau_H(b)}{1-\tau_H(b)(1+\varepsilon)} \geq \alpha_{LH}(\frac{1-\tau_H(x_H(b))}{1-\tau_H(x_H(b))(1+\varepsilon)}) + \alpha_{LL}(\frac{1-\tau_L(x_H(b))}{1-\tau_L(x_H(b))(1+\varepsilon)}) \ (= \text{ if } x_H(b) < \overline{x}),$$

Since $\tau_H(x_H(b)) < \tau_L(x_H(b))$, this equation implies that

$$\frac{1-\tau_H(b)}{1-\tau_H(b)(1+\varepsilon)} > (\frac{1-\tau_H(x_H(b))}{1-\tau_H(x_H(b))(1+\varepsilon)}).$$

But this is a contradiction because the facts that $\tau_H(\cdot)$ is increasing and $b \leq x_H(b)$, imply that $\tau_H(b) \leq \tau_H(x_H(b))$. ■

A.10 Proof of Proposition 6

The dynamic pattern of debt described in the proposition follows immediately from Lemma 3. Thus, to prove the proposition we must show that the debt distribution converges strongly to a unique invariant distribution. To this end, define the state space $S = [\underline{x}, \overline{x}] \times \{L, H\}$ with associated σ-algebra $\mathcal{F} = \mathcal{B} \times \mathcal{H}$, where $\mathcal{B}$ is the family of Borel sets that are subsets of $[\underline{x}, \overline{x}]$, and $\mathcal{H}$ is the family of subsets of $\{L, H\}$. For any subset $A \in \mathcal{F}$, let $\mu_t(A)$ denote the probability that the state lies in A in period t. The probability measure μ_t describes the debt distribution in period t; for example, the probability that in period t the debt level lies between x_a and x_b in a boom is given by $\mu_t(([x_a, x_b], H))/\mu_t(([\underline{x}, \overline{x}], H))$. We are thus interested in the long run behavior of μ_t.

The probability distribution μ_1 depends on the initial level of debt b_0 and the initial state of the economy. To describe the probability distribution in periods $t \geq 2$ we must first describe the transition function implied by the equilibrium. This transition function is given by:

$$Q(A\,|b,\theta) = \begin{cases} \sum_{\{\theta' :\ \text{s.t.}(x_{\theta'}(b),\theta') \in A\}} \alpha_{\theta\theta'} & \text{if } \exists\ \theta' \ \ s.t. \ \left(x_{\theta'}(b), \theta'\right) \in A \\ 0 & otherwise \end{cases}.$$

Intuitively, $Q(A\,|b,\theta)$ is the probability that a set A is reached in one step if the initial state is (b, θ). Using this notation, the probability distribution in period $t \geq 2$ is defined inductively as:

$$\mu_t(A) = \sum_\theta \int_b Q(A\,|b,\theta)\mu_{t-1}(db,\theta).$$

The probability distribution μ^* is an invariant distribution if

$$\mu^*(A) = \sum_{\theta} \int_b Q(A\,|b, \theta\,)\mu^*(db, \theta).$$

We now show that the sequence of distributions $\langle \mu_t \rangle_{t=1}^{\infty}$ converges strongly to a unique invariant distribution.

By Theorem 11.12 in Stokey, Lucas and Prescott (1989), it is enough to show that the transition function Q satisfies the M *condition* (see the definition in Stokey, Lucas and Prescott (1989)). To this end, let $Q^1(A\,|b, \theta\,) = Q(A\,|b, \theta\,)$ and define recursively:

$$Q^n(A\,|b, \theta\,) = \sum_{\theta'} \int_{b'} Q(A\,|b', \theta'\,)Q^{n-1}(db', \theta'\,|b, \theta\,).$$

Thus, $Q^n(A\,|b, \theta\,)$ is the probability that a set A is reached in n steps if the initial state is (b, θ). To establish that Q satisfies the M condition, it is sufficient to prove that there exists a state (x^*, θ^*), an integer $N \geq 1$ and a number $\varepsilon > 0$, such that for any initial state (b, θ), $Q^N((x^*, \theta^*)\,|b, \theta\,) > \varepsilon$ (See Exercises 11.5 and 11.4 in Stokey, Lucas and Prescott (1989)).

Consider the state (x_H^*, H). Define $\eta = \min_{b \in [b_H^*, \overline{x}]} [b - x_H(b)]$. Since, by Lemma 3, $x_H(b) < b$ for any $b \in [b_H^*, \overline{x}]$, we have that $\eta > 0$. Let N be the smallest integer larger than $\frac{\overline{x} - b_H^*}{\eta} + 1$. Then, we claim that for any initial state (b, θ), we have that:

$$Q^N((x_H^*, H)\,|b, \theta\,) \geq \alpha_{LH}\,(\alpha_{HH})^{N-1} > 0.$$

If this claim is true, then by choosing $\varepsilon \in \left(0, \alpha_{LH}\,(\alpha_{HH})^{N-1}\right)$, we have the desired condition.

To see that the claim is true, suppose first that the initial state (b, θ) is such that $b \leq b_H^*$. With probability of at least α_{LH} the state will be (x_H^*, θ_H) in the next period and it will remain there for as long as the economy remains in a boom (which happens with probability α_{HH}). Next suppose that the initial state (b, θ) is such that $b > b_H^*$. With probability of at least α_{LH} the economy will be in a boom the next period and, again, it will remain in a boom thereafter with probability α_{HH}. If it does remain in a boom, then for as long as the debt level remains above b_H^*, debt will be reduced by at least η in each period. Thus, after N periods, the debt level must have gone below b_H^* in some period and therefore will have reached x_H^*. ■

A.11 Proof of Proposition 9

The primary surplus in state θ is given by:

$$PS_\theta(b) = (1 + \rho)b - x_\theta(b).$$

Note first that the primary surplus in state θ is increasing in b. This is immediate if $b < b_\theta^*$ since in that case $x_\theta(b) = x_\theta^*$. To see the result if $b > b_\theta^*$ note first that when the mwc is not providing pork

$$PS_\theta(b) = R_\theta(\tau_\theta(b)) - pg_\theta(b).$$

Now recall that $\tau_\theta(b)$ is increasing in b and $g_\theta(b)$ is decreasing in b.

To understand the long run behavior of the primary surplus when the economy enters a boom, let the level of debt when the economy enters a boom be b. By Proposition 6, we know that this debt level must exceed x_H^*. To show that the primary surplus jumps up when the economy enters the boom, we need to show that

$$(1+\rho)b - x_H(b) > (1+\rho)x_L^{-1}(b) - b.$$

We have that, by definition,

$$(1+\rho)x_L^{-1}(b) - b = (1+\rho)x_L^{-1}(b) - x_L(x_L^{-1}(b))$$

Since debt levels are increasing in a recession, we have that $b > x_L^{-1}(b)$. Thus, using the fact that PS_L is increasing, we have that

$$(1+\rho)x_L^{-1}(b) - x_L(x_L^{-1}(b)) < (1+\rho)b - x_L(b).$$

From the fact that $b > x_H^*$, we know that $x_H(b) < x_L(b)$ and hence

$$(1+\rho)b - x_L(b) < (1+\rho)b - x_H(b).$$

The fact that, after the initial jump, the primary surplus starts gradually declining until either it reaches a minimal level of ρx_H^* or the boom ends follows from Proposition 6 and the fact that $PS_H(b)$ is increasing in b.

To understand the long run behavior of the primary surplus when the economy enters a recession, let the level of debt when the economy enters a recession be b. By Proposition 6, we know that this debt level must be at least as big as x_H^*. To show that the primary surplus jumps down when the economy enters the boom, we need to show that

$$(1+\rho)b - x_L(b) < (1+\rho)x_H^{-1}(b) - b.$$

We have that, by definition,

$$(1+\rho)x_H^{-1}(b) - b = (1+\rho)x_H^{-1}(b) - x_H(x_H^{-1}(b)).$$

Since, in the long run, debt levels are decreasing or constant in a recession, we have that $b \leq x_H^{-1}(b)$. Thus, using the fact that PS_H is increasing, we have that

$$(1+\rho)x_H^{-1}(b) - x_H(x_H^{-1}(b)) \geq (1+\rho)b - x_H(b).$$

From the fact that $b > x_H^*$, we know that $x_H(b) < x_L(b)$ and hence that

$$(1+\rho)b - x_H(b) > (1+\rho)b - x_L(b).$$

The fact that, after the initial jump, the primary surplus starts increasing follows from Proposition 6 and the fact that $PS_L(b)$ is increasing in b. ■

A.12 Data Sources and Definitions

1. Output: seasonally adjusted real GDP, chained, base year is 2005. *Source: National Income and Product Accounts.*

   Available on-line at http://www.bea.gov/national/nipaweb/SelectTable.asp

2. The public spending/GDP ratio: total federal government expenditures less "Net Interest" as a % of GDP. *Source as above.*

3. The tax revenue/GDP ratio: total federal tax revenue as a % of GDP. *Source as above.*

4. The debt/GDP ratio: the end of the year outstanding U.S. government debt not "held by Federal Government Accounts" as a % of GDP. *Source: Historical Tables of the office of Management and Budget, the White House.*

   Available on-line at http://www.whitehouse.gov/omb/budget/Historicals

5. Length and frequency of recessions are constructed from NBER data on US Business Cycle Expansions and Contractions.

   Available on-line at http://www.nber.org/cycles.html